DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :4

Topic :-LIMITS & DERIVATIVES

1 **(a)**

We have,

$$\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$$

$$=\left\{\frac{d}{dx}(x^2\sin x)\right\}_{\text{at } x=a}=2a\sin a+a^2\cos a$$

2 **(a)**

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$$

$$=\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x}$$

$$=\lim_{x\to 0}\frac{2f''(x)-12f''(2x)+16f''(4x)}{2}$$

$$=\frac{2f''(0)-12f''(0)+16f''(0)}{2}=\frac{6a}{2}=3a$$

3 **(b)**

$$\lim_{x\to 0}\frac{e^x+\log(1+x)-(1-x)^{-2}}{x^2}\left[\frac{0}{0}\text{ form}\right]$$

$$=\lim_{x\to 0}\frac{e^x+(1+x)^{-1}-2(1-x)^{-3}}{2x}$$

[by L' Hospital's rule]

$$=\lim_{x\to 0}\frac{e^x-(1+x)^{-2}-6(1-x)^{-4}}{2}$$

[by L' Hospital's rule]

$$=\frac{e^0-1-6}{2}=-3$$

4 **(a)**

Given, $\lim_{x\to 0}kx\operatorname{cosec}(x)=\lim_{x\to 0}x\operatorname{cosec}(kx)$

$$\Rightarrow k\lim_{x\to 0}\frac{x}{\sin x}=\lim_{x\to 0}\frac{x}{\sin kx}\times\frac{k}{k}$$

$\Rightarrow \quad k = \frac{1}{k}$

$\Rightarrow \quad k = \pm 1$

5 **(d)**

We have,

$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$

and, $\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{|-x|}{-x} = \lim_{x \to 0^+} \frac{x}{-x} = -1$

Hence, $\lim_{x \to 0} \frac{|x|}{x}$ does not exist

6 **(b)**

We have,

$\lim_{x \to 0} x^2 \sin \frac{\pi}{x}$

$= 0 \times (\text{A finite oscillating number}) = 0$

7 **(c)**

$$\lim_{x \to \infty} \left(\frac{x^2 + bx + 4}{x^2 + ax + 5} \right) = \lim_{x \to \infty} \frac{\left(1 + \frac{b}{x} + \frac{4}{x^2}\right) x^2}{\left(1 + \frac{a}{x} + \frac{5}{x^2}\right) x^2} = 1$$

8 **(b)**

Since, $f(x)$ is the integral function of $\frac{2 \sin x - \sin 2x}{x^3}$, therefore by definition

$$f'(x) = \frac{2 \sin x - \sin 2x}{x^3}$$

$$\Rightarrow \quad \lim_{x \to 0} f'(x) = \lim_{x \to 0} \frac{2 \sin x - \sin 2x}{x^3}$$

$$= \lim_{x \to 0} = \frac{2 \sin x}{x} . \frac{1 - \cos x}{x^2} = 1$$

9 **(a)**

We have,

$$\lim_{x \to \infty} \left(\frac{3x - 4}{3x + 4} \right)^{\left(\frac{x+1}{3}\right)} = \lim_{x \to \infty} \left(1 + \frac{-6}{3x + 2}\right)^{\frac{x+1}{2}}$$

$$= e^{\lim_{x \to \infty} \frac{-6}{3x+2} \times \frac{x+1}{3}} = e^{\lim_{x \to \infty} \frac{-2x-2}{3x+2}} = e^{-2/3}$$

10 **(c)**

$$\lim_{x \to 0} \frac{(1 + x)^8 - 1}{(1 + x)^2 - 1}$$

$$= \lim_{x \to 0} \frac{[(1 + x)^4 + 1][(1 + x)^2 + 1][(1 + x)^2 - 1]}{(1 + x)^2 - 1}$$

$= 2 \times 2 = 4$

Alternate

$$\lim_{x\to 0}\frac{(1+x)^8-1}{(1+x)^2-1}\left(\frac{0}{0}\text{ form}\right)$$

$\lim_{x\to 0}\frac{8(1+x)^7}{2(1+x)}$ (by L' Hospital's rule)

$= 4$

11 **(a)**

We have,

$$\lim_{x\to \pi/4}\frac{2\sqrt{2}-(\cos x+\sin x)^3}{1-\sin 2x}$$

$$= \lim_{x\to \pi/4}\frac{2^{3/2}-\{(\cos x+\sin x)^2\}^{3/2}}{2-(1+\sin 2x)}$$

$$= \lim_{x\to \pi/4}\frac{2^{3/2}-(1+\sin 2x)^{3/2}}{2-(1+\sin 2x)}$$

$= \lim_{y\to 2}\frac{y^{3/2}-2^{3/2}}{y-2}$, where $y = 1+\sin 2x$

$$= \frac{3}{2}(2)^{3/2-1} = \frac{3}{2}\times\sqrt{2} = \frac{3}{\sqrt{2}}$$

12 **(b)**

We have,

$$\lim_{x\to -\infty}\left(3x+\sqrt{9x^2-x}\right)$$

$= \lim_{y\to\infty}\left(-3y+\sqrt{9y^2+y}\right)$, where $y = -x$

$$= \lim_{y\to\infty}\frac{-9y^2+9y^2+y}{(3y+\sqrt{9y^2+y})} = \lim_{y\to\infty}\frac{y}{3y+\sqrt{9y^2+y}} = \frac{1}{3+3} = \frac{1}{6}$$

13 **(a)**

$$\lim_{x\to 0}\frac{e^{5x}-e^{4x}}{x}$$

$$= \lim_{x\to 0}\frac{\left[\left(1+\frac{5x}{1!}+\frac{(5x)^2}{2!}+\ldots\right)-\left(1+\frac{4x}{1!}+\frac{(4x)^2}{2!}+\ldots\right)\right]}{x}$$

$$= \lim_{x\to 0}\frac{x\left[\left(\frac{5}{1!}+\frac{25x}{2!}+\ldots\right)-\left(\frac{4}{1!}+\frac{16}{2!}+\ldots\right)\right]}{x}$$

$= 1$

14 **(a)**

We have,

$$L=\lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}$$

$$\Rightarrow L=\lim_{x\to 0}\frac{a-a\left(1-\frac{x^2}{a^2}\right)^{1/2}-\frac{x^2}{4}}{x^4}$$

$$\Rightarrow L=\lim_{x\to 0}\frac{a-a\left\{1-\frac{1}{2}\cdot\frac{x^2}{a^2}-\frac{1}{8}\cdot\frac{x^4}{a^4}+\frac{1}{16}\cdot\frac{x^6}{a^6}\dots\right\}-\frac{x^2}{4}}{x^4}$$

$$\Rightarrow L=\lim_{x\to 0}\frac{\left(\frac{1}{2}\cdot\frac{x^2}{a}+\frac{1}{8}\cdot\frac{x^4}{a^3}-\frac{1}{16}\cdot\frac{x^6}{a^5}\dots\right)-\frac{x^2}{4}}{x^4}$$

$$\Rightarrow L=\lim_{x\to 0}\frac{x^2}{2}\left(\frac{1}{a}-\frac{1}{2}\right)+\frac{1}{8a^3}-\frac{1}{16}\cdot\frac{x^2}{a^5}+\dots$$

$\Rightarrow \frac{1}{a}-\frac{1}{2}=0$ and in that case $L=\frac{1}{8a^3}$ $[\because L$ is finite$]$

$\Rightarrow a=2$ and $L=\frac{1}{64}$

15 **(b)**

$$\lim_{x\to 0}\log_e(\sin x)^x=\log_e\left[\lim_{x\to 0}(\sin x)^x\right]$$

$$=\log_e\left[\lim_{x\to 0}(1+\sin x-1)^{\frac{x(\sin x-1)}{(\sin x-1)}}\right]$$

$$=\log_e\left[e^{\lim_{x\to 0}x(\sin x-1)}\right]$$

$$=\log_e 1$$

16 **(a)**

We have,

$$\lim_{x\to 0^+}x^m(\log x)^n=\lim_{x\to 0^+}\frac{(\log x)^n}{x^{-m}}$$

$$\Rightarrow\lim_{x\to 0^+}x^m(\log x)^n=\lim_{x\to 0^+}\frac{n(\log x)^{n-1}\frac{1}{x}}{-m\,x^{-m-1}}$$ [By L' Hospital's Rule]

$$\Rightarrow\lim_{x\to 0^+}x^m(\log x)^n=\lim_{x\to 0^+}\frac{n(\log x)^{n-1}}{-m\,x^{-m}}$$

$$\Rightarrow\lim_{x\to 0^+}x^m(\log x)^n=\lim_{x\to 0^+}\frac{n(n-1)(\log x)^{n-2}\frac{1}{x}}{(-m)^2x^{-m-1}}$$ [By L' Hospital's Rule]

$$\Rightarrow\lim_{x\to 0^+}x^m(\log x)^n=\lim_{x\to 0^+}\frac{n(n-1)(\log x)^{n-2}}{m^2x^{-m}}$$

= ....

$$=\lim_{x\to 0^+}\frac{n!}{(-m)^n x^{-m}}\left[\begin{matrix}\text{Diff. numerator and}\\ \text{denominator } n \text{ times}\end{matrix}\right]$$

$=0$

17 **(a)**

$$\lim_{x\to\infty}\frac{\sqrt{1+x^4}-(1+x^2)}{x^2} = \lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x^4}}-\left(1+\frac{1}{x^2}\right)}{1}$$

$$=\frac{1-1}{1}=0$$

18 **(c)**

Let $y=\lim_{x\to 0}(\operatorname{cosec} x)^{1/\log x}$

$$\Rightarrow \quad \log y=\lim_{x\to 0}\frac{\log \operatorname{cosec} x}{\log x}$$

$$=\lim_{x\to 0}\frac{-\cot x}{1/x} \qquad \text{[by L'Hospital's rule]}$$

$$=-\lim_{x\to 0}\frac{x}{\tan x}=-1$$

$$\Rightarrow \quad \log y=-1$$

$$\Rightarrow \quad y=\frac{1}{e}$$

19 **(c)**

$$\lim_{x\to 1}\frac{2x-f(x)}{x-1}$$

$$=\lim_{x\to 1}\frac{2-f'(x)}{1} \qquad \text{(by L' Hospital's rule)}$$

$$=2-f'(1)$$

$$=2-(1)=1$$

20 **(d)**

$$f(0)=\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$$

$$=\lim_{x\to 0}\frac{\sin(1-\cos x).\sin x}{4x^3}$$

$$=\frac{1}{4}.\lim_{x\to 0}\frac{\sin(1-\cos x)}{1-\cos x}.\frac{1-\cos x}{x^2}.\frac{\sin x}{x}$$

$$=\frac{1}{4}.1.\frac{1}{2}.1=\frac{1}{8}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	B	A	D	B	C	B	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	A	A	B	A	A	C	C	D