DPP

DAILY PRACTICE PRODUCTS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :1

Topic :-LINEAR INEQUALITIES

1 **(a)**

We have,

$3^x + 2^{2x} \geq 5^x$

$\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x \geq 1$

$\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x \geq \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$

$\Rightarrow x \leq 2 \Rightarrow x \in (-\infty, 2] \left[\begin{array}{c}\text{If } a^x + b^x \geq 1 \text{ and } a^2 + b^2 = 1, \\ \text{then } x \in (-\infty, 2)\end{array}\right]$

2 **(c)**

$x^2 - 3|x| + 2 < 0$

$\Rightarrow |x|^2 - 3|x| + 2 < 0$

$\Rightarrow (|x| - 1)(|x| - 2) < 0$

$\Rightarrow 1 < |x| < 2$

$\Rightarrow -2 < x < -1$ or $1 < x < 2$

$\therefore x \epsilon (-2, -1) \cup (1,2)$

3 **(d)**

We have, $2^x + 2^{|x|} \geq 2\sqrt{2}$

If $x \geq 0$,then$2^x + 2^x \geq 2\sqrt{2}$

$\Rightarrow 2^x \geq \sqrt{2} \Rightarrow x \geq \frac{1}{2}$

and if $x < 0$, then$2^x + 2^{-x} \geq 2\sqrt{2}$

$\Rightarrow t + \frac{1}{t} \geq 2\sqrt{2}$(where $t = 2^x$)

$\Rightarrow t^2 - 2\sqrt{2}t + 1 \geq 0$

$\Rightarrow \left(t - (\sqrt{2} - 1)\right)\left(t - (\sqrt{2} + 1)\right) \geq 0$

$\Rightarrow t \leq \sqrt{2} - 1$ or $t \geq \sqrt{2} + 1$

But $t > 0$

$\Rightarrow 0 < 2^x \leq \sqrt{2} - 1$

Or $2^x \geq \sqrt{2} + 1$

$\Rightarrow -\infty < x \leq \log_2(\sqrt{2} - 1)$

Or $x \geq \log_2(\sqrt{2} + 1)$

Which is not possible, because $x > 0$

$\therefore x \in (-\infty, \log_2(\sqrt{2} - 1)) \cup \left[\frac{1}{2}, \infty\right)$

4 **(b)**

Using G.M. $\leq$ A.M., we have

$\sin x_i \cos x_{i+1} \leq \frac{\sin^2 x_i + \cos^2 x_{i+1}}{2}$ for $i = 1,2,3,\ldots,n$,

where $x_{n+1} = x_1$

$\therefore \sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \ldots + \sin x_n \cos x_{n+1}$

$\leq \frac{\sin^2 x_1 + \cos^2 x_2}{2} + \frac{\sin^2 x_2 + \cos^2 x_3}{2} + \ldots + \frac{\sin^2 x_n + \cos^2 x_1}{2}$

$\Rightarrow \sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \ldots + \sin x_n \cos x_1 \leq \frac{n}{2}$

5 **(c)**

We have, $x^2 + (a + b)x + ab < 0$

$\Rightarrow (x + a)(x + b) < 0$

$\Rightarrow -b < x < -a$

6 **(a)**

Given, $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$

$$\Rightarrow \log_{10} \frac{(x^3 + y^3)}{x^2 + y^2 - xy} \leq 2$$

$$\Rightarrow \log_{10}(x + y) \leq 2 \Rightarrow x + y \leq 100$$

Using AM $\geq$ GM

$$\therefore \frac{x + y}{2} \geq \sqrt{xy}$$

$$\Rightarrow \sqrt{xy} \leq \frac{x + y}{2} \leq \frac{100}{2}$$

$$\Rightarrow xy \leq 2500$$

7 **(c)**
By trial,

$3^{x/2} + 2^x \leq 25$ for $x = 1,2,3,4$

But $3^{x/2} + 2^x > 25$ for $x > 4$

Hence, solution set for $3^{x/2} + 2^x > 25$ is $(4, \infty)$

8 **(c)**
Since,

AM $\geq$ GM

$\Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$

$\Rightarrow \frac{a+b}{2} \geq \sqrt{4}$ $(\because ab = 4, \text{given})$

$\Rightarrow a + b \geq 4$

9 **(a)**
When $x > 0$, $P_n(x) > 0$ and so $P_n(x) = 0$ can have no positive real roots
Now,
$P_n(x) = 1 + 2x + 3\,x^2 + \dots + (n+1)x^n$
$\Rightarrow xP_n(x) = x + 2x^2 + 3\,x^3 + \dots + n\,x^n + (n+1)x^{n+1}$
$\Rightarrow (1-x)P_n(x) = 1 + x + x^2 + \dots + x^n - (n+1)x^{n+1}$
$\Rightarrow P_n(x) = \frac{1 - (n+2)x^{n+1} + (n+1)x^{n+2}}{(1-x)^2}$
For negative values of x, $P_n(x)$ will vanish whenever
$f(x) = 1 - (n+2)x^{n+1} + (n+1)x^{n+2} = 0$
Now,
$f(-x) = 1 - (n+2)(-1)^{n+1}x^{n+1} + (n+1)(-1)^{n+2}x^{n+2}$...(i)
If n is even, there is no change of sign in this expression and so there is no negative real root of $f(x)$
10 **(c)**
$(x-1)(x^2 - 5x + 7) < (x-1)$

$\Rightarrow (x-1)(x^2 - 5x + 6) < 0$

$\Rightarrow (x-1)(x-2)(x-3) < 0$

$\therefore x \in (-\infty, 1) \cup (2, 3)$

11 **(b)**
Using A.M. $\geq$ G.M., we have
$\frac{x}{n} \geq \left\{\log_{2^2} 2 \times \log_{2^3} 2^2 \times \log_{2^4} 2^3 \times \dots \times \log_{2^{n+1}} 2^n\right\}^{1/n}$

$\Rightarrow x \geq n(\log_{2^{n+1}} 2)^{1/n}$

$\Rightarrow x \geq n\left(\frac{1}{n+1}\log_2 2\right)^{1/n} \Rightarrow x \geq n\left(\frac{1}{n+1}\right)^{1/n}$

12 **(c)**

We have,

$\frac{1}{2}\left|a+\frac{11}{a}\right| \geq \sqrt{11}$,equality holding iff $a = \pm\sqrt{11}$

$\therefore |x| \geq \sqrt{11}, |y| \geq \sqrt{11}, |z| \geq \sqrt{11}, |t| \geq \sqrt{11}$

Let $x \geq 0$, then $x \geq \sqrt{11}, y \geq \sqrt{11}, z \geq \sqrt{11}$ and $t \geq \sqrt{11}$

Now, $y-\sqrt{11} = \frac{1}{2}\left(\frac{11}{x}+x\right)-\sqrt{11}$

$\Rightarrow y-\sqrt{11} = \frac{(x-\sqrt{11})^2}{2x} = \left(\frac{x-\sqrt{11}}{2x}\right)(x-\sqrt{11})$

$\Rightarrow y-\sqrt{11} = \frac{1}{2}\left(1-\frac{\sqrt{11}}{x}\right)(x-\sqrt{11}) < (x-\sqrt{11})$

$\Rightarrow y < x$ i.e. $x > y$

Similarly, we have

$y > z, z > t$ and $t > x \Rightarrow y > x$

$\therefore x = y = z = t = \sqrt{11}$ is the only solution for $x > 0$

We observe that (x,y,z,t) is a solution iff $(-x,-y,-z,-t)$ is a solution

Thus, $x = y = z = t = -\sqrt{11}$ is the only other solution

13 **(b)**

Given, $3+\frac{3}{3^x}-4 < 0 \Rightarrow 3^{2x}+3-4.3^x < 0$

$\Rightarrow (3^x-1)(3^x-3) < 0$

$$1 < 3^x < 3 \Rightarrow 0 < x < 1$$

$\therefore$ The solution set is (0, 1)

14 **(b)**

We have, $|a| \leq 1$ and $a+b=1$

i.e. $-1 \leq a \leq 1$ and $b = 1-a$

$\Rightarrow -1 \leq a \leq 1$ and $0 \leq b \leq 2 \Rightarrow -2 \leq ab \leq 2$ ...(i)

Now,

$ab \leq \left(\frac{a+b}{2}\right)^2 \Rightarrow ab \leq \frac{1}{4}$...(ii)

From (i) and (ii), we have

$-2 \leq ab \leq \frac{1}{4} \Rightarrow ab \in \left[-\frac{2,1}{4}\right]$

15 **(d)**

Given, $\sqrt{(3x+1)^2} < (2-x)$

$\Rightarrow (3x+1) < 2-x$

$\Rightarrow 3x+1 < 2-x \Rightarrow x < \frac{1}{4}$

16 **(c)**

Given, inequality can be rewritten as $\left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x \geq 1$

$\therefore \cos^x \alpha + \sin^x \alpha \geq 1$

Where, $\cos \alpha = \frac{5}{13}$

If $x = 2$, the above equality holds

If $x < 2$ both $\cos \alpha$and $\sin \alpha$increases in positive fraction

Hence, above inequality holds for $x \in (-\infty, 2]$

17 **(b)**
Let $f(x) = \log_e \frac{x-2}{x-3}$
$f(x)$is defined either $(x-2) > 0, (x-3) > 0$ or $(x-2) < 0$
$(x-3) < 0$or$x \neq 2, 3$
$\Rightarrow f(x)$is defined either $x > 3$ or $x < 2$ or $x \neq 2, 3$
$ie, x \in (-\infty, 2) \cup (3, \infty)$
18 **(c)**
$3 \leq 3t - 18 \leq 18$
$\Rightarrow 21 \leq 3t \leq 36$
$\Rightarrow 7 \leq t \leq 12$
$\Rightarrow 8 \leq t + 1 \leq 13$
19 **(b)**
$\because f(-1) < 1 \Rightarrow a - b + c < 1$...(i)

and$f(1) > -1, f(3) < -4$, then

$a + b + c > -1$...(ii)

$9a + 3b + c < -4$...(iii)

From Eq. (ii),

$-a - b - c < 1$ ....(iv)

On solving Eqs. (i), (iii) and (iv), we get $a < -\frac{1}{8} \Rightarrow a$ is negative

20 **(d)**
Given, $\frac{2x+3}{2x-9} < 0$
$\Rightarrow 2x + 3 < 0$and$2x - 9 > 0$
Or $2x + 3 > 0$ and $2x - 9 < 0$ and $x \neq \frac{9}{2}$
$\Rightarrow x < -\frac{3}{2}andx > \frac{9}{2}$ or $x > -\frac{3}{2}$ and $x < \frac{9}{2}$ and $x \neq \frac{9}{2}$

$$\Rightarrow x \in \left(-\frac{3}{2}, \frac{9}{2}\right)$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	D	B	C	A	C	C	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	B	D	C	B	C	B	D