

Topic :-LIMITS AND DERIVATIVES

1. If $a = \min\{x^2 + 4x + 5 : x \in R\}$ and $b = \lim_{\theta \rightarrow 0} \frac{1 - \cos 2\theta}{\theta^2}$, then the value of $\sum_{r=0}^n {}^n C_r a^r b^{n-r}$, is
 - a) $2n$
 - b) 3^n
 - c) 2^{n+1}
 - d) 2^{n-1}

2. If $0 < x < y$, then $\lim_{n \rightarrow \infty} (y^n + x^n)^{1/n}$ is equal to
 - a) e
 - b) x
 - c) y
 - d) None of these

3. $\lim_{x \rightarrow 1} (\log ex)^{1/\log x}$ is equal to
 - a) e^{-1}
 - b) e
 - c) e^2
 - d) 0

4. If $\lim_{x \rightarrow \infty} \left\{ ax - \frac{x^2 + 1}{x + 1} \right\} = b$, a finite number, then
 - a) $a = 1, b = 1$
 - b) $a = 0, b = 1$
 - c) $a = -1, b = 1$
 - d) $b = -1, a = -1$

5. The value of $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t \log(1+t)}{t^4 + 4} dt$ is
 - a) 0
 - b) $\frac{1}{12}$
 - c) $\frac{1}{24}$
 - d) $\frac{1}{64}$

6. For the function $f(x) = \lim_{n \rightarrow \infty} \frac{\log(2+x) + x^{2n} \sin x}{1 + x^{2n}}$ Which of the following is true?
 - a) $\lim_{x \rightarrow 1^-} f(x)$ does not exist
 - b) $\lim_{x \rightarrow 1^+} f(x)$ does not exist
 - c) Both limits exist and $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$
 - d) Both limits exist and $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$

7. $\lim_{x \rightarrow \infty} \frac{\int_0^{2x} x e^{x^2} dx}{e^{4x^2}}$ equals
 - a) 0
 - b) ∞
 - c) 2
 - d) $1/2$

8. The value of $\lim_{x \rightarrow \pi/2} \tan^2 x (\sqrt{2 \sin^2 x + 3 \sin x + 4} - \sqrt{\sin^2 x + 6 \sin x + 2})$ is equal to

a) $\frac{1}{10}$ b) $\frac{1}{11}$ c) $\frac{1}{12}$ d) $\frac{1}{8}$

9. $\lim_{x \rightarrow \infty} \left(\frac{x+2}{x+1}\right)^{x+3}$ is

a) 1 b) e c) e^2 d) e^3

10. If $f(x) = \cot^{-1}[(3x - x^3)/(1 - 3x^2)]$ and $g(x) = \cos^{-1}[(1 - x^2)/(1 + x^2)]$, then

$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{g(x) - g(a)}$ ($0 < a < \frac{1}{2}$) is

a) $-\frac{3}{2}$ b) $\frac{1}{2}$ c) $\frac{3}{2}$ d) None of these

11. Let $f(x) = \begin{cases} x^2, & x \in Z \\ \frac{k(x^2 - 4)}{2 - x}, & x \notin Z \end{cases}$ Then, $\lim_{x \rightarrow 2} f(x)$

- a) Exists only when $k = 1$
- b) Exists for every real k
- c) Exists for every real k except $k = 1$
- d) Does not exist

12. The value of $\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$ is

a) $2/5$ b) $3/5$ c) $3/2$ d) $3/4$

13. $\lim_{x \rightarrow \infty} \sqrt{\frac{x + \sin x}{x - \cos x}} =$

a) 0 b) 1 c) -1 d) None of these

14. It is given that $f'(a)$ exists, then $\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a}$ is equal to

a) $f(a) - a f'(a)$ b) $f'(a)$ c) $-f'(a)$ d) $f(a) + a f'(a)$

15. $\lim_{h \rightarrow 0} \frac{\sin(a + 3h) - 3 \sin(a + 2h) + 3 \sin(a + h) - \sin a}{h^3}$ is equal to

a) $\sin a$ b) $-\sin a$ c) $\cos a$ d) $-\cos a$

16. $\lim_{x \rightarrow 0} \left\{ \frac{1^x + 2^x + 3^x + \dots + n^x}{n} \right\}^{1/x}$ is equal to

a) $(n!)^n$ b) $(n!)^{1/n}$ c) $n!$ d) $\ln n!$

17. The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \cos t^2 dt}{x \sin x}$ is

a) $3/2$ b) 1 c) -1 d) None of these

18. If $f(x) = \frac{2}{x-3}$, $g(x) = \frac{x-3}{x+4}$ and $h(x) = -\frac{2(2x+1)}{x^2+x-12}$, then $\lim_{x \rightarrow 3} \{f(x) + g(x) + h(x)\}$, is

a) -2 b) -1 c) $-2/7$ d) 0

19. $\lim_{x \rightarrow 0} \left[\frac{8 \sin x + x \cos x}{3 \tan x + x^2} \right]$ is equal to

a) 3

b) 2

c) -1

d) 4

20. If $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is

a) $-2/3$

b) 0

c) $-1/3$

d) $2/3$

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