

## Topic :-LIMITS & DERIVATIVES

1      (b)

We have,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x^3} \\ = \lim_{x \rightarrow 0} \frac{1 + \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right) - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots\right) + \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\right)}{x^3} \\ = \lim_{x \rightarrow 0} \left(-\frac{1}{3!} - \frac{1}{3}\right) + x^2 \left(\frac{1}{5!} - \frac{1}{5}\right) + \dots = -\frac{1}{6} - \frac{1}{3} = -\frac{1}{2} \end{aligned}$$

2      (b)

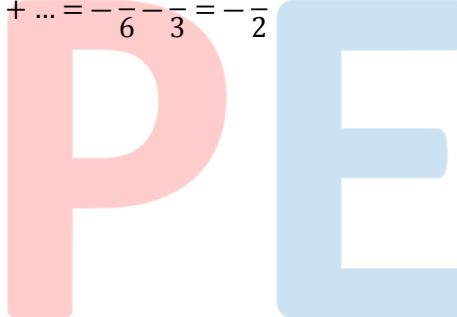
$$\begin{aligned} \lim_{x \rightarrow 0} \frac{(1 + \tan x)^{\text{cosec } x}}{1 + \sin x} \\ = \lim_{x \rightarrow 0} \frac{\left[1 + \frac{\sin x}{\cos x}\right]^{\frac{\cos x}{\sin x}}}{(1 + \sin x)^{1/\sin x}} \\ = \frac{\lim_{x \rightarrow 0} \frac{1}{\cos x}}{e} \\ = \frac{e}{e} = 1 \end{aligned}$$

3      (b)

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1} \\ = \lim_{x \rightarrow \frac{\pi}{6}} \frac{4 \sin x \cos x + \cos x}{4 \sin x \cos x - 3 \cos x} \end{aligned}$$

[by L'Hospital's rule]

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cos x (4 \sin x + 1)}{\cos x (4 \sin x - 3)}$$



$$= \frac{4 \sin \frac{\pi}{6} + 1}{4 \sin \frac{\pi}{6} - 3} = -3$$

4      (a)

$$\lim_{x \rightarrow 0} \left( \frac{1+5x^2}{1+3x^2} \right)^{1/x^2} = \lim_{x \rightarrow 0} \left( 1 + \frac{2x^2}{1+3x^2} \right)^{1/x^2}$$

$$= e^{\lim_{x \rightarrow 0} \frac{1}{x^2} \left( \frac{2x^2}{1+3x^2} \right)} = e^2$$

5      (a)

We have,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0} x = 0 \text{ and, } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0} x^2 = 0$$

$$\text{Hence } \lim_{x \rightarrow 0} f(x) = 0$$

6      (a)

If  $x \in Q$ , then  $n! \pi x$  will be an integral multiple of  $\pi$  for large values of  $n$ . Therefore,  $\cos(n! \pi x)$  will be either 1 or -1 and so  $\cos^{2m}(n! \pi x) = 1$

$$\therefore \lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} [1 + \cos^{2m}(n! \pi x)] = 1 + 1 = 2$$

If  $x \notin Q$ ,  $n! \pi x$  will not be an integral multiple of  $\pi$  and so  $\cos(n! \pi x)$  will lie between -1 and 1

$$\text{Thus, } \lim_{m \rightarrow \infty} \cos^{2m}(n! \pi x) = 0$$

$$\Rightarrow \lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} [1 + \cos^{2m}(n! \pi x)] = 1 + 0 = 1$$

7      (c)

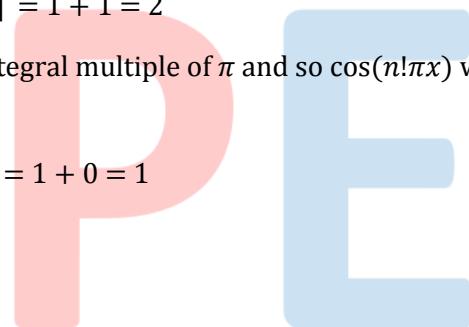
We have,

$$\lim_{x \rightarrow 1} (1 + \cos \pi x) \cot^2 \pi x$$

$$= \lim_{x \rightarrow 1} \frac{(1 + \cos \pi x)(\cos^2 \pi x)}{(1 - \cos^2 \pi x)} = \lim_{x \rightarrow 1} \frac{\cos^2 \pi x}{1 - \cos \pi x} = \frac{1}{2}$$

8      (c)

$$\lim_{x \rightarrow 0} \frac{(e^{kx} - 1) \sin kx}{x^2} = 4$$



$$\Rightarrow \lim_{x \rightarrow 0} \frac{e^{kx} - 1}{kx} \times k \times \frac{\sin kx}{kx} \times k = 4$$

$$\Rightarrow k^2 = 4 \Rightarrow k = \pm 2$$

9      (b)

$$\lim_{x \rightarrow 0} \frac{\log(1 + x^3)}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\left( \frac{x^3}{1} - \frac{(x^3)^2}{2} + \frac{(x^3)^3}{3} - \dots \infty \right)}{\left( x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \infty \right)^3}$$

$$= \lim_{x \rightarrow 0} \frac{\left(1 - \frac{x^3}{2} + \frac{(x^3)^2}{3} - \dots \infty\right)}{\left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots \infty\right)^3} = 1$$

10      **(c)**  

$$l_1 = \lim_{x \rightarrow 2^+} (x + [x])$$

$$= \lim_{h \rightarrow 0} 2 + h + [2 + h] = 4$$

$$l_2 = \lim_{x \rightarrow 2^-} (2x - [x])$$

$$= \lim_{h \rightarrow 0} \{2(2 - h) - [2 - h]\}$$

$$= \lim_{h \rightarrow 0} \{2(2 - h) - 1\} = 3$$

$$l_3 = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\cos x}{x - \frac{\pi}{2}} = \lim_{x \rightarrow \frac{\pi}{2}^-} -\sin x = -1$$

[by L'Hospital's rule]

Thus,  $l_3 < l_2 < l_1$

11      **(b)**  

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{\sin(1 + [x])}{[x]}$$

$$= \frac{\sin(1 - 1)}{-1} = 0$$

12      **(a)**  

$$\lim_{x \rightarrow 0} \frac{(1 - \cos 2x) \sin 5x}{x^2 \sin 3x}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} \lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 3x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \cdot \frac{3x}{\sin 3x} \cdot \frac{5x}{3x}$$

$$= \lim_{x \rightarrow 0} 2 \left(\frac{\sin x}{x}\right)^2 \cdot \frac{5}{3} = \frac{10}{3}$$

13      **(d)**

We have,

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{\sin(e^{x-2} - 1)}{\log(x-1)}$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \left\{ \frac{\sin(e^{x-2} - 1)}{e^{x-2} - 1} \cdot \frac{e^{x-2} - 1}{x-2} \cdot \frac{x-2}{\log(1 + (x-2))} \right\}$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = 1 \times 1 \times 1 = 1$$

14      **(a)**

We have,



$$\lim_{n \rightarrow \infty} \frac{S_{n+1} - S_n}{\sqrt{\sum_{k=1}^n k}} = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{\sqrt{\frac{n(n+1)}{2}}} = 0$$

15      **(c)**

$$\lim_{h \rightarrow 0} \frac{\sin \sqrt{x+h} - \sin \sqrt{x}}{h}$$

Applying L'Hospital's rule,

$$= \lim_{h \rightarrow 0} \frac{\frac{\cos \sqrt{x+h}}{2\sqrt{x+h}}}{1} = \frac{\cos \sqrt{x}}{2\sqrt{x}}$$

16      **(a)**

$$\text{Let } y = \lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$$

$$\Rightarrow \log y = \lim_{x \rightarrow \frac{\pi}{2}} \tan x \log \sin x$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\log \sin x}{\cot x}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{1}{\sin x} \cdot \cos x}{-\operatorname{cosec}^2 x}$$

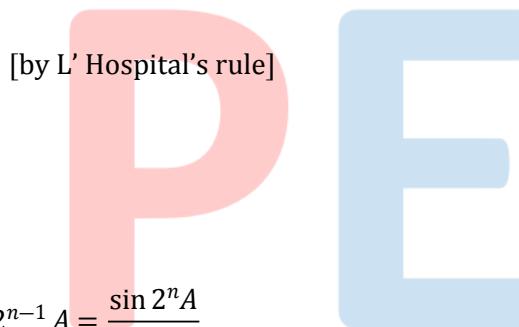
$$= 0$$

$$\Rightarrow y = e^0 = 1$$

17      **(c)**

We know that

$$\cos A \cos 2A \cos 4A \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$$



$$\lim_{n \rightarrow \infty} \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{4}\right) \dots \cos\left(\frac{x}{2^{n-1}}\right) \cos\left(\frac{x}{2^n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{\sin x}{2^n \sin(x/2^n)} \left[ \text{put } A = \frac{x}{2^n} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{\sin x}{x} \cdot \frac{(x/2^n)}{\sin(x/2^n)}$$

$$= \frac{\sin x}{x}$$

18      **(d)**

$$\lim_{x \rightarrow 1} \frac{\sin(e^{x-1} - 1)}{\log x}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(e^h - 1)}{\log(1+h)}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(e^h - 1)}{(e^h - 1)} \times \frac{(e^h - 1)}{\log(1+h)}$$

$$= 1 \times \lim_{h \rightarrow 0} \frac{\left(h + \frac{h^2}{2!} + \dots\right)}{\left(h - \frac{h^2}{2!} + \dots\infty\right)} = 1 \times 1 = 1$$

19      (c)

We have,

$$l = \lim_{x \rightarrow -2} \frac{\tan \pi x}{x+2} + \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2}\right)^x$$

$$\Rightarrow l = \lim_{x \rightarrow -2} \frac{\tan(2\pi + \pi x)}{x+2} + \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2}\right)^x$$

$$\Rightarrow l = \pi \lim_{x \rightarrow -2} \frac{\tan \pi (x+2)}{\pi (x+2)} + e^{\lim_{x \rightarrow \infty} \frac{x}{x^2}} = \pi + e^0 = \pi + 1$$

20      (d)

$$\text{RHL} = \lim_{h \rightarrow 0} f(1+h)$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{1 - \cos 2h}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2} \sin h}{h} = \sqrt{2}$$

$$\text{LHL} = \lim_{h \rightarrow 0} f(1-h) \lim_{h \rightarrow 0} \frac{\sqrt{1 - \cos(-2h)}}{h}$$

$$= \lim_{h \rightarrow 0} \sqrt{2} \frac{\sin h}{-h} = -\sqrt{2}$$

Here, LHL  $\neq$  RHL

So, limit does not exist.



#### ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	A	A	C	C	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	D	A	C	A	C	D	C	D