DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 8

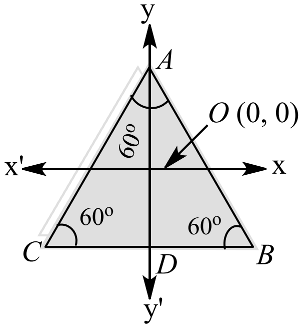
Topic :- CONIC SECTION

1 **(d)**

Centre of triangle is (0, 0)

Since, triangle is an equilateral, the centre of circumcircle is also (0, 0)

$AD = a$ (given)



$\therefore AC = BC = AB$

$= \dfrac{a}{\sin 60°} = \dfrac{2a}{\sqrt{3}}$

$\therefore$ Circumradius $= \frac{AC}{2 \sin B}$

$= \dfrac{2a}{2\sqrt{3}} \times \dfrac{2}{\sqrt{3}} = \dfrac{2a}{3}$ $\quad [\because B = 60°]$

$\therefore$ required equation of circumcircle is

$x^2 + y^2 = \dfrac{4a^2}{9}$

$\Rightarrow 9x^2 + 9y^2 = 4a^2$

2 **(a)**

The coordinates of end point of latusrectum are $(a, 2a)$ and $(a, -2a)$ ie, $(3, 6)$ and $(3, -6)$

The equation of directrix is $x = -3$

The equation of tangents from the above points are $6y = 6(x + 3)$ and $-6y = 6(x + 3)$

$\Rightarrow x - y + 3 = 0$ and $x + y + 3 = 0$

The intersection point is $(-3, 0)$

The equation of directrix of the parabola $y^2 = 12x$ is $x = -3$

$\Rightarrow$ Intersection point $(-3, 0)$ lies on the directrix

3 **(c)**

We have, $\frac{x^2}{25}+\frac{y^2}{9}=1$

The eccentricity of this ellipse is $\frac{4}{5}$. So, the coordinates of foci S and S' are $(4,0)$ and $(-4,0)$

$\therefore$ Area of rhombus $=\frac{1}{2}\times$ Product of diagonals

$\Rightarrow$ Area of rhombus $=\frac{1}{2}(BB'\times SS')$

$\Rightarrow$ Area of rhombus $=\frac{1}{2}\times 6\times 8$ sq. units $=24$ sq.units

4 **(d)**

Let the equation of ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\because\ b^2=a^2(1-e^2)$$

$$\therefore\ \frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1$$

5 **(d)**

Any point on the line $x-y-5=0$ will be of the form $(t,t-5)$ Chord of contact of this point with respect to curve $x^2+4y^2=4$ is

$tx+4(t-5)y-4=0$

$\Rightarrow(-20y-4)+t(x+4y)=0$

Which is a family of straight lines, each member of this family pass through point of intersection of straight lines $-20y-4=0$ and $x+4y=0$ which is $\left(\frac{4}{5},-\frac{1}{5}\right)$

6 **(a)**

The combined equation of the lines joining the origin (vertex) to the points of intersection of $y^2=4\,ax$ and $y=mx+c$ is

$$y^2=4\,ax\left(\frac{y-mx}{c}\right)\Rightarrow cy^2-4\,axy+4\,am\,x^2=0$$

This represents a pair of perpendicular lines

$\therefore c+4\,am=0\Rightarrow c=-4\,am$

7 **(b)**

Let the point on $x^2+y^2=a^2$ is $(a\cos\theta, a\sin\theta)$

Equation of chord of contact is

$ax\cos\theta+ay\sin\theta=b^2$

It touches circle $x^2+y^2=c^2$

$$\therefore\ \left|\frac{-b^2}{\sqrt{a^2\cos^2\theta+a^2\sin^2\theta}}\right|=c$$

$\Rightarrow b^2=ac$

$\therefore a,b,c$ are in GP

8 **(d)**

We have,

$y^2 = 4ax \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{2a}{y_1}$

$\therefore$ Length of the sub-normal at $P(x_1, y_1)$

$= y_1\left(\frac{dy}{dx}\right)_P = y_1 \times \frac{2a}{y_1} = 2a$

9 **(b)**

Let $P(h,k)$ be the point such that the ratio of the squares of the lengths of the tangents from P to the circles $x^2 + y^2 + 2x - 4y - 20 = 0$ and $x^2 + y^2 - 4x + 2y - 44 = 0$ is 2 :3.

Then,

$\frac{h^2 + k^2 + 2h - 4k - 20}{h^2 + k^2 + 4h + 2k - 44} = \frac{2}{3}$

$\Rightarrow h^2 + k^2 + 14h - 16k + 22 = 0$

So, the locus of $P(h,k)$ is $x^2 + y^2 + 14x - 16y + 22 = 0$

Clearly, it represents a circle having its centre at $(-7,8)$

10 **(a)**

The intersection of given line and circle is

$x^2 + y^2 - 2x = 0$

$\Rightarrow 2x(x-1) = 0$

$\Rightarrow x = 0, x = 1$

And $y = 0, 1$

Let coordinates of A are (0, 0) and coordinates of B are (1, 1).

$\therefore$ Equition of circle (AB as a diameter) is

$(x - x_1)(x - x_2) + (y_- y_1)(y - y_2) = 0$

$\Rightarrow (x-0)(x-1) + (y-0)(y-1) = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$

11 **(c)**

Equation of normal to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at

$(a\sec\theta, b\tan\theta)$ is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$

12 **(c)**

The equation of tangent to the given circle $2x^2 + 2y^2 - 2x - 5y + 3 = 0$ at point (1, 1) is

$2x + 2y - (x+1) - \frac{5}{2}(y+1) + 3 = 0$

$\Rightarrow x - \frac{1}{2}y - \frac{1}{2} = 0$

$\Rightarrow 2x - y - 1 = 0$

$\Rightarrow y = 2x - 1$

Slope of tangent=2, therefore slope of normal $= -\frac{1}{2}$

Hence, equation of normal at point (1, 1) and having slope $\left(-\frac{1}{2}\right)$ is

$y-1=-\frac{1}{2}(x-1)$

$\Rightarrow 2y-2=-x+1$

$\Rightarrow x+2y=3$

13 **(b)**

The product of perpendicular distance from any point on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to its asymptotes is $\frac{a^2b^2}{a^2+b^2}$ (See illustration 3 on page 26.12)

$\therefore$ Required product $=\frac{16\times 9}{16+9}=\frac{144}{25}$

14 **(c)**

$x^2=4\,y$ and $y^2=4x$ intersect at $O(0,0)$ and (4,4). Therefore, the coordinates of P are (4,4). The equations of the tangents to the two parabolas at (4,4) are :

$2\,x-y-4=0$ ...(i)

and, $x-2\,y+4=0$ ...(ii)

Now, $m_1=$ Slope of (i) $=2$, $m_2=$ Slope of (ii) $=1/2$

Clearly, $m_1m_2=1$

$\Rightarrow \tan\theta_1\tan\theta_2=1$

$\Rightarrow \tan\theta_1=\cot\theta_2$

$\Rightarrow \theta_1$ and θ_2 are such that $\theta_1+\theta_2=\pi/2$

15 **(c)**

The equation of a second degree curve passing through the points of intersection of the lines $2x-y+11=0$ and $x-2y+3=0$ with the coordinate axes is

$(2x-y+11)(x-2y+3)+\lambda\,xy=0$ ...(i)

This equation will represent a circle, if

Coeff. of $x^2=$ Coeff. of y^2 and Coeff.of $xy=0$

$\Rightarrow\lambda-5=0\Rightarrow\lambda=5$

Putting the value of λ in (i), we obtain that the equation of the circle is

$(2x-y+11)(x-2y+3)+5xy=0$

$\Rightarrow 2x^2+2y^2+7x-5y+3=0$

The coordinates of its centre are $(-7/2, 5/2)$

16 **(a)**

Given , $\frac{x^2}{16}+\frac{y^2}{9}=1$

$\therefore e=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

$\therefore$ Coodinates of foci are $(\pm\sqrt{7},0)$

Since, centre of circle is (0, 3) and passing through foci $(\pm 7, 0)$

$\therefore$ Radius of circle $=\sqrt{(0\pm\sqrt{7})^2+(3-0)^2}$

$=\sqrt{7+9}=4$

17 **(b)**

Given equation of curve is $x=\alpha+5\cos\theta$, $y=\beta+4\sin\theta$

Or $\cos\theta=\frac{x-\alpha}{5}$,$\sin\theta=\frac{y-\beta}{4}$

$\because \cos^2\theta+\sin^2\theta=1$

$\Rightarrow\left(\frac{x-\alpha}{5}\right)^2+\left(\frac{y-\beta}{4}\right)^2=1$

This represents the equation of an ellipse.

18 **(b)**

Let PQ be a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ having focus S. Then,

$\frac{2\,SP\cdot SQ}{SP+SQ}=\frac{b^2}{a}\Rightarrow\frac{2pq}{p+q}=\frac{b^2}{a}\Rightarrow b^2(p+q)=2apq$

19 **(c)**

Given, parametric equations are $x=e^t+e^{-t}$ and $y=e^t-e^{-t}$

Now, on squaring and then on subtracting, we get

$x^2-y^2=4$

20 **(c)**

Intersection points of given circles are (0, 0) and (3, 3) let equation of required circle whose centre $\left(\frac{3}{2},\frac{3}{2}\right)$, is

$x^2+y^2-3x-3y+c=0$

Since, this circle passes through (0, 0), thus equation of circle becomes,

$x^2+y^2-3x-3y=0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	A	C	D	D	A	B	D	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	B	C	C	A	B	B	C	C