# DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

**Solutions**

SUBJECT : MATHS
DPP NO. :5

## Topic :-LIMITS & DERIVATIVES

1 **(a)**

We have,

$$\lim_{x\to\infty} x^{3/2}\left(\sqrt{x^3+1}-\sqrt{x^3-1}\right)$$

$$= \lim_{x\to\infty} \frac{2x^{3/2}}{\sqrt{x^3+1}+\sqrt{x^3-1}}$$

$$= \lim_{x\to\infty} \frac{2x^{3/2}}{\sqrt{1+\frac{1}{x^3}}+\sqrt{1-\frac{1}{x^3}}} = \frac{2}{1+1} = 1$$

2 **(a)**

Given, $\lim_{x\to a} \frac{a^x - x^a}{x^x - a^a} = -1$

$\Rightarrow \lim_{x\to a} \frac{a^x \log_e a - a x^{a-1}}{x^x(1+\log_e x) - 0} = -1$ [by L' Hospital's rule]

$\Rightarrow \frac{a^a \log_e a - a^a}{a^a(1+\log_e a)} = -1$

$\Rightarrow 2\log_e a = 0 \quad \Rightarrow \quad a = 1$

3 **(c)**

We have,

$$\lim_{x\to\infty}\left\{\frac{x^3+1}{x^2+1}-(ax+b)\right\} = 2$$

$$\Rightarrow \lim_{x\to\infty} \frac{x^3(1-a)-bx^2-ax+(1-b)}{x^2+1} = 2$$

$\Rightarrow 1-a=0$ and $-b=2 \Rightarrow a=1, b=-2$

4 **(d)**

$$\lim_{x\to 0} \frac{e^{x^2}-\cos x}{x^2}\left(\frac{0}{0}\text{ from}\right)$$

$$= \lim_{x\to 0} \frac{2xe^{x^2}-\sin x}{2x}\left(\frac{0}{0}\text{ from}\right)$$

$$= \lim_{x \to 0} \frac{2e^{x^2} + 4x^2 e^{x^2} + \cos x}{2}$$

$$= \frac{2 + 0 + 1}{2} = \frac{3}{2}$$

5 **(d)**

$$\lim_{n \to \infty} \frac{1}{1 - n^3} \sum_{r=1}^{n} r^2 = \lim_{n \to \infty} \frac{1}{n^3\left(\frac{1}{n^3} - 1\right)} \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \to \infty} \frac{n^3\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)}{n^3\left(\frac{1}{n^3} - 1\right)6} = -\frac{1}{3}$$

6 **(a)**

$$\lim_{x \to \frac{\pi}{6}} \frac{\sin 2x}{\sin x} = \lim_{x \to \frac{\pi}{6}} \frac{2 \sin x \cos x}{\sin x}$$

$$= 2 \lim_{x \to \frac{\pi}{6}} \cos x = \sqrt{3}$$

7 **(a)**

We have,

$$\lim_{x \to 0} \frac{1 - \cos(1 - \cos x)}{x^4} = \lim_{x \to 0} \frac{1 - \cos(2 \sin^2 x/2)}{x^4}$$

$$= \lim_{x \to 0} \frac{2 \sin^2(\sin^2 x/2)}{x^4} = 2 \lim_{x \to 0} \left\{\frac{\sin(\sin^2 x/2)}{x^2}\right\}^2$$

$$= 2 \lim_{x \to 0} \left\{\frac{\sin(\sin^2 x/2)}{\sin^2 x/2} \times \frac{\sin^2 x/2}{x^2/4} \times \frac{1}{4}\right\}^2 = 2\left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

8 **(d)**

Let $f(x) = ax^2 + bx + c$

We have,

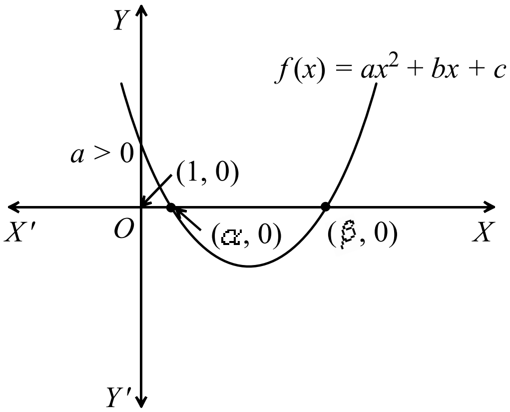
$$\lim_{x \to m} \frac{|ax^2 + bx + c|}{ax^2 + bx + c} = 1$$

$\Rightarrow ax^2 + bm + c > 0$

$\Rightarrow f(m) > 0$

$\Rightarrow$ Point $(m, f(m))$ must be on darkened part of the curve $y = f(x)$

Thus, options (a), (b) and (c) are true




9 **(b)**

$\lim_{x\to 1} \frac{x^m - 1}{x^n - 1} = \lim_{x\to 1} \frac{mx^m - 1}{nx^n - 1}$ [by L' Hospital's rule]

$= \frac{m}{n}$

10 **(d)**

$$\lim_{x\to\alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x-\alpha)^2} = \lim_{x\to\alpha} \frac{2\sin^2\left(\frac{ax^2 + bx + c}{2}\right)}{(x-\alpha)^2}$$

[Since $\alpha$ and $\beta$ are the roots of $ax^2 + bx + c = 0$, so it can be written as $a(x-\alpha)(x-\beta) = 0$]

$$= \lim_{x\to\alpha} \frac{2\sin^2\left(\frac{a(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha} \frac{2\sin^2\left(\frac{a}{2}(x-\alpha)(x-\beta)\right)\left(\frac{a}{2}\right)^2(x-\beta)^2}{\left[\left(\frac{a}{2}\right)(x-\alpha)(x-\beta)\right]^2}$$

$$= \lim_{x\to\alpha} 2\left(\frac{a}{2}\right)^2(x-\beta)^2 = \frac{a^2}{2}(\alpha-\beta)^2$$

11 **(c)**

$\lim_{x\to 0}\left[\frac{2^x - 1}{\sqrt{1+x} - 1}\right] = \lim_{x\to 0} \frac{2^x \log_e 2}{\frac{1}{2\sqrt{1+x}}}$ [by L' Hospital's rule]

$= 2\log_e 2 = \log_e 4$

12 **(b)**

$\lim_{x\to 0}\left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}}\right)$

$= \lim_{x\to 0}\left(\frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x}\right)$

$= 1$

13 **(c)**

Given, $\lim_{x\to 0} \frac{\log(3+x) - \log(3-x)}{x} = k$

using L' Hospital's rule

$$\Rightarrow \quad \lim_{x\to 0}\frac{\left(\frac{1}{3+x}+\frac{1}{3-x}\right)}{1}=k$$

$$\Rightarrow \quad \frac{1}{3}+\frac{1}{3}=k \quad \Rightarrow \quad k=\frac{2}{3}$$

14 **(a)**

We have,

$$\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}=e^{\lim_{x\to\infty}\frac{c+dx}{a+bx}}=e^{d/b}$$

15 **(c)**

$$\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}\right)=\lim_{x\to 0}\frac{\sec^2 x^2.2x}{\sin x+x\cos x}$$

$$=\lim_{x\to 0}\frac{2x.\sec^2 x^2}{x\left(\frac{\sin x}{x}+\cos x\right)}$$

$$=\frac{2\times 1}{1+1}=1\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

16 **(c)**

It is fundamental co0ncept of indeterminate

$$ie, \quad \lim_{x\to\infty}\frac{\sin x}{x}=\frac{\sin\infty}{\infty}$$

$$=0\times \text{finite term}=0$$

17 **(a)**

Using expressions of $\cos x$ and $\log(1+x)$, the given limit is equal to

$$\lim_{x\to 0}\frac{x\left\{1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\dots\right\}-\left\{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\dots\right\}}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{1}{2}-\frac{x}{2!}-\frac{x}{3}+\dots\right)=\frac{1}{2}$$

18 **(c)**

Let $A=\lim_{n\to\infty}\frac{1}{n^k}\{(n+1)^k(n+2)^k\dots(n+n)^k\dots(n+n)^k\}^{1/n}$

Then,

$$A=\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)^k\left(1+\frac{2}{n}\right)^k\dots\left(1+\frac{1}{n}\right)^k\right\}^{1/n}$$

$$\Rightarrow \log A=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1^n}\log\left(1+\frac{r}{n}\right)^k=k\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\log\left(1+\frac{r}{n}\right)$$

$$\Rightarrow \log A=2k(\log 2-1/2)=\log 4^k-k$$

$\Rightarrow A = \left(\frac{4}{e}\right)^k$

19 **(d)**

$\text{LHL} = \lim_{x \to 0} \frac{-\sin x}{x}$

$= -1$

$\text{RHL} = \lim_{x \to 0} \frac{\sin x}{x}$

$= 1$

$\Rightarrow \quad \text{LHL} \neq \text{RHL}$

$\Rightarrow \quad \lim_{x \to 0} \frac{\sin |x|}{x}$ Does not exist.

20 **(a)**

We have,

$$\lim_{x \to \infty} \left\{ \frac{x^2 \sin\left(\frac{1}{x}\right) - x}{1 - |x|} \right\}$$

$$= \lim_{x \to \infty} \frac{x^2 \sin(x^{-1}) - x}{1 - x}$$

$$= \lim_{x \to \infty} \frac{\left(\frac{\sin(x^{-1})}{x^{-1}}\right) - 1}{x^{-1} - 1} = \frac{1 - 1}{0 - 1} = 0$$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | A | C | D | D | A | A | D | B | D |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | C | B | C | A | C | C | A | C | D | A |
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