DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :1

Topic :-LIMITS & DERIVATIVES

1 **(c)**

We have, $\lim_{\theta \to \frac{\pi}{2}} \frac{\frac{\pi}{2}-\theta}{\cot\theta} = \lim_{\theta \to \frac{\pi}{2}} \frac{-1}{-\operatorname{cosec}^2\theta}$

$= \lim_{\theta \to \frac{\pi}{2}} \sin^2\theta = 1$

2 **(a)**

$\lim_{x\to 0}\frac{a^x-b^x}{e^x-1} = \lim_{x\to 0}\frac{a^x-b^x}{x}\cdot\frac{x}{a^x-1}$

$= \left[\lim_{x\to 0}\left(\frac{a^x-1}{x}\right) - \lim_{x\to 0}\left(\frac{b^x-1}{x}\right)\right].\lim_{x\to 0}\frac{x}{e^x-1}$

$= (\log_e a - \log_e b).\lim_{x\to 0}\frac{1}{\frac{e^x-1}{x}}$

$= \log_e\left(\frac{a}{b}\right)$

3 **(b)**

$\lim_{x\to -\infty}\frac{2x-1}{\sqrt{x^2+2x+1}} = \lim_{y\to\infty}\frac{-2x-\frac{1}{y}}{\sqrt{1-\frac{2}{y}+\frac{1}{y^2}}}$

[put $x=-y \ \therefore \ x\to-\infty$ *ie*, $y\to\infty$]

$= -\frac{2}{1} = -2$

4 **(a)**

$\lim_{x\to 2} = \frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$

$= \lim_{x\to 2}\frac{(1+\sqrt{2+x}-3)}{(x-2)(\sqrt{1+\sqrt{2+x}}+\sqrt{3})}$

$= \lim_{x\to 2}\frac{x-2}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)(\sqrt{2+x}+2)}$

$= \lim_{x\to 2}\frac{1}{(\sqrt{1+\sqrt{2+x}}+\sqrt{3})(\sqrt{2+x}+2)}$

$$= \frac{1}{(\sqrt{1+2}+\sqrt{3})(\sqrt{2+2}+2)} = \frac{1}{8\sqrt{3}}$$

5 **(d)**

We have,

$$\lim_{x\to 0}\left\{\frac{1}{x\sqrt[3]{8+x}} - \frac{1}{2x}\right\} \quad [\infty - \infty \text{ form}]$$

$$= \lim_{x\to 0}\frac{1}{2x}\left\{\left(1+\frac{x}{8}\right)^{-1/3} - 1\right\}$$

$$= \frac{1}{16}\lim_{x\to 0}\frac{\left(1+\frac{x}{8}\right)^{-1/3} - 1^{-1/3}}{\left(1+\frac{x}{8}\right) - 1}$$

$$= \frac{1}{16}\lim_{y\to 1}\frac{y^{-1/3} - 1^{-1/3}}{y-10}, \text{ where } y = 1+\frac{x}{8}$$

$$= \frac{1}{16}\times\frac{-1}{3}(1)^{-1/3-1} = -\frac{1}{48}$$

6 **(a)**

We have,

$$\lim_{x\to 0}\frac{a^x - b^x}{x}$$

$$= \lim_{x\to 0}\left\{\left(\frac{a^x-1}{x}\right) - \left(\frac{b^x-1}{x}\right)\right\} = \log(a) - \log(b) = \log\left(\frac{a}{b}\right)$$

7 **(b)**

We have,

$$\lim_{x\to 1}\frac{\sum_{r=1}^{n} x^r - n}{x-1} = \lim_{x\to 1}\frac{x-1}{x-1} + \frac{x^2-1^2}{x-1} + \frac{x^3-1^3}{x-1} + \dots + \frac{x^n-1^n}{x-1}$$

$$= 1+2+3+\dots+n = \frac{n(n+1)}{2}$$

8 **(b)**

We have,

$$\lim_{x\to 1}(\log_4 5x)^{\log_x 5} = \lim_{x\to 1}(\log_5 5 + \log_5 x)^{\log_x 5}$$

$$= \lim_{x\to 1}(1+\log_5 x)^{\frac{1}{\log_5 x}} = e^{\lim_{x\to 1}\log_5 x \cdot \frac{1}{\log_x 5}} = e^1 = e$$

9 **(a)**

We have,

$$\lim_{x\to 0}\frac{e^{\tan x} - e^x}{\tan x - x} = \lim_{x\to 0}\frac{e^x\{e^{\tan x - x} - 1\}}{\tan x - x}$$

$$= \lim_{x\to 0} e^x \times \lim_{x\to 0}\frac{e^{\tan x - x} - 1}{\tan x - x} e^0 \times 1 = 1$$

10 **(a)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$$

$$=\lim_{x\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x = e^{\lim_{x\to\infty}\frac{(2x-1)x}{x^2-4x+2}} = e^2$$

11 **(c)**

$$\lim_{x\to 0}\frac{\log(x+a)-\log a}{x}+k\lim_{x\to e}\frac{\log x-1}{x-e}=1$$

Using L' Hospital's rule

$$\lim_{x\to 0}\frac{\frac{1}{x+a}}{1}+k\lim_{x\to 0}\frac{\frac{1}{x}}{1}=1$$

$$\Rightarrow \quad \frac{1}{a}+\frac{k}{e}=1$$

$$\Rightarrow \quad k=e\left(1-\frac{1}{a}\right)$$

12 **(b)**

We have,

$$\lim_{x\to\infty}\frac{\sin x}{x} = \lim_{y\to 0} y\sin\left(\frac{1}{y}\right) = 0$$

13 **(a)**

$$\lim_{x\to 0} x\log\sin x = \lim_{x\to 0}\frac{\log\sin x}{1/x}\left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to 0}\frac{\frac{1}{\sin x}\cos x}{\frac{-1}{x^2}} = \lim_{x\to 0} -\frac{x^2}{\tan x} \qquad \text{[by L'Hospital's rule]}$$

$$=\lim_{x\to 0}\frac{-2x}{\sec^2 x} \qquad \text{[by L'Hospital's rule]}$$

$$=0$$

14 **(c)**

$$\lim_{x\to 0}\frac{d}{dx}\int\left(\frac{1-\cos x}{x^2}\right)dx = \lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x/2}{4.x^2/4}$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\frac{\sin x/2}{x/2}\right)^2=\frac{1}{2}$$

15 **(a)**

We have,

$$\lim_{x\to 0}\frac{\left(\int_y^a e^{\sin^2 t}dt-\int_{x+y}^a e^{\sin^2 t}\,dt\right)}{x}$$

$$=\lim_{x\to 0}\frac{\left(\int_y^a e^{\sin^2 t}dt+\int_a^{x+y} e^{\sin^2 t}\,dt\right)}{x}$$

$$= \lim_{x\to 0} \frac{\int_y^{x+y} e^{\sin^2 t}\, dt}{x}$$

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(x+y)e^{\sin^2(x+y)} - 0}{1} \quad \text{[Using L' Hospital's Rule]}$$

$$= \lim_{x\to 0} 1.e^{\sin^2(x+y)} = e^{\sin^2 y}$$

16 **(c)**

We have,

$$\lim_{x\to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2} \left[\text{Form } \frac{0}{0}\right]$$

$$= \lim_{x\to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x} \quad \text{[By L' Hospital's Rule]}$$

$$= \lim_{x\to 0} \frac{2f'(x) - 3f'(2x) + 2f'(4x)}{x} \left[\text{Form } \frac{0}{0}\right]$$

$$= \lim_{x\to 0} \frac{2f''(x) - 6f''(2x) + 8f''(4x)}{1} \quad \text{[By L' Hospital's Rule]}$$

$$= f''(0) - 6f''(0) + 8f''(0) = 3f''(0) = 3 \times 4 = 12$$

17 **(d)**

Given, $\lim_{x\to 0} \frac{\{(a-n)nx - \tan x\} \sin nx}{x^2} = 0$

$$\Rightarrow \quad \lim_{x\to 0} \left((a-n)n - \frac{\tan x}{x}\right) \cdot \frac{\sin nx}{x} = 0$$

$$\Rightarrow \quad [\{a-n\}n - 1]n = 0$$

$$\Rightarrow \quad a = n + \frac{1}{n}$$

18 **(c)**

We have,

$$\lim_{x\to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{x\left\{1 + a\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots\right)\right\} - b\left\{x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots\right\}}{x^3} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{(1+a-b) + x^2\left(\frac{b}{3!} - \frac{a}{2!}\right) + x^4\left(\frac{a}{4!} - \frac{b}{5!}\right) + \dots}{x^2} = 1 \qquad \dots(i)$$

If $1 + a - b \neq 0$, then LHS $\to \infty$ as $x \to 0$ which RHS $= 1$

$\therefore 1 + a - b = 0$

From (i), we have

$$\lim_{x\to 0} \frac{x^2\left(\frac{b}{3!} - \frac{a}{2!}\right) + x^4\left(\frac{a}{4!} - \frac{b}{5!}\right) + \dots}{x^2} = 1$$

$$\therefore \frac{b}{3!} - \frac{a}{2!} = 1 \Rightarrow b - 3a = 6$$

Solving $1 + a - b = 0$ and $b - 3a = 6$, we get $a = -\frac{5}{2}, b = -\frac{3}{2}$

19 **(a)**

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x = \lim_{x\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x$$

$$= e^{\lim_{x\to\infty}\left(\frac{x(2x-1)}{x^2-4x+2}\right)} = e^2$$

20 **(d)**

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{x^2}$$

$$= \lim_{x\to 0}\frac{2\sin^2 x}{x^2}$$

$$= 2\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)^2 = 2$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	B	A	D	A	B	B	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	A	C	A	C	D	C	A	D