DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :3

Topic :-LIMITS & DERIVATIVES

1 **(a)**

We have,

$\lim_{x\to 2} \frac{2x^2 - 4f'(x)}{x-2} = \lim_{x\to 0} \frac{4x - 4f''(x)}{1}$ [Using L' Hospital's Rule]

$\Rightarrow \lim_{x\to 2} \frac{2x^2 - 4f'(x)}{x-2} = 8 - 4f''(2) = 8 - 4 = 4$

2 **(a)**

$$\lim_{x\to \pi/4} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \pi^2/16}$$

$= \lim_{x\to \pi/4} \frac{2\sec^2 x \tan x \ f(\sec^2 x)}{2x}$ [Using Leibniz and L' Hospital's rules]

$$= \frac{\sec^2 \frac{\pi}{2} \ f\left(\sec^2 \frac{\pi}{4}\right) \tan \frac{\pi}{4}}{\pi/4} = \frac{8}{\pi} f(2)$$

3 **(d)**

$$\lim_{x\to a} \frac{f(a)g(x) - f(x)g(a)}{x-a} \left[\frac{0}{0} \text{ from}\right]$$

$= \lim_{x\to a} \frac{f(a)g'(x) - f'(x)g(a)}{1-0}$ [by L' Hospital's rule]

$= f(a)g'(a) - f'(a)g(a)$

$= 2(-1) - 1(3) = -2 - 3 = -5$

4 **(a)**

We have,

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$$

$$\Rightarrow \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \left(1 + \frac{4x+1}{x^2 + x + 2}\right)^x = e^{\lim_{x\to\infty} \frac{x(4x+1)}{x^2+x+2}} = e^4$$

5 **(c)**

$$\lim_{x\to 0} \frac{\sin^{-1} x - x}{x^3 \cos x} = \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}} - 1}{x^3(-\sin x) + 3x^2 \cos x}$$

[using L'Hospital's rule]

$$= \lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{\sqrt{1-x^2}.x^2(-x\sin x + 3\cos x)} \times \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$$

$$= \lim_{x\to 0} \frac{1}{\left[\begin{array}{l}\sqrt{1-x^2}(1+\sqrt{1-x^2})\\(-x\sin x + 3\cos x)\end{array}\right]}$$

$$= \frac{1}{1(1+1)(3)} = \frac{1}{6}$$

6 **(c)**

Here, $\lim_{x\to 0}(\sin x)^{1/x} + \lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin x} = 0 + \lim_{x\to 0} e^{\log\left(\frac{1}{x}\right)^{\sin x}}$

$$\left[\begin{array}{l}\lim_{x\to 0}(\sin x)^{\frac{1}{x}} \to 0\\ \text{as,} \quad 0 < \sin x < 1\end{array}\right]$$

$$= e^{\lim_{x\to 0}\frac{\log(1/x)}{\operatorname{cosec} x}} = e^{\lim_{x\to 0}\frac{x\left(-\frac{1}{x^2}\right)}{-\operatorname{cosec} x \cot x}}$$

[by L'Hospital's rule]

$$= e^{\lim_{x\to 0}\frac{\sin x}{x}\tan x} = e^0 = 1$$

7 **(b)**

$$\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$$

$$= \lim_{x\to\infty}\left[1 + \frac{-6}{3x+2}\right]^{\frac{x+1}{3}}$$

$$= \left[\lim_{x\to\infty}\left\{1 + \frac{-6}{3x+2}\right\}^{\frac{3x+2}{-6}}\right]^{\frac{-6}{3x+2}\times\frac{x+1}{3}}$$

$$= [e]^{\lim_{x\to\infty}\frac{-6}{3x+2}\times\frac{x+1}{3}}\left[\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\right]$$

$$= e^{\lim_{x\to\infty}\frac{-2x-2}{3x+2}} = e^{-2/3}$$

8 **(c)**

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

As $x\to 0 \Rightarrow \theta \to 0$

$$\therefore \lim_{\theta\to 0}\frac{1}{\tan\theta}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \lim_{\theta\to 0}\frac{1}{\tan\theta}\sin^{-1}(\sin 2\theta)$$

$$= \lim_{\theta\to 0}\frac{2\theta}{\tan\theta} = 2$$

9 **(c)**

$$\lim_{x\to 0}\frac{2\sin^2 3x}{x^2} = \lim_{x\to 0} 2\left(\frac{\sin 3x}{3x}\right)^2 \times \frac{9}{1} = 18$$

10 **(a)**

$$\lim_{x\to 0}\frac{a^x + a^{-x} - 2}{x^2} = \lim_{x\to 0}\frac{a^x \log a - a^{-x}\log a}{2x}$$

[by L' Hospital's rule]

$$= \lim_{x\to 0}\frac{a^x(\log a)^2 + a^{-x}(\log a)^2}{2}$$

$= (\log a)^2$ [by L' Hospital's rule]

11 **(b)**

$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = 1$

[$\because (0-h)$ is rational]

$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = 1$

[$\because (0+h)$ is rational]

Hence, LHL=RHL=1

12 **(c)**

$$\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x = \lim_{x\to\infty}\left[\frac{1-\frac{3}{x}}{1+\frac{2}{x}}\right]^x$$

$$= \frac{e^{-3}}{e^2} = e^{-5}$$

13 **(b)**

We have,

$$\lim_{x\to\infty}\frac{x}{2}\sin\left(\frac{\pi}{2x}\right)$$

$$= \lim_{x\to\infty}\frac{\sin\left(\frac{\pi}{2x}\right)}{\frac{\pi}{2x}}\cdot\frac{\pi}{4} = \frac{\pi}{4}\lim_{y\to 0}\frac{\sin y}{y} = \frac{\pi}{4}, \text{ where } y = \frac{\pi}{2x}$$

14 **(b)**

$$\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\frac{f'(x)}{1}$$

$$= \lim_{x\to 0}\frac{\tan^4 x}{1} = 0$$

15 **(c)**

$$\text{RHL} = \lim_{x\to 1^+}\frac{1}{2}\{g(x) + (x)\}\sin x$$

$$= \lim_{x\to 1^+}\frac{1}{2}\{1+x\}\sin x$$

$= \frac{1}{2}.(1+1)\sin 1 = \sin 1$

and LHL $= \lim_{x\to 1^-} \frac{\sin x}{x} = \sin 1$

Since, RHL=LHL=sin 1

$\therefore \quad \lim_{x\to 1} f(x) = \sin 1$

16 **(c)**

Given, $\lim_{x\to\infty} \left[\frac{x^3+1}{x^2+1} - (ax+b)\right] = 2$

$$\Rightarrow \quad \lim_{x\to\infty} \left[\frac{x(1-a) - b - \frac{a}{x} + \frac{(1-b)}{x^2}}{1+\frac{1}{x^2}}\right] = 2$$

This limit will exist, if

$1 - a = 0$

and $b = -2$

$\Rightarrow \quad a = 1$

and $b = -2$

17 **(b)**

$$\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2} \frac{\frac{x-2}{x^2-3x+2} - 1}{x-2}$$

$$= \lim_{x\to 2} \frac{x-2-(x^2-3x+2)}{(x-2)(x^2-3x+2)}$$

$$= \lim_{x\to 2} \frac{-(x-2)^2}{(x-2)(x-2)(x-1)}$$

$$= -\lim_{x\to 2} \frac{1}{x-1}$$

$= -1$

18 **(c)**

$$\lim_{x\to 3} \frac{\int_3^{f(x)} 2t^3 dt}{x-3} = \lim_{x\to 3} \frac{2[f(x)]^3 . f'(x)}{1}$$

$$= 2[f(3)]^3 . f'(3) = 2 \times 3^3 \times \frac{1}{2}$$

$= 27$

19 **(a)**

Given, $\lim_{x\to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$

Applying L' Hospital's rule,

$$\Rightarrow \quad \lim_{x\to a} \frac{f(a)g'(x) - g(a)f'(x)}{g'(x) - f'(x)} = 4$$

$$\Rightarrow \qquad \lim_{x\to a} \frac{kg'^{(x)} - kf'(x)}{g'(x) - f'(x)} = 4$$

$$\Rightarrow \qquad k = 4$$

20 **(d)**

We have,

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{x^2}} = \lim_{x\to 0} \frac{\sin x}{|x|}$$

Now, $\lim_{x\to 0^-} \frac{\sin x}{|x|} = \lim_{x\to 0} \frac{\sin x}{x|} = -1$

and, $\lim_{x\to 0^+} \frac{\sin x}{|x|} = \lim_{x\to 0} \frac{\sin x}{x} = 1$

Hence, $\lim_{x\to 0} \frac{\sin x}{|x|}$ does not exist

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	A	C	C	B	C	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	B	C	C	B	C	A	D