DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 9

Topic :- CONIC SECTION

1 **(b)**

Equation of circle is

$x^2 + y^2 = 25$ ..(i)

Polar equation of a circle with respect to the point $(1, a)$ and $(b, 2)$ is

$x + ay = 25$ ...(ii)

and $bx + 2y = 25$ ...(iii)

since, $(1, a)$ and $(b, 2)$ are the conjugate point of a circle, therefore point $(1, a)$ satisfy the Eq. (iii), we get

$b + 2a = 25 \Rightarrow 2b + 4a = 50$

3 **(a)**

Given, $\frac{x^2}{16} - \frac{y^2}{9} = 1$

We know that the difference of focal distances of any point of the hyperbola is equal to major axis

$\therefore$ Required distance $= 2a = 2 \times 4 = 8$

4 **(a)**

We have,

$y^2 - 6y + 4x + 9 = 0 \Rightarrow (y-3)^2 = -4(x-0)$

The coordinate of the focus of this parabola are $(-1,3)$ and the equation of the directrix is

$$x - 1 = 0$$

We know that the chord of contact of tangents drawn from any point on the directrix always passes through the focus.

Hence, the required point is $(-1,3)$

ALITER Let $P(1,\lambda)$ be an arbitrary point on $x - 1 = 0$. The chord of contact of tangents drawn from $P(1,\lambda)$ to the parabola $y^2 - 6y + 4x + 9 = 0$ is

$\lambda y - 3(y + \lambda) + 2(x + 1) + 9 = 0$

$\Rightarrow (2x - 3y + 11) + \lambda(y - 3) = 0$

Clearly, it represents a family of lines passing through the intersection of the lines

$2x - 3y + 11 = 0$ and $y - 3 = 0$ i.e. $(-1,3)$

5 **(b)**

Equation of circle whose centre is at (2, 2) and radius r is

$(x-2)^2+(y-2)^2=r^2$ ...(i)
This circle passes through $(4, 5)$, then
$(4-2)^2+(5-2)^2=r^2$
$\Rightarrow \quad r^2=13$
On putting this values in Eq. (i), we get
$(x-2)^2+(y-2)^2-13=0$
$\Rightarrow x^2+y^2-4x-4y-5=0$

6 **(d)**

The equations of asymptotes of $x^2-y^2=8$ are given by
$x^2-y^2=0$ or, $x+y=0$ and $x-y=0$
Let (x_1, y_1) be a point on the hyperbola $x^2-y^2=8$. Then, product of perpendicular from (x_1, y_1) on the asymptotes

$$=\left|\frac{x_1-y_1}{\sqrt{2}}\right|\left|\frac{x_1+y_1}{\sqrt{2}}\right|$$

$$=\left|\frac{x_1^2-y_1^2}{2}\right|=\left|\frac{8}{2}\right|=4 \quad [\because x_1^2-y_1^2=8]$$

7 **(d)**

Given foci of ellipse are $(0, -4)$ and $(0, 4)$

$\therefore$ Focal distance is $2be=8$

$be=4$ ...(i)

Also, since equation of directrices are $\frac{b}{e}=\pm 9$ ...(ii)

From, Eqs. (i) and (ii), we get

$b^2=36 \Rightarrow b=6$ and $e=\frac{2}{3}$

$$\because \; a^2=b^2(1-e^2)=36\left(1-\frac{4}{9}\right)=20$$

$$\therefore \; \frac{x^2}{20}+\frac{y^2}{36}=1$$

$\Rightarrow 9x^2+5y^2=180$

8 **(a)**

The equation of tangent is

$$\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$$

$\therefore$ Coordinates of A and B are $(a\cos\theta, 0)$ and $(0, -b\cot\theta)$respectively.
Let coordinates of P are(h, k).
$\therefore \; h=a\cos\theta, \; k=-b\cot\theta$

$\Rightarrow \dfrac{k}{h} = -\dfrac{b}{a \sin \theta}$

$\Rightarrow \sin \theta = \; -\dfrac{bh}{ak}$

$\Rightarrow \dfrac{b^2h^2}{a^2k^2} = \; \sin^2 \theta$

$\Rightarrow \; \dfrac{b^2h^2}{a^2k^2} + \dfrac{h^2}{a^2} = 1$

$\Rightarrow \; \dfrac{b^2}{k^2} + 1 = \dfrac{a^2}{h^2}$

$\Rightarrow \dfrac{a^2}{h^2} - \dfrac{b^2}{k^2} = 1$

Hence, the locus of P is $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

9 **(d)**

The coordinates of P are $(1, 0)$. A gerneral point Q on $y^2 = 8x$ is $(2t^2, 4t)$. Let mid point of PQ is (h, k)

$\therefore 2h = 2t^2 + 1$ and $2k = 4t \Rightarrow t = \frac{k}{2}$

$\therefore 2h = \dfrac{2k^2}{4} + 1 \Rightarrow 4h = k^2 + 2$

Hence, the locus of (h, k) is $y^2 - 4x + 2 = 0$

11 **(a)**

The equation of the ellipse is

$\dfrac{(x+3)^2}{2^2} + \dfrac{(y-5)^2}{(\sqrt{3})^2} = 1$

$\Rightarrow 3x^2 + 4y^2 + 18x - 40y + 115 = 0$

12 **(c)**

Let (h,k) be the pole of the line $9\,x + y - 28 = 0$ with respect to the circle $x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0$.

Then, the equation of the polar is

$hx + ky - \dfrac{3}{4}(x + h) + \dfrac{5}{4}(y + k) - \dfrac{7}{2} = 0$

$\Rightarrow x\left(h - \dfrac{3}{4}\right) + y\left(k + \dfrac{5}{4}\right) - \dfrac{3}{4}h + \dfrac{5}{4}k - \dfrac{7}{2} = 0$

$\Rightarrow x(4\,h - 3) + y(4\,k + 5) - 3\,h + 5\,k - 14 = 0$

This equation and $9\,x + y - 28 = 0$ represent the same line.

$\therefore \dfrac{4\,h - 3}{9} = \dfrac{4\,k + 5}{1} = \dfrac{-3\,h + 5\,k - 14}{-28} = \lambda$ (say)

$\Rightarrow h = \dfrac{3 + 9\,\lambda}{4}, k = \dfrac{\lambda - 5}{4}, -3\,h + 5\,k - 14 = -28\,\lambda$

$\Rightarrow -3\left(\frac{3+9\lambda}{4}\right)+5\left(\frac{\lambda-5}{4}\right)-14=-28\lambda$

$\Rightarrow -9-27\lambda+5\lambda-25-56=-112\lambda$

$\Rightarrow -22\lambda-90=-112\lambda$

$\Rightarrow 90\lambda=90 \ \Rightarrow \lambda=1$

Hence, the pole of the given line is $(3,-1)$

13 **(a)**

Let (h,k) is mid point of chord.

Then, its equation is $T=S_1$

$\therefore 3hx-2ky+2(x+h)-3(y+k)=3h^2-2k^2+4h-6k$

$x(3h+2)+y(-2k-3)=3h^2-2k^2+2h-3k$

Since, this line is parallel to $y=2x$

$\frac{3h+2}{2k+3}=2$

$\Rightarrow 3h-4k=4$

Thus, locus of point is $3x-4y=4$

14 **(b)**

If circle $x^2+y^2-10x-14y+24=0$ cuts an intercept on y-axis, then

Length of intercept $=2\sqrt{f^2-c}=2\sqrt{49-24}=10$

15 **(a)**

Given line $y=ax+\beta$ is a tangent to given hyperbola, if $\beta^2=a^2\alpha^2-b^2$

Hence, locus of (α,β) is $y^2=a^2x^2-b^2$, which represents a hyperbola

16 **(c)**

Let the points are (x_1,y_1),(x_2,y_2) and (x_3,y_3)

$\therefore y_1^2=4ax_1, y_2^2=4ax_2, y_3^2=4ax_3$

$\therefore$ Area of triangle whose vertices are (x_1,y_1),(x_2,y_2) and (x_3,y_3)

$$=\frac{1}{2}\left\|\begin{matrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{matrix}\right\|=\frac{1}{2}\left\|\begin{matrix}\frac{y_1^2}{4a} & y_1 & 1\\ \frac{y_2^2}{4a} & y_2 & 1\\ \frac{y_3^2}{4a} & y_3 & 1\end{matrix}\right\|=\frac{1}{8a}\left\|\begin{matrix}y_1^2 & y_1 & 1\\ y_2^2 & y_2 & 1\\ y_3^2 & y_3 & 1\end{matrix}\right\|$$

$\Rightarrow$ Area of triangle

$=\frac{1}{8a}(y_1-y_2)(y_2-y_3)(y_3-y_1)$

17 **(c)**

Let $y = mx + \frac{a}{m}$ be a tangent to $y^2 = 4ax$ cutting $y^2 = -4ax$ at P and Q. Let (h,k) be mid-point of PQ.
Then, equation of PQ is
$ky + 2a(x + h) = k^2 + 4ah$ [Using : $T = S'$]
or, $ky = -2ax + k^2 + 2ah$
But, equation of PQ is
$y = mx + \frac{a}{m}$
$\therefore m = -\frac{2a}{k}$ and $\frac{k^2 + 2ah}{k} = \frac{a}{m}$
$\Rightarrow -\frac{2a}{k}(k^2 + 2ah) = ak$
$\Rightarrow -2(k^2 + 2ah) = k^2 \Rightarrow 3k^2 + 4ah = 0$
Hence, the locus of (h,k) is $3y^2 + 4ax = 0$ or, $y^2 = -\frac{4a}{3}x$

18 **(b)**

Let $P(x_1,y_1)$ be a point on the hyperbola. Then the coordinates of N are $(x_1,0)$
The equation of the tangent at (x_1,y_1) is $\frac{x x_1}{a^2} - \frac{y y_1}{b^2} = 1$
This meets x-axis at $T\left(\frac{a^2}{x_1},0\right)$
$\therefore OT.ON = \frac{a^2}{x_1} \times x_1 = a^2$

20 **(a)**

The equation of circles whose radius is r and centres (2, 3) and (5, 6) is
$(x - 2)^2 + (y - 3)^2 = r^2$
And $(x - 5)^2 + (y - 6)^2 = r^2$
$\Rightarrow x^2 + y^2 - 4x - 6y + (-r^2 + 13) = 0$
And $x^2 + y^2 - 10x - 12y + (-r^2 + 61) = 0$
Since, circles cut orthogonally, then
$2g_1g_2 + 2f_1f_2 = c_1 + c_2$
$\Rightarrow 2(2)(5) + 2(3)(6) = 13 - r^2 + 61 - r^2$
$\Rightarrow 2r^2 = 18 \Rightarrow r = 3$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	A	A	B	D	D	A	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	A	B	A	C	C	B	B	A