DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :2

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$$\lim_{x\to 0}\frac{1+\sin x-\cos x+\log(1-x)}{x^3}$$

$$=\lim_{x\to 0}\frac{1+\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots\right)-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\dots\right)+\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}\right)}{x^3}$$

$$=\lim_{x\to 0}\left(-\frac{1}{3!}-\frac{1}{3}\right)+x^2\left(\frac{1}{5!}-\frac{1}{5}\right)+\dots=-\frac{1}{6}-\frac{1}{3}=-\frac{1}{2}$$

2 **(b)**

$$\lim_{x\to 0}\left\{\frac{1+\tan x}{1+\sin x}\right\}^{\operatorname{cosec} x}$$

$$=\lim_{x\to 0}\frac{\left[\left(1+\frac{\sin x}{\cos x}\right)^{\frac{\cos x}{\sin x}}\right]^{1/\cos x}}{(1+\sin x)^{1/\sin x}}$$

$$=\frac{e^{\lim_{x\to 0}\frac{1}{\cos x}}}{e}$$

$$=\frac{e}{e}=1$$

3 **(b)**

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{4\sin x\cos x+\cos x}{4\sin x\cos x-3\cos x}$$

[by L'Hospital's rule]

$$=\lim_{x\to\frac{\pi}{6}}\frac{\cos x\,(4\sin x+1)}{\cos x\,(4\sin x-3)}$$

$$= \frac{4\sin\frac{\pi}{6}+1}{4\sin\frac{\pi}{6}-3} = -3$$

4 **(a)**

$$\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2} = \lim_{x\to 0}\left(1+\frac{2x^2}{1+3x^2}\right)^{1/x^2}$$

$$= e^{\lim_{x\to 0}\frac{1}{x^2}\left(\frac{2x^2}{1+3x^2}\right)} = e^2$$

5 **(a)**

We have,

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} x = 0$ and, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0} x^2 = 0$

Hence $\lim_{x\to 0} f(x) = 0$

6 **(a)**

If $x \in Q$, then $n!\pi\, x$ will be an integral multiple of π for large values of n. Therefore, $\cos(n!\pi x)$ will be either 1 or -1 and so $\cos^{2\,m}(n!\pi x) = 1$

$$\therefore \lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\,\pi\,x)\,] = 1+1 = 2$$

If $x \notin Q$, $n!\pi\, x$ will not be an integral multiple of π and so $\cos(n!\pi x)$ will lie between -1 and 1

Thus, $\lim_{m\to\infty}\cos^{2m}(n!\pi x) = 0$

$$\Rightarrow \lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\pi x)\,] = 1+0 = 1$$

7 **(c)**

We have,

$\lim_{x\to 1}(1+\cos\pi\, x)\cot^2\pi\, x$

$$= \lim_{x\to 1}\frac{(1+\cos\pi x)(\cos^2\pi x)}{(1-\cos^2\pi x\,)} = \lim_{x\to 1}\frac{\cos^2\pi x}{1-\cos\pi x} = \frac{1}{2}$$

8 **(c)**

$$\lim_{x\to 0}\frac{(e^{kx}-1)\sin kx}{x^2} = 4$$

$$\Rightarrow \quad \lim_{x\to 0}\frac{e^{kx}-1}{kx}\times k\times\frac{\sin kx}{kx}\times k = 4$$

$$\Rightarrow \quad k^2 = 4 \Rightarrow \quad k = \pm 2$$

9 **(b)**

$$\lim_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x} = \lim_{x\to 0}\frac{\left(\frac{x^3}{1}-\frac{(x^3)^2}{2}+\frac{(x^3)^3}{3}-\ldots\infty\right)}{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\infty\right)^3}$$

$$= \lim_{x\to 0} \frac{\left(1 - \frac{x^3}{2} + \frac{(x^3)^2}{3} - \dots\infty\right)}{\left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots\infty\right)^3} = 1$$

10 **(c)**

$l_1 = \lim_{x\to 2^+} (x + [x])$

$= \lim_{h\to 0} 2 + h + [2 + h] = 4$

$l_2 = \lim_{x\to 2^-} (2x - [x])$

$= \lim_{h\to 0} \{2(2 - h) - [2 - h]\}$

$= \lim_{h\to 0} \{2(2 - h) - 1\} = 3$

$l_3 = \lim_{x\to \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}} = \lim_{x\to \frac{\pi}{2}} - \sin x = -1$

[by L'Hospital's rule]

Thus, $l_3 < l_2 < l_1$

11 **(b)**

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin(1 + [x])}{[x]}$

$= \frac{\sin(1 - 1)}{-1} = 0$

12 **(a)**

$\lim_{x\to 0} \frac{(1 - cos\ 2x)\sin 5x}{x^2 \sin 3x}$

$= \lim_{x\to 0} \frac{1 - cos\ 2x}{x^2} \lim_{x\to 0} \frac{\sin 5x}{\sin 3x}$

$= \lim_{x\to 0} \frac{2\sin^2 x}{x^2} \lim_{x\to 0} \frac{\sin 5x}{5x} \cdot \frac{3x}{\sin 3x} \cdot \frac{5x}{3x}$

$= \lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)^2 \cdot \frac{5}{3} = \frac{10}{3}$

13 **(d)**

We have,

$\Rightarrow \lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{\sin(e^{x-2} - 1)}{\log(x - 1)}$

$\Rightarrow \lim_{x\to 2} f(x) = \lim_{x\to 2} \left\{\frac{\sin(e^{x-2} - 1)}{e^{x-2} - 1} \cdot \frac{e^{x-2} - 1}{x - 2} \cdot \frac{x - 2}{\log(1 + (x - 2))}\right\}$

$\Rightarrow \lim_{x\to 2} f(x) = 1 \times 1 \times 1 = 1$

14 **(a)**

We have,

$$\lim_{n\to\infty}\frac{S_{n+1}-S_n}{\sqrt{\sum_{k=1}^{n}k}} = \lim_{n\to\infty}\frac{a_{n+1}}{\sqrt{\frac{n(n+1)}{2}}} = 0$$

15 **(c)**

$$\lim_{h\to 0}\frac{\sin\sqrt{x+h}-\sin\sqrt{x}}{h}$$

Applying L'Hospital's rule,

$$= \lim_{h\to 0}\frac{\frac{\cos\sqrt{x+h}}{2\sqrt{x+h}}}{1} = \frac{\cos\sqrt{x}}{2\sqrt{x}}$$

16 **(a)**

Let $y = \lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$

$\Rightarrow \quad \log y = \lim_{x\to\frac{\pi}{2}}\tan x\log\sin x$

$$= \lim_{x\to\frac{\pi}{2}}\frac{\log\sin x}{\cot x}$$

$= \lim_{x\to\frac{\pi}{2}}\frac{\frac{1}{\sin x}.\cos x}{-\mathrm{cosec}^2 x}$ [by L' Hospital's rule]

$= 0$

$\Rightarrow \quad y = e^0 = 1$

17 **(c)**

We know that

$$\cos A\cos 2A\cos 4A\ldots\cos 2^{n-1}A = \frac{\sin 2^n A}{2^n\sin A}$$

$$\lim_{n\to\infty}\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\ldots\cos\left(\frac{x}{2^{n-1}}\right)\cos\left(\frac{x}{2^n}\right)$$

$$= \lim_{n\to\infty}\frac{\sin x}{2^n\sin(x/2^n)}\left[\text{put } A = \frac{x}{2^n}\right]$$

$$= \lim_{n\to\infty}\frac{\sin x}{x}.\frac{(x/2^n)}{\sin(x/2^n)}$$

$$= \frac{\sin x}{x}$$

18 **(d)**

$$\lim_{x\to 1}\frac{\sin(e^{x-1}-1)}{\log x}$$

$$= \lim_{h\to 0}\frac{\sin(e^h-1)}{\log(1+h)}$$

$$= \lim_{h\to 0}\frac{\sin(e^h-1)}{(e^h-1)}\times\frac{(e^h-1)}{\log(1+h)}$$

$$= 1 \times \lim_{h\to 0} \frac{\left(h + \frac{h^2}{2!} + ...\right)}{\left(h - \frac{h^2}{2!} + ...\infty\right)} = 1 \times 1 = 1$$

19 **(c)**

We have,

$$l = \lim_{x\to -2} \frac{\tan \pi x}{x+2} + \lim_{x\to\infty} \left(1 + \frac{1}{x^2}\right)^x$$

$$\Rightarrow l = \lim_{x\to -2} \frac{\tan(2\pi + \pi x)}{x+2} + \lim_{x\to\infty} \left(1 + \frac{1}{x^2}\right)^x$$

$$\Rightarrow l = \pi \lim_{x\to -2} \frac{\tan \pi (x+2)}{\pi (x+2)} + e^{\lim_{x\to\infty} \frac{x}{x^2}} = \pi + e^0 = \pi + 1$$

20 **(d)**

$$\text{RHL} = \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0} \frac{\sqrt{1 - \cos 2h}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2} \sin h}{h} = \sqrt{2}$$

$$\text{LHL} = \lim_{h\to 0} f(1-h) \lim_{h\to 0} \frac{\sqrt{1 - \cos(-2h)}}{h}$$

$$= \lim_{h\to 0} \sqrt{2} \frac{\sin h}{-h} = -\sqrt{2}$$

Here, LHL $\neq$ RHL

So, limit does not exist.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	A	A	C	C	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	D	A	C	A	C	D	C	D