Topic :-LIMITS AND DERIVATIVES

1. The value of $\lim\limits_{x \to 0} \dfrac{1 + \sin x - \cos x + \log(1 - x)}{x^3}$, is

   a) $1/2$ b) $-1/2$ c) 0 d) 1

2. $\lim\limits_{x \to 0} \left\{ \dfrac{1 + \tan x}{1 + \sin x} \right\}^{\cosec x}$ is equal to

   a) $\dfrac{1}{e}$ b) 1 c) e d) e^2

3. $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1}$ is equal to

   a) 3 b) -3 c) 6 d) 0

4. The value of $\lim\limits_{x \to 0} \left(\dfrac{1 + 5x^2}{1 + 3x^2} \right)^{1/x^2}$ is

   a) e^2 b) e c) $\dfrac{1}{e}$ d) $\dfrac{1}{e^2}$

5. If $f(x) = \begin{cases} x, & x < 0 \\ 1, & x = 0 \\ x^2, & x > 0 \end{cases}$, then $\lim\limits_{x \to 0} f(x)$ is

   a) 0 b) 1 c) 2 d) Does not exist

6. If x is a real number in $[0, 1]$, then the value of $\lim\limits_{m \to \infty} \lim\limits_{n \to \infty} [1 + \cos^{2m}(n! \, \pi \, x)]$ is given by

   a) 2 or 1 according as x is rational or irrational
   b) 1 or 2 according as x is rational or irrational
   c) 1 for all x
   d) 2 or 1 for all x

7. $\lim\limits_{x \to 1} (1 + \cos \pi x) \cot^2 \pi$ is equal to

   a) 1 b) -1 c) $1/2$ d) $-1/2$

8. If $\lim\limits_{x \to 0} \dfrac{(e^{kx} - 1) \sin kx}{x^2} = 4$, then k is equal to

   a) 2 b) -2 c) ± 2 d) ± 4

9. $\lim\limits_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x}$ is equal to

   a) 0   b) 1   c) 3   d) None of these

10. If $l_1 = \lim\limits_{x\to 2^+}(x+[x])$, $\quad l_2 = \lim\limits_{x\to 2^-}(2x-[x])$ and $l_3 = \lim\limits_{x\to \pi/2}\frac{\cos x}{(x-\pi/2)}$, then

   a) $l_1 < l_2 < l_3$ b) $l_2 < l_3 < l_1$ c) $l_3 < l_2 < l_1$ d) $l_1 < l_3 < l_2$

11.   If $f(x) = \begin{cases} \frac{\sin(1+[x])}{[x]}, & for\ [x] \neq 0 \\ 0, & for\ [x] = 0 \end{cases}$

Where $[x]$ denotes the greatest integer not exceeding x, then $\lim\limits_{x\to 0^-} f(x)$ is equal to

   a) -1   b) 0   c) 1   d) 2

12. $\lim\limits_{x\to 0}\frac{(1-\cos 2x)\sin 5x}{x^2 \sin 3x}$ equals

   a) 10/3   b) 3/10   c) 6/5   d) 5/6

13. If $f(x) = \frac{\sin(e^{x-2}-1)}{\log(x-1)}$, then $\lim\limits_{x\to 2} f(x)$ is given by

   a) -2 b) -1 c) 0   d) 1

14. If $S_n = \sum_{k=1}^{n} a_k$ and $\lim\limits_{n\to\infty} a_n = a$, then $\lim\limits_{n\to\infty}\frac{S_{n+1}-S_n}{\sqrt{\sum_{k=1}^{n}k}}$ is equal to

   a) 0   b) a c) $\sqrt{2}\,a$ d) $2a$

15. $\lim\limits_{h\to 0}\frac{\sin\sqrt{x+h}-\sin\sqrt{x}}{h}$ is equal to

   a) $\cos\sqrt{x}$ b) $1/(2\sin\sqrt{x})$ c) $(\cos\sqrt{x})/2\sqrt{x}$ d) $\sin\sqrt{x}$

16. The value of $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$ is

   a) 1   b) 0   c) e d) None of these

17.   The value of

$\lim\limits_{n\to\infty}\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\dots\cos\left(\frac{x}{2^n}\right)$ is

   a) $\frac{x}{\sin x}$ b) $\frac{x}{\cos x}$ c) $\frac{(\sin x)}{x}$ d) $\frac{(\cos x)}{x}$

18. The value of the limit $\lim\limits_{x\to 1}\frac{\sin(e^{x-1}-1)}{\log x}$ is

   a) 0   b) e c) $\frac{1}{e}$ d) 1

19. If $l = \lim\limits_{x\to -2}\frac{\tan \pi x}{x+2} + \lim\limits_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x$, then which one of the following is not correct?

   a) $l > 3$

   b) $l > 4$

   c) $l < 4$

   d) l is a transcendental number

20. $\lim\limits_{x \to 1} \dfrac{\sqrt{1 - \cos 2(x-1)}}{x-1}$

    a) Exists and is equals $\sqrt{2}$

    b) Exists and is equals $-\sqrt{2}$

    c) Does not exist because $x - 1 \to 0$

    d) Does not exist because left hand limit is not equal to right hand limit