

Topic :-LINEAR INEQUALITIES

1 (c)

We have,

$$\begin{aligned} 27^{1/x} + 12^{1/x} &= 2 \times 8^{1/x} \\ \Rightarrow 3^{3/x} + 2^{2/x} \times 3^{1/x} &= 2 \times 2^{3/x} \\ \Rightarrow \left(\frac{3}{2}\right)^{3/x} + \left(\frac{3}{2}\right)^{1/x} &= 2 \end{aligned}$$

$$\Rightarrow y^3 + y - 2 = 0, \text{ where } y = \left(\frac{3}{2}\right)^{1/x}$$

$$\Rightarrow (y-1)(y^2+y+2) = 0$$

$$\Rightarrow y = 1, y = -2 \Rightarrow \left(\frac{3}{2}\right)^{1/x} = 1 \Rightarrow \left(\frac{3}{2}\right)^{1/x} = \left(\frac{3}{2}\right)^0$$

But, there is no value of x for which $\frac{1}{x}$ is zero

Hence, the given equation has no solution

2 (c)

Let x_1, x_2, x_3 and x_4 be four positive roots of the equation $x^4 - 8x^3 + bx^2 + cx + 16 = 0$. Then,

$$x_1 + x_2 + x_3 + x_4 = 8 \text{ and } x_1 x_2 x_3 x_4 = 16$$

$$\Rightarrow \frac{x_1 + x_2 + x_3 + x_4}{4} = 2 \text{ and } (x_1 x_2 x_3 x_4)^{1/4} = 2$$

\Rightarrow A.M. and G.M. of x_1, x_2, x_3 and x_4 are equal

$$\Rightarrow x_1 = x_2 = x_3 = x_4$$

$$\Rightarrow x_1 = x_2 = x_3 = x_4 = 2$$

$$\therefore x^4 - 8x^3 + bx^2 + cx + 16 = (x-2)^4$$

$$\Rightarrow b = {}^4C_2 \times 2^2 = 24 \text{ and } c = -{}^4C_3 \times 2^3 = -32$$

3 (a)

We have, $3 < |x| < 6 \Rightarrow -6 < x < -3$ or $3 < x < 6$

$$\therefore x \in (-6, -3) \cup (3, 6)$$

4 (a)

We have,

$$a^4 + b^4 - a^3 b - ab^3 = a^3(a-b) - b^3(a-b) = (a^3 - b^3)(a-b)$$

$$\Rightarrow a^4 + b^4 - a^3 b - ab^3 > 0 \quad \left[\begin{array}{l} \because a^3 - b^3 \text{ and} \\ a-b \text{ are of the same sign} \end{array} \right]$$

$$\Rightarrow a^4 + b^4 > a^3 b + ab^3$$

5 (a)

The given inequation is

$$4^{-x+0.5} - 7 \cdot 2^{-x} < 4, x \in R$$

Let $2^{-x} = t$

$$\therefore 2t^2 - 7t < 4$$

$$\Rightarrow 2t^2 - 7t - 4 < 0$$

$$\Rightarrow (2t + 1)(t - 4) < 0$$

$$\Rightarrow -\frac{1}{2} < t < 4$$

$$\Rightarrow 0 < t < 4 \quad (\because t = 2^{-x} > 0)$$

$$\Rightarrow 0 < 2^{-x} < 2^2$$

As 2^x is an increasing function $-x < 2$ or $x > -2$

$$\therefore x = (-2, \infty)$$

6 (c)

Given condition are $\frac{a}{b} > 1$ and $\frac{a}{c} < 0$

1. $a > 0$ iff $c < 0$ and also $b > 0$

2. $a < 0$ iff $c > 0$ and also $b < 0$

7 (b)

Proceeding as in the solution of Q. no. 10, we have

$$(a + b)(b + c)(c + a) \geq 8abc$$

$$\Rightarrow (p - a)(p - b)(p - c) \geq 8abc \quad [\because a + b + c = p]$$

8 (a)

We have,

$$\frac{(1 + e^{x^2})\sqrt{1 + x^2}}{\sqrt{1 + x^4 - x^2}} = 1 + \cos x$$

$$\Rightarrow (1 + e^{x^2})\sqrt{1 + x^2}(\sqrt{1 + x^4} + x^2) = 1 + \cos x$$

Clearly, LHS ≥ 2 and RHS ≤ 2 . So, the equation exists when each side is equal to 2. This is for $x = 0$ only. Hence, it has only one solution

10 (c)

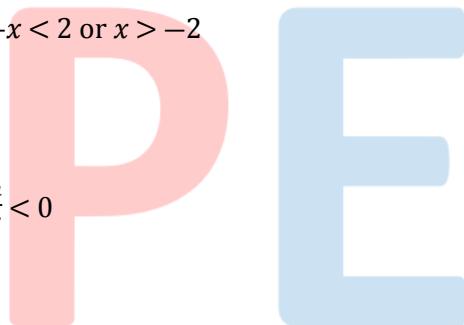
Let $f(x) = c_0 + c_1x + c_2x^2 + \dots + c_nx^n$. Then,

$$f(b) - f(a) = 1$$

$$\Rightarrow c_1(b - a) + c_2(b^2 - a^2) + \dots + c_n(b^n - a^n) = 1$$

$$\Rightarrow (b - a)\{c_1 + c_2(b + a) + \dots + c_n(b^{n-1} + b^{n-2}a + \dots + a^{n-1})\} = 1$$

$\Rightarrow (b - a)I = 1$, where I is an integer



$$\Rightarrow b - a = \pm 1$$

12 **(d)**

Using, AM > GM

$$\therefore \frac{a+b+c}{3} > \sqrt[3]{abc}$$

$$\Rightarrow a+b+c > 3 \dots (\text{i})$$

[$\because abc = 1$ given]

Also, GM > HM

$$\sqrt[3]{abc} > \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$\Rightarrow (1)^{1/3} > \frac{3abc}{bc+ac+ab}$$

$$\Rightarrow ab+bc+ac > 3 \dots (\text{ii})$$

\therefore From Eqs. (i) and (ii), we get

$$a+b+c + ab+bc+ac > 6$$

13 **(d)**

We have,

$$\sin(2^x) \cos(2^x) = \frac{2^x + 2^{-x}}{2}$$

$$\Rightarrow \sin(2^{x+1}) = 2^x + 2^{-x}$$

Clearly, RHS ≥ 2 and LHS lies between -1 and 1 . So, the given equation has no solution

14 **(d)**

$$x^{12} - x^9 + x^4 - x + 1 > 0$$

When $0 < x < 1$; $x^4 > x^9$ and $1 > x$

$$\therefore x^{12} + (x^4 - x^9) + (1 - x) > 0$$

\Rightarrow Positive for all x

Again, when $x > 1$:

$$x^{12} - x^9 + x^4 - x + 1 > 0$$

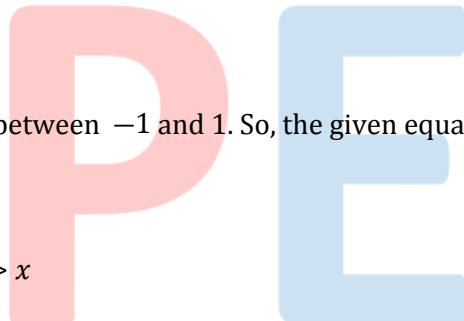
\therefore Largest interval $(0, \infty)$, also the above inequality is true for $x < 0$

16 **(d)**

\because AM \geq GM

$$\Rightarrow \frac{\cos^3 x + \sin^3 x}{2} \geq \left(\frac{\cos^3 x}{\sin x} \cdot \frac{\sin^3 x}{\cos x} \right)^{1/2}$$

$$\Rightarrow \frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq 2 \sin x \cos x \geq 1$$



Hence, option (d) is correct

17 (a)

We have,

$$\sqrt{3x^2 + 6x + 7} = \sqrt{3(x+1)^2 + 4} \geq 2 (= 2 \text{ when } x = -1)$$

$$\sqrt{5x^2 + 10x + 14} = \sqrt{5(x+1)^2 + 9} \geq 3 (= 3 \text{ when } x = -1)$$

and,

$$4 - 2x - x^2 = 5 - (x+1)^2 \leq 5 (= 5 \text{ when } x = -1)$$

Thus, LHS ≥ 5 and RHS ≤ 5

So, the given equation is valid when each sides is equal to 5.

This happens only when $x = -1$

Hence, the given equation has only one solution

18 (d)

$$||x| - 1| < |1 - x|$$

Case I $x \geq 0$

\therefore Inequality (i) becomes $|x - 1| < x - 1$ or $|1 - x| < 1 - x$ which is not satisfied by any x , because

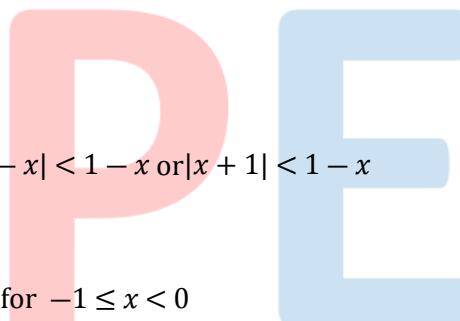
$$|a| \geq \forall a \in R$$

Case II $-1 \leq x < 0$

\therefore Inequality (i) becomes $|-1 - x| < 1 - x$ or $|x + 1| < 1 - x$

Or $x + 1 < 1 - x$ or $x < 0$

Thus, inequality (i) is satisfied for $-1 \leq x < 0$



Case III $x < -1$

Inequality (i) becomes $|-1 - x| < 1 - x \Rightarrow |1 + x| < 1 - x$

$\Rightarrow -(1 + x) < 1 - x \Rightarrow -2 < 0$, which is true

So, solution set is $(-\infty, 0)$

19 (c)

Minimum value of $f(x)$

Is attained at $x = 3$

\therefore Minimum value of $f(x) = 7$

20 (a)

$$\frac{x+11}{x-3} > 0$$

$$\Rightarrow (x-3)(x+11) > 0$$

$$\Rightarrow x < -11, x > 3$$

$$\Rightarrow x \in (-\infty, -11) \cup (3, \infty)$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	A	A	A	C	B	A	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	D	D	C	D	A	D	C	A

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