DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :10

Topic :-LIMITS & DERIVATIVES

1 **(a)**

$$\left(\lim_{x\to\infty}\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\times\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

$$=\lim_{y\to 0}\frac{(\sqrt{1+\sqrt{y}})/\sqrt{y}}{\sqrt{\frac{1}{y}+\frac{\sqrt{1+\sqrt{y}}}{\sqrt{y}}}+\sqrt{\frac{1}{y}}} \qquad \left[\text{put } x=\frac{1}{y}\right]$$

$$=\lim_{y\to 0}\frac{\sqrt{1+\sqrt{y}}}{\sqrt{1+\sqrt{y(1+\sqrt{y})}}+1}=\frac{1}{2}$$

2 **(d)**

We have,

$$\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}=\lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{x+4}=e^{\lim_{x\to\infty}\frac{5(x+4)}{x+1}}=e^{5}$$

3 **(c)**

We have,

$$\lim_{x\to a}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$$

$$=2\lim_{x\to\alpha}\frac{\sin^2\left\{\frac{(ax^2+bx+c)}{2}\right\}}{(x-\alpha)^2}$$

$$=2\lim_{x\to\alpha}\frac{\sin^2\left\{\frac{a(x-\alpha)(x-\beta)}{2}\right\}}{(x-\alpha)^2} \qquad \left[\begin{array}{c}\because \alpha,\beta \text{ are roots of}\\ ax^2+bx+c=0\\ \therefore ax^2+bx+c\\ =a(x-\alpha)(x-\beta)\end{array}\right]$$

$$=2\lim_{x\to\alpha}\left[\frac{\sin\left\{\frac{a(x-\alpha)(x-\beta)}{2}\right\}}{\frac{a(x-\alpha)(x-\beta)}{2}}\right]^2\times\frac{a^2}{4}(x-\beta)^2$$

$= 2(1)^2 \times \frac{a^2}{4}(\alpha-\beta)^2 = \frac{a^2}{2}(\alpha-\beta)^2$

4 **(a)**

$$\lim_{x\to 0}(1-ax)^{1/x} = \lim_{x\to 0}\left[(1-ax)^{-\frac{1}{ax}}\right]^{-a} = e^{-a}$$

5 **(c)**

$$\lim_{n\to\infty}\frac{1-(10)^n}{1+(10)^{n+1}}$$

$$= \lim_{n\to\infty}\frac{10^n}{10^{n+1}}\left(\frac{\frac{1}{10^n}-1}{\frac{1}{10^{n+1}}+1}\right)$$

$\Rightarrow \quad -\frac{\alpha}{10} = \frac{1}{10}\left(\frac{0-1}{0+1}\right) = -\frac{1}{10}$ [given]

$\Rightarrow \quad \alpha = 1$

6 **(d)**

We have,

$$\lim_{n\to\infty}\frac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt[4]{n^3+n}-\sqrt[4]{n}}$$

$$= \lim_{n\to\infty}\frac{n\left\{\sqrt{1+\frac{1}{n^2}}+\frac{1}{\sqrt{n}}\right\}}{n^{3/4}\left\{\sqrt[4]{1+\frac{1}{n^2}}-\frac{1}{\sqrt{n}}\right\}}$$

$$= \lim_{n\to\infty} n^{1/4}\frac{\left\{\sqrt{1+\frac{1}{n^2}}+\frac{1}{\sqrt{n}}\right\}}{\left\{\sqrt[4]{1+\frac{1}{n^2}}-\frac{1}{\sqrt{n}}\right\}} = \infty \times 2 = \infty$$

7 **(d)**

$\text{LHL} = \lim_{x\to 2^-}\frac{5}{\sqrt{2}-\sqrt{x}}$

$$= \lim_{h\to 0}\frac{5}{(\sqrt{2}-\sqrt{2-h})}\times\frac{(\sqrt{2}+\sqrt{2-h})}{(\sqrt{2}+\sqrt{2-h})}$$

$$= \lim_{h\to 0}\frac{5(\sqrt{2}+\sqrt{2-h})}{2-2+h} = \infty$$

$\text{RHL} = \lim_{x\to 2^+}\frac{5}{\sqrt{2}-\sqrt{x}}$

$$= \lim_{h\to 0}\frac{5}{(\sqrt{2}-\sqrt{2+h})}\times\frac{(\sqrt{2}+\sqrt{2+h})}{(\sqrt{2}+\sqrt{2+h})}$$

$$= \lim_{h\to 0}\frac{5(\sqrt{2}+\sqrt{2+h})}{2-2-h} = -\infty$$

$\therefore$ LHL $\neq$ RHL

Hence, limit does not exist.

8 **(a)**

$$\lim_{n\to\infty}\frac{2^{-n}(n^2+5n+6)}{(n+4)(n+5)} = \lim_{n\to\infty}\frac{n^2(1+\frac{5}{n}+\frac{6}{n^2})}{2^n.n^2\left(1+\frac{4}{n}\right)(1+\frac{5}{n})}$$

$= 0$

9 **(b)**

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} x.\sin\frac{1}{x}$$

$= 0 \times$ finite term $= 0$

10 **(b)**

$$\lim_{x\to a}\frac{\log(x-a)}{\log(e^x-e^a)} = \lim_{x\to a}\frac{\frac{1}{x-a}}{\frac{e^x}{e^x-e^a}}$$

[by L' Hospital's rule]

$$= \lim_{x\to a}\frac{e^x-e^a}{e^x(x-a)}$$

$$= \lim_{x\to a}\frac{e^x}{e^x(x-a)+e^x}$$

[by L' Hospital's rule]

$$= \frac{e^a}{0+e^a} = 1$$

11 **(c)**

We have,

$$\lim_{x\to\infty}\left(1-\frac{2}{x+1}\right)^x = e^{\lim_{x\to\infty}\frac{-2x}{x+1}} = e^{-2}$$

12 **(d)**

$$\lim_{x\to 0}\left[\frac{3^x+3^{-x}-2}{x^2}\right]$$

$$= \lim_{x\to 0}\frac{\begin{bmatrix}1+x\log 3+\frac{x^2}{2!}(\log 3)^2+\ldots+\\ 1-x\log 3+\frac{x^2}{2!}(\log 3)^2-\ldots-2\end{bmatrix}}{x^2} = (\log 3)^2$$

13 **(d)**

We have,

$$\lim_{x\to 0}\left(\frac{e^x+e^{-x}-2}{x^2}\right)^{\frac{1}{x^2}}$$

$$= \lim_{x\to 0}\left\{\frac{2\left(\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\dots\right)}{x^2}\right\}^{\frac{1}{x^2}}$$

$$= \lim_{x\to 0}\left(1+2\frac{x^2}{4!}+2\frac{x^4}{6!}+\dots\right)^{\frac{1}{x^2}} = e^{\lim_{x\to 0}\left(\frac{x^2}{4!}+\frac{2x^4}{6!}+\dots\right)\times\frac{1}{x^2}} = e^{1/12}$$

14 **(b)**

Let $y = \lim_{x\to\infty}\left(\frac{\pi}{2} - \tan^{-1}x\right)^{1/x}$

$\Rightarrow \quad \log y = \lim_{x\to\infty}\frac{1}{x}\log\left(\frac{\pi}{2} - \tan^{-1}x\right)$ $\left[\frac{\infty}{\infty}\text{ form}\right]$

$= \lim_{x\to\infty}\frac{\left(-\frac{1}{1+x^2}\right)}{\left(\frac{\pi}{2} - \tan^{-1}x\right)}$ [using L'Hospital's rule]

$$= \lim_{x\to\infty}\frac{\frac{2x}{(1+x^2)^2}}{-\left(\frac{1}{1+x^2}\right)} = \lim_{x\to\infty}\frac{-2x}{1+x^2}$$

[using L'Hospital's rule]

$\Rightarrow \quad \log y = 0 \quad \Rightarrow \quad y = 1$

15 **(d)**

We have,

$$\lim_{x\to 1}\left\{\frac{x^3+2x^2+x+1}{x^2+2x+3}\right\}^{\frac{1-\cos(x-1)}{(x-1)^2}}$$

$$= \lim_{x\to 1}\left\{\frac{x^3+2x^2+x+1}{x^2+2x+3}\right\}^{\frac{2\sin^2(x-1)/2}{(x-1)^2}} = \left(\frac{5}{6}\right)^{\frac{2}{4}\left\{\frac{\sin\left(\frac{x-1}{2}\right)}{\frac{x-1}{2}}\right\}^2} = \sqrt{\frac{5}{6}}$$

16 **(a)**

Here, $\lim_{x\to -3} x^2 + 2x - 3 = 0$

$\therefore \lim_{x\to -3} 3x^2 + ax + a - 7$ must be zero, in order to limit exist.

$\Rightarrow \quad 3(-3)^2 + a(-3) + a - 7 = 0$

$\Rightarrow \quad 27 - 2a - 7 = 0$

$\Rightarrow \quad 2a = 20$

$\Rightarrow \quad a = 10$

17 **(b)**

$$\lim_{x\to\infty}\left(1-\frac{4}{x-1}\right)^{3x-1} = \lim_{x\to\infty}\left[\left(1-\frac{4}{x-1}\right)^{\frac{-(x-1)}{4}}\right]^{-4\left(\frac{3x-1}{x-1}\right)}$$

$$= e^{-4\lim_{x\to\infty}(3-1/x)(1-1/x)} = e^{-12}$$

18 **(b)**

We have,

$$\lim_{n\to\infty}\left\{\frac{1}{1-n^2}+\frac{2}{1-n^2}+\frac{3}{1-n^2}+\ldots+\frac{n}{1-n^2}\right\}$$

$$=\lim_{n\to\infty}\frac{1+2+3+\ldots+n}{1-n^2}=\lim_{n\to\infty}\frac{n}{2(1-n)}=-\frac{1}{2}$$

20 **(c)**

$$\lim_{n\to\infty}\frac{3.2^{n+1}-4.5^{n+1}}{5.2^n+7.5^n}$$

$$=\lim_{n\to\infty}\frac{5^n\left(6.\left(\frac{2}{5}\right)^n-20\right)}{5^n\left(5.\left(\frac{2}{5}\right)^n+7\right)}=-\frac{20}{7}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	C	A	C	D	D	A	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	D	B	D	A	B	B	C	C