DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :2

Topic :-LINEAR INEQUALITIES

1 **(b)**

We have,

$\sqrt{x^2+\sqrt{x^2+11}}-\sqrt{x^2-\sqrt{x^2+11}}=4$

Putting $x^2+11=t^2$, we get

$\sqrt{t^2+t-11}+\sqrt{t^2-t-11}=4$ ...(i)

But, $(t^2+t-11)-(t^2-t-11)=2t$ ...(ii)

Dividing (ii) by (i), we get

$\sqrt{t^2+t-11}-\sqrt{t^2-t-11}=\frac{t}{2}$ ...(iii)

Adding (i) and (iii), we get

$2\sqrt{t^2+t-11}=4+\frac{t}{2}$

$\Rightarrow t^2+t-11=4+t+\frac{t^2}{16}$

$\Rightarrow t^2=16 \Rightarrow t=4\left[\because t=\sqrt{x^2+11}>0\right]$

$\therefore x=\pm\sqrt{5}$

2 **(a)**

We have,

$\log_{x-3}(x^3-3x^2-4x+8)=3$ ...(i)

$\Rightarrow x^3-3x^2-4x+8=(x-3)^3$

$\Rightarrow 6x^2-31x+35=0 \Rightarrow (3x-5)(2x-7)=0 \Rightarrow x=\frac{5}{3},\frac{7}{2}$

The equation (i) exists, if

$x-3>0, x-3\neq 1$ and $x^3-3x^2-4x+8>0$

Clearly, $x=\frac{7}{2}$ satisfies these conditions

3 **(a)**

Curves $y=\log_{0.5}x$ and $y=|x|$ intersect at one point in first quadrant. So, the equation $\log_{0.5}x=|x|$ has one real root

5 **(b)**

$\cos^x\alpha+\sin^x\alpha\geq 1$

Equality holds when $x=2$

If $x<2$, both $\cos\alpha$ and $\sin\alpha$ are increasing

$\therefore \cos^x \alpha + \sin^x \alpha > 1$, if $x < 2$

If $x > 2$, then $\cos^x \alpha + \sin^x \alpha < 1$

$\therefore x \in (-\infty, 2]$

7 **(a)**
We have,
$2\cos(e^x) = 3^x + 3^{-x}$
We observe that $2\cos(e^x) < 2$ and $3^x + 3^{-x} \geq 2$. So, the given equation has no solution
8 **(b)**
Graphs of $y = 1 - x$ and $y = [\cos x]$ cut each other at point $(0, 1)$ and at a point whose x-coordinate lie in $(\pi/2, \pi)$. So, the given equation has two real roots
10 **(b)**
If $a^2 + b^2 = 1$, then $a^x + b^x \geq 1$ is true for all $x \in (-\infty, 2]$
$\therefore (\sin\alpha)^x + (\cos\alpha)^x \geq 1 \Rightarrow x \in (-\infty, 2]$
11 **(a)**
If $f(x) > 0$,then $D < 0$
$4a^2 - 4(10 - 3a) < 0$
$\Rightarrow (a + 5)(a - 2) < 0$
$\Rightarrow -5 < a < 2$
12 **(b)**
The given inequality is

$$49.4 - \left(\frac{27 - x}{10}\right) < 47.4 - \left(\frac{27 - 9x}{10}\right)$$

$$\Rightarrow 49.4 - 47.4 < \left(\frac{27 - x}{10}\right) - \left(\frac{27 - 9x}{10}\right)$$

$$\Rightarrow 2 < \frac{8x}{10} \Rightarrow x > \frac{5}{2}$$

$\therefore$ Least integer is 3

13 **(a)**
Since, AM $\geq$ GM

$$\therefore \frac{a^2 + b^2}{2} \geq \sqrt{a^2 b^2} = ab, \frac{b^2 + c^2}{2} \geq bc$$

and $\frac{c^2 + a^2}{2} \geq ca$

On adding, we get

$a^2 + b^2 + c^2 \geq ab + bc + ca$

$\Rightarrow(a)$holds

Next, $\frac{b+c}{2} \geq \sqrt{bc}$, $\frac{c+a}{2} \geq \sqrt{ca}$,$\frac{a+b}{2} \geq \sqrt{ab}$

$\Rightarrow\left(\frac{b+c}{2}\right)\left(\frac{c+a}{2}\right)\left(\frac{a+b}{2}\right) \geq \sqrt{a^2b^2c^2}$

$\Rightarrow(b+c)(c+a)(a+b) \geq 8abc$

$\Rightarrow(b)$ does not hold

Again, $\frac{1}{3}\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right) \geq \left(\frac{a}{b}\cdot\frac{b}{c}\cdot\frac{c}{a}\right)^{1/3}$

$\Rightarrow \frac{a}{b}+\frac{b}{c}+\frac{c}{a} \geq 3$

$\Rightarrow(c)$does not hold

Again, $\frac{a^3+b^3+c^3}{3} \geq (a^3b^3c^3)^{1/3}$

$\Rightarrow a^3+b^3+c^3 \geq 3\,abc$

$\Rightarrow(d)$ does not hold

14 **(d)**

If s is the semi-perimeter of a cyclic quadrilateral of sides a,b,c and d units in length, then its area A is given by

$A=\sqrt{(s-a)(s-b)(s-c)(s-d)}$

Using A.M. ≥ G.M., we have

$\frac{s-a+s-b+s-c+s-d}{4} \geq \{(s-a)(s-b)(s-c)(s-d)\}^{1/4}$

$\Rightarrow \frac{4s-2s}{4} \geq \sqrt{A} \Rightarrow 2s \geq 4\sqrt{A}$

Hence, the least perimeter is $4\sqrt{A}$

15 **(c)**

Two curves

$y=[\sin x+\cos x]$

and, $y=3+[-\sin x]+[-\cos x]=1+[\sin x]+[\cos x]$

intersect at infinitely many points in $[0, 2\pi]$

So, the given equation has infinitely many solutions

16 **(b)**

Two curves $y=3^{|x|}$ and $y=|2-|x||$ intersect at two points only. So, the equation $3^{|x|}=|2-|x||$ has only two real roots

17 **(a)**

Since, angle C is obtuse, angle A and B are actute

$\therefore \tan C < 0$and $\tan A > 0, \tan B > 0$

Now, $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \ \tan B}$

$\Rightarrow \tan(\pi - C) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow - \tan C = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow 1 - \tan A \tan B > 0$ ($\because$ Numerator are positive)

$\Rightarrow \tan A \tan B < 1$

18 **(a)**

We have,

$x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$

$\Rightarrow y + z = 4 - x$ and $y^2 + z^2 = 6 - x^2$

$\therefore yz = \frac{1}{2}\{(y+z)^2 - (y^2 + z^2)\} = \frac{1}{2}\{(4 - x)^2 - (6 - x)^2\}$

$\Rightarrow yz = x^2 - 4x + 5$

Thus, y and z are roots of the equation

$t^2 - (4 - x)\ t + x^2 - 4x + 5 = 0$

As y,z are real

$\therefore (4 - x)^2 - 4(x^2 - 4x + 5) \geq 0$

$\Rightarrow 3x^2 - 8x + 4 \leq 0 \Rightarrow \frac{2}{3} \leq x \leq 2$

20 **(b)**

We have,

$3^{x/2} + 2^x > 25 \Rightarrow 3^{x/2} + 4^{x/2} > 25$

Clearly, $x \in (4,\infty)$ satisfies the above inequation

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	A	B	B	C	A	B	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	A	D	C	B	A	A	B	B