DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 7

Topic :- CONIC SECTION

1 **(d)**
Centre of the given circle is $(4, -2)$. Therefore, the equation of the unit circle concentric with the given circle is $(x-4)^2+(y+2)^2=1 \Rightarrow x^2+y^2-8x+4y+19=0$

2 **(a)**
Since, the point $(9a, 6a)$ is bounded in the region formed by the parabola $y^2=16x$ and $x=9$, then
$y^2-16x<0, x-9<0$
$\Rightarrow 36a^2-16\cdot 9a<0, 9a-9<0$
$\Rightarrow 36a(a-4)<0, a<1$
$0<a<4, a<1 \Rightarrow 0<a<1$

3 **(b)**
It is given that the coordinates of the vertices are $A'(-6,1)$ and $A(4,1)$. So, centre of the ellipse is at $C(-1,1)$ and length of major axis is $2a=10$
Let e be the eccentricity of the ellipse. Then, coordinates its focus on the right side of centre ar$(ae,1)$ or $(5e,1)$
It is given that $2x-y-5=0$ is a focal chord of the ellipse.
So, it passes through $(5e,1)$
$\therefore 10e-1-5=0 \Rightarrow e=\frac{3}{5}$
So, $b^2=a^2(1-e^2)=25\left(1-\frac{9}{25}\right)=16$
Hence, the equation of the ellipse is
$$\frac{(x+1)^2}{25}+\frac{(y-1)^2}{16}=1$$

4 **(a)**
Given, $r=\sqrt{3}\sin\theta+\cos\theta$
Put $x=r\cos\theta, y=r\sin\theta$
$\therefore \quad r=\sqrt{3}\frac{y}{r}+\frac{x}{r}$
$\Rightarrow \quad r^2=\sqrt{3}y+x$
$\Rightarrow x^2+y^2-\sqrt{3}y-x=0$
$\therefore$ Radius $=\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2}=1$

5 **(b)**
We have,

$2\left(\frac{b^2}{4}\right)=\frac{9}{2}\Rightarrow b^2=9\Rightarrow 16(e^2-1)=9$

$\Rightarrow 16\,e^2=25\Rightarrow e=\frac{5}{4}$

6 **(c)**

Form right $\Delta\,OSB$

$\tan 0° = \frac{b}{ae}$

$\Rightarrow\ \sqrt{3}=\frac{b}{ae}$

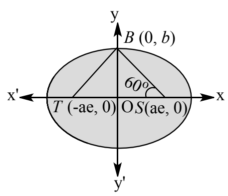
$\Rightarrow\ \ b=\sqrt{3}\ ae$

Also, $\quad b^2=a^2(1-e^2)$

$\Rightarrow 3a^2e^2=a^2(1-e^2)$

$\Rightarrow 3e^2=1-e^2\Rightarrow\ 4e^2=1$

$\Rightarrow e=\frac{1}{2}$



7 **(b)**

The eccentricity of a hyperbola is never less than or equal to 1. So option (b) is correct

8 **(d)**

The equation of the tangent at (α,β) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{\alpha\,x}{a^2}-\frac{\beta\,y}{b^2}=1$

The ordinates of the points of intersection of this tangent and the auxiliary circle $x^2+y^2=a^2$ are the roots of the equation

$\left\{\frac{a^2}{\alpha}\left(1+\frac{\beta\,y}{b^2}\right)\right\}^2+y^2=a^2$

$\Rightarrow\frac{a^4}{\alpha^2}\left(1+\frac{\beta^2y^2}{b^4}+\frac{2\,\beta\,y}{b^2}\right)+y^2=a^2$

$\Rightarrow y^2\left(\frac{\alpha^2}{a^4}+\frac{\beta^2}{b^4}\right)+\frac{2\,\beta}{b^2}y-\frac{\alpha^2}{a^2}+1=0$

Clearly, y_1 and y_2 are the roots of this equation

$$\therefore y_1+y_2=-\frac{2\beta/b^2}{\frac{\alpha^2}{a^4}+\frac{\beta^2}{b^4}} \text{ and, } y_1y_2=\frac{1-\frac{\alpha^2}{a^2}}{\frac{\alpha^2}{a^4}+\frac{\beta^2}{b^4}}$$

$$\Rightarrow \frac{1}{y_1}+\frac{1}{y_2}=\frac{-2\,\beta/b^2}{1-\frac{\alpha^2}{a^2}}=\frac{-2\,\beta/b^2}{-\frac{\beta^2}{b^2}} \quad \left[\because \frac{\alpha^2}{a^2}-\frac{\beta^2}{b^2}=1\right]$$

$$\Rightarrow \frac{1}{y_1}+\frac{1}{y_2}=\frac{2}{\beta}$$

9 **(a)**

Given hyperbola is a rectangular hyperbola whose eccentricity is $\sqrt{2}$

10 **(a)**

Since, the given line touches the given circle, the length of the perpendicular from the centre (2, 4) of the circle to the line $3x-4y-k=0$ is equal to the radius $\sqrt{4+16+5}=5$ of the circle

$$\therefore \frac{3\times 2-4\times 4-k}{\sqrt{9+16}}=\pm 5$$

$\Rightarrow k=15 \quad (\because k>0)$

Now, equation of the tangent at (a,b) to the given circle is

$xa+yb-2(x+a)-4(y+b)-5=0$

$\Rightarrow (a-2)x+(b-4)y-(2a+4b+5)=0$

If it represents the given line $3x-4y-k=0$

Then, $\frac{a-2}{3}=\frac{b-4}{-4}=\frac{2a+4b+5}{k}=l$ (say)

$\Rightarrow a=3l+2, b=4-4l$

and $2a+4b+5=kl$

$\Rightarrow 2(3l+2)+4(4-4l)+5=15l \quad (\because k=15)$

$\Rightarrow l=1 \Rightarrow a=5, b=0$

$\therefore k+a+b=15+5+0=20$

11 **(a)**

Since, the distance between the focus and directrix of the parabola is half of the length of the latusrectum. Therefore length of latusrectum =2 (length of the perpendicular from (3, 3) to $3x-4y-2=0$)

$$=2\left|\frac{9-12-2}{\sqrt{9+16}}\right|=2\cdot\frac{5}{5}=2$$

12 **(a)**

Given equation of circle is

$x^2+y^2-2x-6y+6=0$ ...(i)

Its centre is (1, 3) and radius $=\sqrt{1+9-6}=2$

Equation of any line through (0, 1) is

$y-1=m(x-0)$

$\Rightarrow mx-y+1=0$ ...(ii)

If it touches the circle (i), then the length of perpendicular from centre (1, 3) to the circle is equal to radius 2

$\therefore \quad \dfrac{m-3+1}{\sqrt{m^2+1}} = \pm 2$

$\Rightarrow \quad (m-2)^2 = 4(m^2+1)$

$\therefore \quad m = 0, \ -\dfrac{4}{3}$

On substituting these values of m in Eq. (ii), the required tangent are $y-1=0$ and $4x+3y-3=0$

13 **(d)**

The centres of given circles are $C_1(-3, -3)$ and $C_2(6, 6)$respectively and radii are $r_1 = \sqrt{9+9+0}$ $= 3\sqrt{2}$ and $r_2 = \sqrt{36+36+0} = 6\sqrt{2}$ respectively

Now, $C_1C_2 = \sqrt{(6+3)^2+(6+3)^2} = 9\sqrt{2}$

and $r_1 + r_2 = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}$

$\Rightarrow \quad C_1C_2 = r_1 + r_2$

$\therefore$ Both circles touch each other externally

14 **(a)**

Let $A \equiv (at_1^2, 2at_1)$, $B \equiv (at_2^2, 2at_2)$

Tangents, at A and B will intersect at the point C, whose coordinate is given by $\{at_1t_2, a(t_1+t_2)\}$.

Clearly, ordinates of A, C and B are always in AP

15 **(c)**

The pair of asymptotes and second degree curve differ by a constant.

$\therefore$ Pair of asymptotes is

$2x^2 + 5xy + 2y^2 - 11x - 7y + \lambda = 0$ ...(i)

Hence, Eq. (i) represents a pair of straight lines.

$\therefore \Delta = 0$

$\Rightarrow \ 2 \times 2 \times \lambda + 2 \times -\dfrac{7}{2} \times -\dfrac{11}{2} \times \dfrac{5}{2} - 2 \times \left(-\dfrac{7}{2}\right)^2 - 2 \times \left(-\dfrac{11}{2}\right)^2 - \lambda \times \left(\dfrac{5}{2}\right)^2 = 0$

$\Rightarrow \lambda = 5$

From Eq.(i), pair of asymptotes is

$2x^2 + 5xy + 2y^2 - 11x - 7y + 5 = 0$

16 **(b)**

Since, the given circles cut each other orthogonally

$\therefore g_1g_2 + a^2 = 0$ ...(i)

If $lx + my = 1$ is a common tangent of these circles, then

$\dfrac{-lg_1 - 1}{\sqrt{l^2+m^2}} = \pm\sqrt{g_1^2 + a^2}$

$\Rightarrow (lg_1 + 1)^2 = (l^2+m^2)(g_1^2 + a^2)$

$\Rightarrow m^2g_1^2 - 2lg_1 + a^2(l^2+m^2) - 1 = 0$

Similarly, $m^2g_2^2 - 2lg_2 + a^2(l^2+m^2) - 1 = 0$

So, that g_1,g_2 are the roots of the equation

$m^2g^2 - 2lg + a^2(l^2+m^2) - 1 = 0$

$\Rightarrow\ g_1g_2 = \frac{a^2(l^2+m^2)-1}{m^2} = -a^2$ [from Eq. (i)]

$\Rightarrow a^2(l^2+m^2) = 1 - a^2m^2$ ...(ii)

Now, $p_1p_2 = \frac{|ma-1|}{\sqrt{l^2+m^2}} \cdot \frac{|-ma-1|}{\sqrt{l^2+m^2}}$

$= \frac{|1-m^2a^2|}{l^2+m^2} = a^2$ [from Eq. (ii)]

17 **(b)**

If $(a\cos\alpha, b\sin\alpha)$ and $(a\cos\beta, b\sin\beta)$ are the end points of chord, then equation of chord is

$$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$$

If it is a focal chord, it passes through $(ae, 0)$, so

$$e\cos\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\Rightarrow e = \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}$$

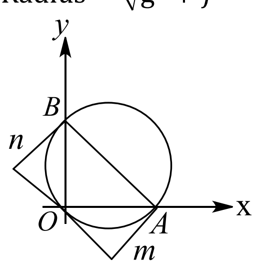
18 **(d)**

Let the equation of circle be

$x^2 + y^2 + 2gx + 2fgy = 0$

(passing through origin)

Radius $= \sqrt{g^2+f^2}$



Now, equation of tangents at $O(0,0)$ is

$x(0) + y(0) + g(x) + f(y) = 0$

$\Rightarrow gx + fy = 0$

Distance from $A(2g, 0) = \frac{2g^2}{\sqrt{g^2+f^2}} = m$

and distance from $B(0, 2f) = \frac{2f^2}{\sqrt{g^2+f^2}} = n$

$\Rightarrow \frac{2r^2}{r} = m + n \Rightarrow 2r = m + n$

19 **(c)**

We know that every line passing through the focus of a parabola intersects the parabola in two distinct points except lines parallel to the axis.

The equation $(y-2)^2=4(x+1)$ represents a parabola with vertex $(-1,2)$ and axis parallel to x-axis. So, the line of slope m will cut the parabola in two distinct points if $m \neq 0$ i.e.
$m \in (-\infty,0) \cup (0,\infty)$

20 **(a)**

Given that, any tangent to the circle $x^2+y^2=b^2$ is$y=mx-b\sqrt{1+m^2}$. It touches the circle $(x-a)^2+y^2=b^2$, then

$$\frac{ma-b\sqrt{1+m^2}}{\sqrt{m^2+1}}=b$$

$$\Rightarrow ma=2b\sqrt{1+m^2}$$

$$\Rightarrow m^2a^2=4b^2+4b^2m^2$$

$$\therefore \quad m=\pm\frac{2b}{\sqrt{a^2-4b^2}}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	A	B	A	B	C	B	D	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	D	A	C	B	B	D	C	A