CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :3

Topic :-LINEAR INEQUALITIES

1 **(c)**

We have,

$27^{1/x}+12^{1/x}=2\times 8^{1/x}$

$\Rightarrow 3^{3/x}+2^{2/x}\times 3^{1/x}=2\times 2^{3/x}$

$\Rightarrow\left(\frac{3}{2}\right)^{3/x}+\left(\frac{3}{2}\right)^{1/x}=2$

$\Rightarrow y^3+y-2=0$, where $y=\left(\frac{3}{2}\right)^{1/x}$

$\Rightarrow(y-1)(y^2+y-2)=0$

$\Rightarrow y=1, y=-2\Rightarrow\left(\frac{3}{2}\right)^{1/x}=1\Rightarrow\left(\frac{3}{2}\right)^{1/x}=\left(\frac{3}{2}\right)^{0}$

But, there is no value of x for which $\frac{1}{x}$ is zero

Hence, the given equation has no solution

2 **(c)**

Let x_1,x_2,x_3 and x_4 be four positive roots of the equation $x^4-8x^3+bx^2+cx+16=0$.Then,

$x_1+x_2+x_3+x_4=8$ and $x_1x_2x_3x_4=16$

$\Rightarrow\frac{x_1+x_2+x_3+x_4}{4}=2$ and $(x_1x_2x_3x_4)^{1/4}=2$

$\Rightarrow$A.M. and G.M. of x_1,x_2,x_3 and x_4 are equal

$\Rightarrow x_1=x_2=x_3=x_4$

$\Rightarrow x_1=x_2=x_3=x_4=2$

$\therefore x^4-8x^3+bx^2+cx+16=(x-2)^4$

$\Rightarrow b={}^4C_2\times 2^2=24$ and $c=-{}^4C_3\times 2^3=-32$

3 **(a)**

We have, $3<|x|<6\Rightarrow-6<x<-3$ or $3<x<6$

$\therefore x\in(-6,-3)\cup(3,6)$

4 **(a)**

We have,

$a^4+b^4-a^3b-ab^3=a^3(a-b)-b^3(a-b)=(a^3-b^3)(a-b)$

$\Rightarrow a^4+b^4-a^3b-ab^3>0\quad\left[\begin{matrix}\because a^3-b^3\text{and}\\ a-b\text{ are of the same sign}\end{matrix}\right]$

$\Rightarrow a^4 + b^4 > a^3 b + ab^3$

5 **(a)**

The given inequation is

$4^{-x+0.5} - 7 \cdot 2^{-x} < 4, x \in R$

Let $2^{-x} = t$

$\therefore 2t^2 - 7t < 4$

$\Rightarrow 2t^2 - 7t - 4 < 0$

$\Rightarrow (2t+1)(t-4) < 0$

$\Rightarrow -\frac{1}{2} < t < 4$

$\Rightarrow 0 < t < 4 \quad (\because t = 2^{-x} > 0)$

$\Rightarrow 0 < 2^{-x} < 2^2$

As $2^x$is an increasing function $-x < 2$ or $x > -2$

$\therefore x = (-2, \infty)$

6 **(c)**

Given condition are $\frac{a}{b} > 1$ and $\frac{a}{c} < 0$

1. $a > 0$iff$c < 0$ and also $b > 0$
2. $a < 0$iff$c > 0$ and also $b < 0$

7 **(b)**

Proceeding as in the solution of Q. no. 10, we have

$(a+b)(b+c)(c+a) \geq 8abc$

$\Rightarrow (p-a)(p-b)(p-c) \geq 8abc \quad [\because a+b+c = p]$

8 **(a)**

We have,

$\frac{(1+e^{x^2})\sqrt{1+x^2}}{\sqrt{1+x^4} - x^2} = 1 + \cos x$

$\Rightarrow (1+e^{x^2})\sqrt{1+x^2}(\sqrt{1+x^4} + x^2) = 1 + \cos x$

Clearly, LHS ≥ 2 and RHS ≤ 2.So, the equation exists when each side is equal to 2. This is for $x = 0$ only. Hence, it has only one solution

10 **(c)**

Let $f(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n$. Then,

$f(b) - f(a) = 1$

$\Rightarrow c_1(b-a) + c_2(b^2 - a^2) + \ldots + c_n(b^n - a^n) = 1$

$\Rightarrow (b-a)\{c_1 + c_2(b+a) + \ldots + c_n(b^{n-1} + b^{n-2}a + \ldots + a^{n-1})\} = 1$

$\Rightarrow (b-a)I = 1$,where$I$ is an integer

$\Rightarrow b-a=\pm 1$

12 **(d)**

Using, AM > GM

$\therefore \quad \frac{a+b+c}{3}>\sqrt[3]{abc}$

$\Rightarrow a+b+c>3$...(i)

[$\therefore abc=1$given]

Also, GM > HM

$\sqrt[3]{abc}>\frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\Rightarrow \quad (1)^{1/3}>\frac{3abc}{bc+ac+ab}$

$\Rightarrow \ ab+bc+ac>3$...(ii)

$\therefore$ From Eqs. (i) and (ii), we get

$a+b+c+ab+bc+ac>6$

13 **(d)**

We have,

$\sin\left(2^x\right)\cos\left(2^x\right) = \frac{2^x+2^{-x}}{2}$

$\Rightarrow \sin(2^{x+1}) = 2^x+2^{-x}$

Clearly, RHS ≥ 2 and LHS lies between -1 and 1. So, the given equation has no solution

14 **(d)**

$x^{12}-x^9+x^4-x+1>0$

When $0<x<1; x^4>x^9$ and $1>x$

$\therefore x^{12}+\left(x^4-x^9\right)+(1-x)>0$

$\Rightarrow$ Positive for all x

Again, when $x>1$:

$x^{12}-x^9+x^4-x+1>0$

$\therefore$ Largest interval $(0,\infty)$, also the above inequality is true for $x<0$

16 **(d)**

$\because$ AM $\geq$ GM

$$\Rightarrow \frac{\frac{\cos^3 x}{\sin x}+\frac{\sin^3 x}{\cos x}}{2} \geq \left(\frac{\cos^3 x}{\sin x}\cdot\frac{\sin^3 x}{\cos x}\right)^{1/2}$$

$$\Rightarrow \frac{\cos^3 x}{\sin x}+\frac{\sin^3 x}{\cos x} \geq 2\sin x\cos x \geq 1$$

Hence, option (d) is correct

17 **(a)**

We have,

$\sqrt{3x^2+6x+7}=\sqrt{3(x+1)^2+4}\geq 2(=2 \text{ when } x=-1)$

$\sqrt{5x^2+10x+14}=\sqrt{5(x+1)^2+9}\geq 3(=3 \text{ when } x=-1)$

and,

$4-2x-x^2=5-(x+1)^2\leq 5(=5 \text{ when } x=-1)$

Thus, LHS ≥ 5 and RHS ≤ 5

So, the given equation is valid when each sides is equal to 5.

This happens only when $x=-1$

Hence, the given equation has only one solution

18 **(d)**

$||x|-1|<|1-x|$

Case I $x\geq 0$

$\therefore$ Inequality (i) becomes $|x-1|<x-1$ or $|1-x|<1-x$ which is not satisfied by any x, because

$|a|\geq \forall a\in R$

Case II $-1\leq x<0$

$\therefore$ Inequality (i) becomes$|-1-x|<1-x$ or$|x+1|<1-x$

Or $x+1<1-x$ or $x<0$

Thus, inequality (i) is satisfied for $-1\leq x<0$

Case III $x<-1$

Inequality (i) becomes $|-1-x|<1-x\Rightarrow|1+x|<1-x$

$\Rightarrow -(1+x)<1-x\Rightarrow -2<0$, which is true

So, solution set is $(-\infty,0)$

19 **(c)**

Minimum value of $f(x)$

Is attained at $x=3$

$\therefore$ Minimum value of $f(x)=7$

20 **(a)**

$\frac{x+11}{x-3}>0$

$\Rightarrow (x-3)(x+11)>0$

$\Rightarrow x<-11, x>3$

$\Rightarrow x\in(-\infty,-1)\cup(3,\infty)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	A	A	A	C	B	A	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	D	D	C	D	A	D	C	A