DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :9

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2+6}{x^2-6}\right)^x = \lim_{x\to\infty}\left(1+\frac{12}{x^2-6}\right)^x = e^{\lim_{x\to\infty}\frac{12x}{x^2-6}} = e^0 = 1$$

2 **(a)**

$$\lim_{x\to\infty}\left[1+\frac{4x+1}{x^2+x+2}\right]^x = \lim_{x\to\infty} e^{\lim_{x\to\infty}\frac{4x^2+x}{x^2+x+2}} = e^4$$

3 **(a)**

We have,

$\lim_{x\to 1}\frac{G(x)-G(1)}{x-1} = G'(1)$ [By def. of derivative]

Now,

$$G(x) = -\sqrt{25-x^2} \Rightarrow G'(x) = \frac{x}{\sqrt{25-x^2}} \Rightarrow G'(1) = \frac{1}{\sqrt{24}}$$

4 **(a)**

$$\lim_{x\to 1}\cos^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$$

$$= \lim_{x\to 1}\cos^{-1}\left(\frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})}\right)$$

$$= \lim_{x\to 1}\cos^{-1}\left(\frac{1}{1+\sqrt{x}}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3}$$

5 **(a)**

We have,

$$\lim_{x\to 1}\frac{\sin(e^{x-1}-1)}{\log x} = \lim_{h\to 0}\frac{\sin(e^h-1)}{\log(1+h)}$$

$$= \lim_{h\to 0}\frac{\sin(e^h-1)}{(e^h-1)} \times \frac{(e^{-h}-1)}{h} \times \frac{h}{\log(1+h)} = 1$$

6 **(b)**

$$\lim_{x\to\frac{\pi}{6}}\frac{3\sin x-\sqrt{3}\cos x}{6x-\pi} = \lim_{x\to\frac{\pi}{6}}\frac{3\cos x+\sqrt{3}\cos x}{6}$$

$$=\frac{3\cos\frac{\pi}{6}+\sqrt{3}\sin\frac{\pi}{6}}{6}$$

$$=\frac{3.\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{6}$$

$$=\frac{1}{\sqrt{3}}$$

7 **(c)**

We have,

$$\lim_{x\to 2}\int_2^{f(x)}\frac{4\,t^3}{x-2}dt = \lim_{x\to 2}\frac{[t^4]_2^{f(x)}}{x-2} = \lim_{x\to 2}\frac{\{f(x)\}^4-16}{x-2}$$

$$=\lim_{x\to 2}\frac{4\{f(x)\}^3 f'(x)}{1}\quad \text{[Applying L' Hospital's Rule]}$$

$$=4(f(2))^3f'(2)=32f'(2)$$

8 **(a)**

$$\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec^2 x}f(t)dt}{x^2-\frac{\pi^2}{16}}\qquad\left[\frac{0}{0}\text{ from}\right]$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{f(\sec^2 x)2\sec x\sec x\tan x}{2x}$$

$$\therefore\quad L=\frac{2f(2)}{\pi/4}=\frac{8f(2)}{\pi}$$

9 **(c)**

We have,

$$\lim_{x\to\infty}f(x) = \lim_{x\to\infty}\sqrt{\frac{x-\sin x}{x+\cos^2 x}} = \lim_{x\to\infty}\frac{\sqrt{1-\frac{\sin x}{x}}}{1+\frac{\cos^2 x}{x}}$$

$$\Rightarrow \lim_{x\to\infty}f(x) = \sqrt{\frac{1-0}{1+0}}=1\left[\because \frac{\sin x}{x}\to 0,\frac{\cos^2 x}{x}\to 0\text{ as }x\to\infty\right]$$

10 **(a)**

We have,

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}\qquad\left[\frac{0}{0}\text{ form}\right]$$

$= \lim_{x\to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2}$ [Using L' Hospital's Rule]

$= \lim_{x\to 0} \frac{2f''(x)12f''(2x) + 16f''(4x)}{2}$ [Using L' Hospital's Rule]

$= \frac{2\,f''(0) - 12f''\,(0) + 16f''\,(0)}{2}$

$= 3f''(0) = 3 \times 2 = 6$

11 **(b)**

$\lim_{x\to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

$= \lim_{h\to 0} \frac{\cot\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2} + h\right)}{(-2h)^3}$

$= \lim_{h\to 0} \frac{-\tan h + \sin h}{(-2h)^3}$

$= \lim_{h\to 0} \frac{\sin h(1 - \cos h)}{\cos h \times 8h^3}$

$= \frac{1}{8} \lim_{h\to 0} \frac{\sin h}{h} \times \frac{2\sin^2 h/2}{4(h/2)^2} \times \frac{1}{\cos h} = \frac{1}{16}$

12 **(c)**

We have,

$\lim_{x\to\infty} \left(\frac{x+3}{x-1}\right)^{x+3} = \lim_{x\to\infty} \left(1 + \frac{4}{x-1}\right)^{x+3} = e^{\lim_{x\to\infty} \frac{4(x+3)}{(x+1)}} = e^4$

13 **(d)**

$\lim_{x\to 7} \frac{2 - \sqrt{x-3}}{x^2 - 49} = \lim_{x\to 7} \frac{-\frac{1}{2\sqrt{x-3}}}{2x}$

$= -\frac{1}{4.7.2} = -\frac{1}{56}$

14 **(b)**

$\lim_{x\to\infty} \frac{\sqrt{x^2+1} - \sqrt[3]{x^3+1}}{\sqrt[4]{x^4+1} - \sqrt[5]{x^4+1}}$

$= \lim_{x\to\infty} \frac{\sqrt{1 + \frac{1}{x^2}} - \sqrt[3]{1 + \frac{1}{x^3}}}{\sqrt[4]{1 + \frac{1}{x^4}} - \sqrt[5]{\frac{1}{x} + \frac{1}{x^5}}} = \frac{1-1}{1-0} = 0$

15 **(a)**

$\lim_{x\to 2} \frac{\int_6^{f(x)} 4\,t^3\,dt}{x - 2} \lim_{x\to 2} \frac{4\{f(x)\}^3}{1} . f'(x)$

$= 4\{f(2)\}^3 . f'(2)$

$= 4 \times (6)^3 . \dfrac{1}{48}$

$= 18$

16 **(b)**

Since, $g(x)g(y) = g(x) + g(y) + g(xy) - 2$ ...(i)

Now, at $x = 0$, $y = 2$, we get

$g(0)g(2) = g(0) + g(2) + g(0) - 2$

$\Rightarrow \quad 5g(0) = 5 + 2g(0) - 2 \quad [\because \; g(2) = 5]$

$\Rightarrow \quad g(0) = 1$

$g(x)$ is given in a polynomial and by the relation given $g(x)$ cannot be linear.

Let $g(x) = x^2 + k$

$\Rightarrow \quad g(x) = x^2 + 1 \quad [\because \; g(0) = 1]$

$\therefore \quad g(x)$ is satisfied in Eq. (i)

$\therefore \quad \lim_{x\to 3} g(x) = g(3) = 3^2 + 1 = 10$

17 **(b)**

We have,

$$\lim_{x\to\pi/4} \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$$

$= \lim_{y\to 1} \frac{1 - y^3}{2 - y - y^3}$, where $y = \cot x$

$$= \lim_{y\to 1} \frac{y^3 - 1}{y^3 + y - 2}$$

$$= \lim_{y\to 1} \frac{(y-1)(y^2 + y + 1)}{(y-1)(y^2 + y + 2)} = \lim_{y\to 1} \frac{y^2 + y + 1}{y^2 + y + 2} = \frac{3}{4}$$

18 **(c)**

We have,

$\lim_{x\to 1^-} \{1 - x + [x - 1] + [1 - x]\}$

$= \lim_{h\to 0} \{1 - (1 - h) + [1 - h - 1] + [1 - (1 - h)]\}$

$= \lim_{h\to 0} \{h + [-h] + [h]\} = \lim_{h\to 0} (h - 1 + 0) = -1$

and,

$\lim_{x\to 1^+} \{1 - x + [x - 1] + [1 - x]\}$

$= \lim_{h\to 0} \{1 - (1 + h) + [1 + h - 1] + [1 - (1 + h)]\}$

$= \lim_{h\to 0} \{-h + [h] + [-h]\} = \lim_{h\to 0} (-h + 0 - 1) = -1$

Hence, $\lim_{x\to 1} f(x) = -1$

19 **(a)**

Given, $\lim_{x\to 1} \frac{ax^2 + bx + c}{(x-1)^2} = 2$

This limit will exist, if

$ax^2 + bx + c = 2(x - 1)^2$

$\Rightarrow \quad ax^2 + bx + c = 2x^2 - 4x + 2$

$\Rightarrow \quad a = 2, \quad b = -4, \quad c = 2$

20 **(a)**

$$\lim_{x \to 1} \frac{2x^2 + x - 3}{3x^3 - 3x^2 + 2x - 2} = \lim_{x \to 1} \frac{(2x + 3)(x - 1)}{(3x^2 + 2)(x - 1)}$$

$$= \lim_{x \to 1} \frac{2x + 3}{3x^2 + 2} = 1$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.										
Q.	11	12	13	14	15	16	17	18	19	20
A.										