DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :7

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$$\lim_{x\to 0}\frac{e^x-1-x}{x^2}=\lim_{x\to 0}\frac{e^x-1}{2x}=\frac{1}{2}\quad \text{[Using L' Hospital's Rule]}$$

2 **(c)**

$$\lim_{x\to\infty}\frac{(2x+1)^{40}(4x-1)^5}{(2x+3)^{45}}=\lim_{x\to\infty}\frac{x^{40}\left(2+\frac{1}{x}\right)^{40}x^5\left(4-\frac{1}{x}\right)^5}{x^{45}\left(2+\frac{3}{x}\right)^{45}}$$

$$=\frac{(2+0)^{40}(4-0)^5}{(2+0)^{45}}=\frac{2^{50}}{2^{45}}=32$$

3 **(c)**

$$\therefore \lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}([x-3]+|x-4|)$$

$$=\lim_{h\to 0}([3-h-3]+|3-h-4|)$$

$$=\lim_{h\to 0}([-h]+1+h)$$

$$=-1+1+0=0$$

4 **(d)**

We have,

$$\lim_{x\to 2^-}\{x+(x-[x]^2\}=\lim_{x\to 2^-}x+\lim_{x+2^-}(x-[x])^2$$

5 **(d)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2+2x+3}{2x^2+x+5}\right)^{\frac{3x-2}{3x+2}}=\lim_{x\to\infty}\left(\frac{1+\frac{2}{x}+\frac{3}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}}\right)^{\frac{1-2/3x}{1+2/3x}}=\left(\frac{1}{2}\right)^1=\frac{1}{2}$$

6 **(c)**

$$\lim_{x\to 0}\left[\frac{e^{\sin x}-e^x}{\sin x-x}\right]=\lim_{x\to 0}\left[\frac{e^x(e^{\sin x-x}-1)}{\sin x-x}\right]$$

$$=\lim_{x\to 0}e^x\lim_{x\to 0}\left[\frac{e^{\sin x-x}-1}{\sin x-x}\right]$$

$= e^0 \times 1 = 1$

7 **(b)**

We have,

$\lim_{x\to 1} (\log_2 2x)^{\log_x 5}$

$= \lim_{x\to 1} (\log_2 2 + \log_2 x)^{\log_x 5}$

$= \lim_{x\to 1} (1 + \log_2 x)^{1/\log_5 x} = e^{\lim_{x\to 1} \log_2 x \cdot \frac{1}{\log_5 x}}$

$= e^{\lim_{x\to 1} \log_2 5} = e^{\log_2 5}$

8 **(a)**

$f(x)$ is a positive increasing function

$$\Rightarrow \quad 0 < f(x) < f(2x) < f(3x)$$

$$\Rightarrow \quad 0 < 1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$$

$$\Rightarrow \quad \lim_{x\to\infty} 1 < \lim_{x\to\infty} \frac{f(2x)}{f(x)} < \lim_{x\to\infty} \frac{f(3x)}{f(x)}$$

By Sandwich theorem

$$\lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1$$

9 **(c)**

$\text{LHL} = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2 - 3$

$= 9 - 3 = 6$

And $\text{RHL} = \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x + 5$

$= 2 \times 3 + 5 = 11$

$\therefore$ 6 and 11 are the roots of equation

$\therefore$ Required equation is

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$\Rightarrow \quad x^2 - (11 + 6)x + (11 \times 6) = 0$

$\Rightarrow \quad x^2 - 17x + 66 = 0$

10 **(c)**

We have,

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(0 - h) = \lim_{x\to 0} -h \sin\left(-\frac{1}{h}\right)$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0} h \sin\left(\frac{1}{h}\right) = 0$

Similarly, we have $\lim_{x\to 0^+} f(x) = 0$

Hence, $\lim_{x\to 0} f(x) = 0$

11 **(d)**

We have, $f(x)\frac{|x+\pi|}{\sin x}$

$$\therefore \lim_{x\to -\pi^-} f(x) = \lim_{h\to 0} f(-\pi-h) = \lim_{h\to 0}\frac{|-\pi-h+\pi|}{\sin(-\pi-h)}$$

$$\Rightarrow \lim_{x\to -\pi^-} f(x) = -\lim_{h\to 0}\frac{h}{\sin(\pi+h)} = \lim_{h\to 0}\frac{h}{\sin h} = 1$$

and,

$$\lim_{x\to -\pi^+} f(x) = \lim_{h\to 0} f(-\pi+h) = \lim_{h\to 0}\frac{|-\pi+h+\pi|}{\sin(-\pi+h)}$$

$$\Rightarrow \lim_{x\to -\pi^+} f(x) = -\lim_{h\to 0}\frac{h}{\sin h} = -1$$

Hence, $\lim_{x\to -\pi} f(x)$ does not exist

12 **(d)**

We have,

$$\lim_{x\to 3}\frac{f(x)-f(3)}{x-3} = f'(3)$$

Now, $f'(x) = \frac{x}{(18-x^2)^{3/2}} \Rightarrow f'(3) = \frac{3}{(9)^{3/2}} = \frac{1}{9}$

13 **(c)**

We have,

$$\lim_{x\to\infty}\frac{5^{x+1}-7^{x+1}}{5^x-7^x} = \lim_{x\to\infty}\frac{5.\left(\frac{5}{7}\right)^x-7}{\left(\frac{5}{7}\right)^x-1} = \frac{5\times 0-7}{0-1} = 7$$

14 **(d)**

We have,

$A_i = \frac{x-a_i}{|x-a_i|}$, $i = 1, 2, \ldots, n$ and $a_1 < a_2 < \ldots < a_{n-1} < a_n$

Let x be in the left neighbourhood of a_m. Then,

$x - a_i < 0$ for $i = m, m+1, \ldots, n$

and,

$x - a_i > 0$ for $i = 1, 2, \ldots, m-1$

$$A_i = \begin{cases} = \dfrac{(x-a_i)}{-(x-a_i)} = -1 \text{ for } i = m, m+1, \ldots, n \\ = \dfrac{x-a_i}{x-a_i} = 1 \text{ for } i = 1, 2, \ldots, m-1 \end{cases}$$

Similarly, if x is in the right neighbourhood of a_m. Then,

$x - a_i < 0$ for $i = m+1, \ldots, n$ and $x - a_i > 0$ for $i = 1, 2, \ldots, m$

$$\therefore A_i = \begin{cases} A_i = \dfrac{x-a_i}{-(x-a_i)} = -1 \text{ for } i = m+1,\ldots,n \\ A_i = \dfrac{x-a_i}{x-a_i} = 1 \text{ for } i = 1, 2,\ldots, m \end{cases}$$

Thus, we have

$$\lim_{x\to a_m^-} (A_1 A_2 \ldots A_n) = (-1)^{n-m+1}$$

and,

$$\lim_{x\to a_m^+} (A_1 A_2 \ldots A_n) = (-1)^{n-m}$$

Hence, $\lim\limits_{x\to a_m} (A_1 A_2 \ldots A_n)$ does not exist

15 **(a)**

We have,

$$\lim_{x\to -1} \frac{(1+x)(1-x^2)(1+x^3)(1-x^4)\ldots(1+x^{4n-1})(1-x^{4n})}{[(1+x)(1-x^2)(1+x^3)(1-x^4)\ldots(1-x^{2n})]^2}$$

$$= \lim_{x\to -1} \frac{(1+x^{2n+1})(1-x^{2n+2})\ldots(1+x^{4n-1})(1-x^{4n})}{(1+x)(1-x^2)(1+x^3)(1-x^4)\ldots(1-x^{2n})}$$

$$= \lim_{x\to -1} \left\{\frac{1+x^{2n+1}}{1+x} \times \frac{1-x^{2n+2}}{1-x^2} \times \frac{1+x^{2n+3}}{1+x^3} \times \ldots \times \frac{1-x^{4n}}{1-x^{2n}}\right\}$$

$$= \lim_{x\to -1} \left\{\frac{x^{2n+1}+1}{x+1} \times \frac{x^{2n+2}-1}{x^2-1} \times \frac{x^{2n+3}+1}{x^3+1} \times \ldots \times \frac{x^{4n}-1}{x^{2n}-1}\right\}$$

$$= \lim_{x\to -1} \left\{\frac{x^{2n+1}-(-1)^{2n+1}}{x-(-1)} \times \frac{x^{2n+2}-(-1)^{2n+2}}{x^2-(-1)^2}\right\}$$

$$\times \left\{\frac{x^{2n+3}-(-1)^{2n+3}}{x^3-(-1)^3} \times \ldots \times \frac{x^{4n}-(-1)^{4n}}{x^{2n}-(-1)^{2n}}\right\}$$

$$= \frac{2n+1}{1} \times \frac{2n+2}{2} \times \frac{2n+3}{3} \times \ldots \times \frac{4n}{2n}$$

$$= \frac{4n!}{\{(2n)!\}^2} = {}^{4n}C_{2n}$$

16 **(a)**

We have,

$$\lim_{x\to\infty} \frac{\log x}{x^n} = \lim_{x\to\infty} \frac{1}{n\,x^n} = 0 \quad \text{[By L' Hospital's Rule]}$$

17 **(b)**

$$\lim_{x\to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3} = \lim_{x\to 9} \frac{f(x)-9}{x-9} \times \frac{\sqrt{x}+3}{\sqrt{f(x)}+3}$$

$$= f'(9) \times \left(\frac{\sqrt{9}+3}{\sqrt{f(9)}+3}\right)$$

$$= f'(9) \times \frac{3+3}{3+3} = 3$$

18 **(a)**

We have,

$$\lim_{n\to\infty} x^{2n} = \begin{cases} 0, \text{ if } |x| < 1 \\ 1, \text{ if } |x| = 1 \\ \infty, \text{ if } |x| > 1 \end{cases}$$

$$\therefore f(x) = \lim_{n\to\infty} \frac{1 - \frac{1}{x^{2n}}}{1 + \frac{1}{x^{2n}}} = \lim_{n\to\infty} \begin{cases} -1, |x| < 1 \\ 0, |x| = 1 \\ 1, |x| > 1 \end{cases}$$

19 **(b)**

$$\lim_{x\to 1} \frac{e^{-x} - e^{-1}}{x - 1} = \frac{1}{e} \lim_{x\to 1} \frac{e^{-(x-1)} - 1}{x - 1} = -\frac{1}{e}$$

20 **(c)**

$$\lim_{n\to\infty} \frac{x^n}{x^n + 1} = \lim_{n\to\infty} \frac{x^n}{(1 + 1/x^n)x^n}$$

$$= \lim_{n\to\infty} \frac{1}{1 + \frac{1}{x^n}} = 1$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	D	D	C	B	A	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	D	C	D	A	A	B	A	B	C