DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :6

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$l_1 = \lim_{x \to -2} (x + |x|) = -2 + 2 = 0$

$l_2 = \lim_{x \to -2} (2x + |x|) = -4 + 2 = -2$

and $l_3 = \lim_{x \to \pi/2} \frac{\cos x}{x - \pi/2} = \lim_{x \to \pi/2} \frac{\sin(\pi/2)}{-(\pi/2 - x)} = -1$

$\therefore l_2 < l_3 < l_1$

2 **(b)**

We have,

$$\lim_{x \to \infty} \left(\frac{x+2}{x+1}\right)^{x+3} = \lim_{x \to \infty} \left(1 + \frac{1}{x+1}\right)^{x+3} = e^{\lim_{x \to \infty} \frac{x+3}{x+1}} = e$$

3 **(b)**

We have,

$$\lim_{x \to 0} \frac{1}{x^{12}} \left\{1 - \cos\frac{x^2}{2} - \cos\frac{x^4}{4} + \cos\frac{x^2}{2}\cos\frac{x^4}{4}\right\}$$

$$= \lim_{x \to 0} \frac{\left(1 - \cos\frac{x^2}{2}\right)\left(1 - \cos\frac{x^4}{4}\right)}{x^8}$$

$$= \frac{1}{64} \lim_{x \to 0} \frac{1 - \cos\frac{x^2}{2}}{\left(\frac{x^2}{2}\right)^2} \times \frac{1 - \cos\frac{x^4}{4}}{\left(\frac{x^4}{4}\right)^2}$$

$$= \frac{1}{64} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{256} \quad \left[\because \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2} = \frac{1}{2}\right]$$

4 **(c)**

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$$

$$= \lim_{x \to \infty} \sqrt{\frac{x - \frac{\sin x}{x}}{1 + \frac{\cos^2 x}{x}}}$$

$= \sqrt{\dfrac{1-0}{1+0}}$

$\left[\because \dfrac{\sin x}{x}\to 0,\ \dfrac{\cos^2 x}{x}\to 0 \text{ as } x\to\infty\right]$

$= 1$

5 **(c)**

We have,

$$\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}} = \lim_{x\to 1}\{1+(1-x)\}^{\tan\frac{\pi x}{2}} = e^{\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}}$$

$$= e^{\lim_{h\to 0} -h\tan\left(\frac{\pi}{2}+\frac{\pi h}{2}\right)} = e^{\lim_{h\to 0} h\cot\frac{\pi h}{2}} = e^{\lim_{h\to 0}\frac{h}{\tan\left(\frac{\pi h}{2}\right)}} = e^{2/\pi}$$

6 **(a)**

We know that, if $r<1$, then

$\lim_{n\to\infty} r^n = 0$

And if $r>1$, then

$\lim_{n\to\infty} r^n = \infty$

Here, $\lim_{n\to\infty} r^n = 0$

$\therefore \quad r<1 \quad ie, \quad r=\dfrac{4}{5}$

7 **(a)**

$$\lim_{x\to 0}\frac{5^x-5^{-x}}{2x} = \lim_{x\to 0}\frac{5^x\log 5 - 5^{-x}\log 5}{2}$$

[by L' Hospital's rule]

$= \dfrac{\log 5 + \log 5}{2}$

$= \log 5$

8 **(d)**

$$\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x} = \lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x}$$

$= \lim_{x\to 0}\dfrac{\sin|x|}{x} = f(x)$ [say]

Now, $f(0+0) = \lim_{h\to 0}\dfrac{|\sin(0+h)|}{0+h} = 1$

$f(0-0) = \lim_{h\to 0}\dfrac{|\sin(0-h)|}{-h} = -1$

$\because \quad f(0+0) \neq f(0-0)$

$\therefore$ The limit of function does not exist.

9 **(c)**

We have,

$$\lim_{x\to\infty} a^x \sin\left(\frac{b}{a^x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{b}{a^x}\right)}{\left(\frac{b}{a^x}\right)} . b = 1 \cdot b = b$$

10 **(d)**

$$\lim_{x\to\infty}\left(\frac{x^3}{3x^2-4} - \frac{x^2}{3x+2}\right)$$

$$= \lim_{x\to\infty}\frac{x^3(3x+2) - x^2(3x^2-4)}{(3x^2-4)(3x+2)}$$

$$= \lim_{x\to\infty}\frac{2x^3+4x^2}{9x^3+6x^2-12x-8}$$

$$= \lim_{x\to\infty}\frac{2+4/x}{9+6/x-12/x^2-8/x^3} = \frac{2}{9}$$

11 **(a)**

We have,

$$\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2} = \lim_{x\to 0}\left\{\frac{(e^{x^2}-1)}{x^2} + \frac{(1-\cos x)}{x^2}\right\}$$

$$= \lim_{x\to 0}\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0}\frac{1-\cos x}{x^2} = 1 + \frac{1}{2} = \frac{3}{2}$$

12 **(d)**

Let $x = \frac{1}{y}$. Then,

$$\lim_{x\to\infty}\left\{\frac{a_1^{1/x}+a_2^{1/x}+\ldots+a_n^{1/x}}{n}\right\}^{nx}$$

$$= \lim_{y\to 0}\left\{\frac{a_1^y+a_2^y+\ldots+a_n^y}{n}\right\}^{n/y}$$

$$= \lim_{y\to 0}\left\{\frac{1+a_1^y+a_2^y+\ldots+a_n^y-n}{n}\right\}^{n/y}$$

$$= e^{\lim_{y\to 0}\left\{\frac{a_1^y-1}{y}+\frac{a_2^y-1}{y}+\ldots+a_n^y-1\right\}^{n/y}}$$

$$= e^{\log a_1+\log a_2+\ldots+\log a_n} = e^{\log(a_1a_2\ldots\ldots a_n)} = a_1a_2a_3\ldots a_n$$

13 **(b)**

$$\lim_{x\to 0}\frac{\sin^2 x + \cos x - 1}{x^2}$$

$$= \lim_{x\to 0}\frac{\cos x + \cos^2 x}{x^2}$$

$$= \lim_{x\to 0}\cos x \,.\, \frac{1-\cos x}{x^2}$$

$= 1.\frac{1}{2} = \frac{1}{2}$

14 **(b)**

We have,

$$\lim_{x\to 0}\frac{\sin 4x}{1-\sqrt{1-x}} = \lim_{x\to 0}\frac{\sin 4x}{x}(1+\sqrt{1-x})$$

$$= \lim_{x\to 0} 4\left(\frac{\sin 4x}{4x}\right)(1+\sqrt{1-x}) = 8$$

15 **(a)**

$$\lim_{x\to 1}\frac{x^8-2x+1}{x^4-2x+1} = \lim_{x\to 1}\frac{8x^7-2}{4x^3-2}$$

$= \frac{8-2}{4-2} = 3$ [using L' Hospital's rule]

166 **(a)**

$$\lim_{x\to\infty}\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}$$

$$= \lim_{x\to\infty}\frac{a^2x^2+ax+1-a^2x^2-1}{\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1}}$$

$$= \lim_{x\to\infty}\frac{ax}{x\left[\sqrt{a^2+\frac{a}{x}+\frac{1}{x^2}}+\sqrt{a^2+\frac{1}{x^2}}\right]}$$

$$= \frac{a}{\sqrt{a^2}+\sqrt{a^2}} = \frac{1}{2}$$

17 **(d)**

$\text{LHL} = \lim_{x\to 0^-}(-1)^{[x]}$

$= \lim_{h\to 0}(-1)^{[0-h]} = (-1)^{-1} = -1$

$\text{RHL} = \lim_{x\to 0^+}(-1)^{[x]} = \lim_{h\to 0}(-1)^{[0+h]}$

$= (-1)^0 = 1$

$\because$ LHL $\neq$ RHL

$\therefore$ Limit does not exist.

18 **(a)**

$$\lim_{x\to 0}\frac{(1-e^x)\sin x}{(x+x^2)x}$$

$$= \lim_{x\to 0}\frac{\left(-x-\frac{x^2}{2!}-\frac{x^3}{3!}-\ldots\right)}{x(1+x)}\times\lim_{x\to 0}\frac{\sin x}{x}$$

$= -1\times 1 = -1$

19 **(c)**

$$\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+2} = \lim_{x\to\infty}\left[\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+2}{2}\right)}\right]^{\frac{2}{x+2}\times(x+2)}$$

$$= \lim_{x\to\infty}\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+2}{2}\right)\left[\frac{2(x+2)}{x+1}\right]}$$

$$= e^{\lim_{x\to\infty}\frac{(2+4/x)}{(1+1/x)}}$$

$$= e^2$$

20 **(b)**

Since, α is a repeated root.

$\therefore \quad ax^2 + bx + c = a(x-\alpha)^2$

Now, $\lim_{x\to\alpha}\frac{\sin(ax^2+bx+c)}{(x-\alpha)^2}$

$$= \lim_{x\to\alpha}\frac{\sin a(x-\alpha)^2}{a(x-\alpha)^2}\times a$$

$$= \lim_{x\to\alpha} a(1) = a$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	C	C	A	A	D	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	B	B	A	A	D	A	C	B