DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :8

Topic :-LIMITS & DERIVATIVES

1 **(b)**
We have,
$x^2 + 4x + 5 = (x + 2)^2 + 1 \geq 1$ for all x
$\therefore a = 1$

$$b = \lim_{\theta \to 0} \frac{1 - \cos^2 \theta}{\theta^2} = 1$$

$$\therefore \sum_{r=0}^{n} {}^nC_r \, a^r \, b^{n-r} = (a + b)^n = 2^n$$

2 **(c)**
We have,

$$\lim_{n \to \infty} \left\{1 + \left(\frac{x}{y}\right)^n\right\}^{1/n} = \lim_{n \to \infty} y(1 + 0)^{1/n} = y \times 1^0 = y$$

3 **(b)**

$$\lim_{x \to 1} (\log ex)^{1/\log x} = \lim_{x \to 1} [\log e + \log x]^{1/\log x}$$

$$= \lim_{x \to 1} [1 + \log x]^{1/\log x}$$

$$= e^{\lim_{x \to 1} \frac{\log x}{\log x}} = e$$

4 **(a)**
We have,

$$\lim_{x \to \infty} \left\{ax - \frac{x^2 + 1}{x + 1}\right\} = b \Rightarrow \lim_{x \to \infty} \frac{(a - 1)x^2 + ax - 1}{x + 1} = b$$

Since b is a finite number. Therefore, degree of numerator must be less than or equal to that of the denominator
$\therefore a - 1 = 0 \Rightarrow a = 1$
Now,

$$\lim_{x \to \infty} \frac{(a - 1)x^2 + ax - 1}{x + 1} = b \Rightarrow \lim_{x \to \infty} \frac{ax - 1}{x + 1} = b \Rightarrow a = b$$

Hence, $a = b = 1$

5 **(b)**
Given limit

$$= \lim_{x \to 0} \frac{\int_0^x \frac{t \log(1 + t)}{t^4 + 4} dt}{x^3}$$

Using L' Hospital's rule,

$$= \lim_{x \to 0} \frac{\frac{x \log(1 + x)}{x^4 + 4}}{3x^2}$$

$$= \lim_{x \to 0} \frac{\log(1 + x)}{3x} \cdot \frac{1}{x^4 + 4}$$

$$= \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

6 **(d)**

We know, $\lim_{n\to\infty} x^{2n} = \begin{cases} \infty, & \text{if } x^2 > 1 \\ 1, & \text{if } x^2 = 1 \\ 0, & \text{if } x^2 < 1 \end{cases}$

Given, $f(x) = \lim_{n\to\infty} \frac{\log(2+x) - x^{2n}\sin x}{1 + x^{2n}}$

For $x^2 = 1$, $f(x) = \lim_{n\to\infty} \frac{\log 3 - \sin 1}{2}$

$= \frac{1}{2}(\log 3 - \sin 1)$

For $x^2 < 1$,

$f(x) = \log(2+x)$

Foe $x^2 > 1$,

$$f(x) = \lim_{n\to\infty} \frac{\left(\frac{1}{x^{2n}}\right)\log(2+x) - \sin x}{\left(1 + \frac{1}{x^{2n}}\right)}$$

$= -\sin x$

$$\therefore \quad f(x) = \begin{cases} \log(2+x), & x^2 < 1 \\ \frac{1}{2}(\log 3 - \sin 1), & x = 1 \\ -\sin x, & x^2 > 1 \end{cases}$$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{h\to 0} \log(2 + 1 - h)$

$= \log 3$

$\lim_{x\to 1^+} f(x) = \lim_{h\to 0}[-\sin(1+h)]$

$= -\sin 1$

It is clear that both limits exist and $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

7 **(d)**

We have,

$$\lim_{x\to\infty} \frac{\int_0^{2x} x\, e^{x^2}\, dx}{e^{4x^2}} = \lim_{x\to\infty} \frac{\int_0^{2x} e^{x^2} d\,(x^2)}{2e^{4x^2}} = \lim_{x\to\infty} \frac{\left[e^{x^2}\right]^{2x}}{2e^{4x^2}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{\int_0^{2x} x\, e^{x^2}\, dx}{e^{4x^2}} = \lim_{x\to\infty} \frac{e^{4x^2} - 1}{2\, e^{4x^2}} = \lim_{x\to\infty}\left(\frac{1}{2} - \frac{1}{e^{4x^2}}\right) = \frac{1}{2}$$

8. **(c)**

We have,

$$\lim_{x\to\pi/2} \tan^2 x \left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2}\right)$$

$$= \lim_{x\to\pi/2} \tan^2 x \frac{(2\sin^2 x + 3\sin x + 4 - \sin^2 x - 6\sin x - 2)}{\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}}$$

$$= \lim_{x\to\pi/2} \frac{\tan^2 x\,(\sin^2 x - 3\sin x + 2)}{\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}}$$

$$= \lim_{x\to\pi/2} \frac{\sin^2 x\,(\sin x - 1)(\sin x - 2)}{(1 - \sin^2 x)(\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2})}$$

$$= \lim_{x\to\pi/2} \frac{-\sin^2 x\,(\sin x - 2)}{(1+\sin x)\left(\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}\right)}$$

$$= \frac{1}{2(\sqrt{9}+\sqrt{9})} = \frac{1}{12}$$

9 **(b)**

$$\lim_{x\to\infty}\left(1+\frac{1}{x+1}\right)^{x+3}$$

$$= e^{\lim_{x\to\infty}\frac{x+3}{x+1}} = e$$

10 **(a)**

$$f(x) = \cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \frac{\pi}{2} - 3\tan^{-1}x$$

and $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$

$$\therefore \quad \lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)}$$

$$= \lim_{x\to a}\frac{\frac{\pi}{2} - 3\tan^{-1}x - \frac{\pi}{2} + 3\tan^{-1}a}{2\tan^{-1}x - 2\tan^{-1}a}$$

$$= -\frac{3}{2}\lim_{x\to a}\frac{\tan^{-1}x - \tan^{-1}a}{\tan^{-1}x - \tan^{-1}a} = -\frac{3}{2}$$

11 **(b)**

We have,

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{h\to 0}\frac{k((2-h)^2-4)}{2-(2-h)} = k\lim_{h\to 0}\frac{h(h-4)}{h} = -4k$$

$$\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0}\frac{k((2+h)^2-4)}{2-(2+h)}$$

$$\Rightarrow \lim_{x\to 2^+} f(x) = k\lim_{h\to 0}\frac{h(h+4)}{-h} = -4k$$

$$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \text{ for all } k \in R$$

12 **(c)**

We have,

$$\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x\cos x}$$

$$= \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x\sin x\cos x}$$

$$= \lim_{x\to 0}\frac{2\sin^2_{x/2}}{x.2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} \times \frac{(1-\cos x+\cos^2 x)}{\cos x}$$

$$= \lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{2(x/2)} \times \frac{1+\cos x+\cos^2 x}{\cos\left(\frac{x}{2}\right)\cos x} = \frac{1}{2}\times 3 = \frac{3}{2}$$

13 **(b)**

We have,

$$\lim_{x\to\infty}\sqrt{\frac{x+\sin x}{x-\cos x}}=\lim_{x\to\infty}\sqrt{\frac{1+\frac{\sin x}{x}}{1-\frac{\cos x}{x}}}=\sqrt{\frac{1+0}{1-0}}=1$$

14 **(a)**

Since, $f'(a)$ exists.

$$\therefore\ f'(a)=\lim_{h\to 0}\frac{f(x)-f(a)}{x-a}$$

Now, $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$

$$=\lim_{x\to a}\frac{xf(a)-af(a)+af(a)-af(x)}{x-a}$$

$$=\lim_{x\to a}\frac{f(a)-(x-a)}{(x-a)}-a\lim_{x\to a}\frac{f(x)-f(a)}{(x-a)}$$

$$=f(a)-a\,f'(a)$$

15 **(d)**

We have,

$$\lim_{h\to 0}\frac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3}$$

$$=\lim_{h\to 0}\frac{\{\sin(a+3h)-\sin a\}-3\{\sin(a+2h)-\sin(a+h)\}}{h^3}$$

$$=\lim_{h\to 0}\frac{2\sin\frac{3h}{2}\cos\left(a+\frac{3h}{2}\right)-6\cos\left(a+\frac{3h}{2}\right)\sin\frac{h}{2}}{h^3}$$

$$=\lim_{h\to 0}\frac{2\cos\left(a+\frac{3h}{2}\right)\left(\sin\frac{3h}{2}-3\sin\frac{h}{2}\right)}{h^3}$$

$$=-8\lim_{h\to 0}\cos\left(a+\frac{3h}{2}\right)\frac{\sin^3\frac{h}{2}}{h^3}$$

$$=-\lim_{h\to 0}\cos\left(a+\frac{3h}{2}\right)\left\{\frac{\sin\frac{h}{2}}{h/2}\right\}^3=-\cos a$$

16 **(b)**

We have,

$$\lim_{x\to 0}\left\{\frac{1^x+2^x+\ldots+n^x}{n}\right\}^{1/x}$$

$$=\lim_{x\to 0}\left\{1+\frac{1^x-1}{n}+\frac{2^x-1}{n}+\ldots+\frac{n^x-1}{n}\right\}^{1/x}$$

$$= e^{\lim_{x\to 0}\frac{1}{n}\left\{\frac{1^x-1}{n}+\frac{2^x-1}{n}+\ldots+\frac{n^x-1}{n}\right\}}$$

$$= e^{\frac{1}{n}[\log 1+\log 2+\ldots+\log n]}$$

$$= e^{\frac{1}{n}(\log n!)} = e^{\log (n!)^{\frac{1}{n}}} = (n!)^{\frac{1}{n}}$$

17 **(b)**

We have,

$$\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2 dt}{x\sin x}$$

$$= \lim_{x\to\infty}\frac{2x\cos x^4}{x\cos x + \sin x} \qquad \text{[Using L' Hospital's Rule]}$$

$$= \lim_{x\to\infty}\frac{2\cos x^4 - 8x^4\sin x^4}{2\cos x - x\sin x} = \frac{2-0}{2-0} = 1$$

18 **(c)**

We have,

$$f(x)+g(x)+h(x) = \frac{x^2-4x+17-4x-2}{x^2+x-12} = \frac{(x-3)(x-5)}{(x-3)(x+4)}$$

$$\therefore \lim_{x\to 3}[f(x)+g(x)+h(x)] = \lim_{x\to 3}\frac{(x-3)(x-5)}{(x-3)(x+4)} = -\frac{2}{7}$$

19 **(a)**

$$\lim_{x\to 0}\left[\frac{8\sin x + x\cos x}{3\tan x + x^2}\right] = \lim_{x\to 0}\left[\frac{\frac{8\sin x}{x}+\cos x}{\frac{3\tan x}{x}+x}\right]$$

$$= \frac{\lim_{x\to 0}\left[\frac{8\sin x}{x}+\cos x\right]}{\lim_{x\to 0}\left[\frac{3\tan x}{x}+x\right]}$$

$$= \frac{9}{3} = 3$$

20 **(d)**

We have,

$$\lim_{x\to 0}\frac{\log_e(3+x)-\log_e(3-x)}{x} = k$$

$$\Rightarrow \lim_{x\to 0}\frac{\log_e\left(1+\frac{x}{3}\right)-\log_e\left(1-\frac{x}{3}\right)}{x} = k$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{3}\times\frac{\log_e\left(1+\frac{x}{3}\right)}{\frac{x}{3}} + \frac{1}{3} + \lim_{x\to 0}\frac{\log\left(1-\frac{x}{3}\right)}{\left(-\frac{x}{3}\right)} = k$$

$$\Rightarrow \frac{1}{3}+\frac{1}{3} = k \Rightarrow k = \frac{2}{3}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	B	A	B	D	D	C	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	A	D	B	B	C	A	D