# DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

**Solutions**

SUBJECT : MATHS
DPP NO. : 6

## Topic :- CONIC SECTION

1 **(c)**

We have,

$x^2 + y^2 + ax + (1 - a)y + 5 = 0$

It is given that the radius of this circle is less than or equal to 5

$\therefore \frac{a^2}{4} + \frac{(1-a)^2}{4} - 5 \leq 25$

$\Rightarrow 2a^2 - 2a - 119 \leq 0 \Rightarrow -7.2 \leq a \leq 8.2 \Rightarrow a \in [-7, 8]$

But, $a$ is an integer

$\therefore a = -7, -6, -5, -4, -3, -2, -1, 1, 0, 1, 2, 3, 4, 5, 6, 7, 8$

Hence, these are 16 integral values of $a$

2 **(d)**

Given equation of circles are $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 - 12x - 16y + 91 = 0$ whose centre and radius are $C_1(1, 2), r_1 = 2$ and $C_2(6, 8), r_2 = 3$

$\therefore C_1C_2 = \sqrt{(1-6)^2 + (2-8)^2}$

$= \sqrt{25 + 36} = \sqrt{61}$

And $r_1 + r_2 = 2 + 3 = 5$

$\because C_1C_2 > r_1 + r_2$

$\therefore$ Number of common tangents $= 4$

3 **(c)**

We know that the locus of point $P$ from which two perpendicular tangents are drawn to the parabola, is the directrix of the parabola.

Hence, the required locus is $x = 1$

4 **(b)**

Let two coplanar points be $(0, 0)$ and $(a, 0)$

$\therefore \quad \frac{\sqrt{x^2 + y^2}}{\sqrt{(x-a)^2 + y^2}} = \lambda \quad [\lambda \neq 1]$

[where $\lambda$ is any number]

$\Rightarrow x^2 + y^2 + \left(\frac{\lambda^2}{\lambda^2 - 1}\right)(a^2 - 2ax) = 0$

Which is the equation of circle

5 **(c)**

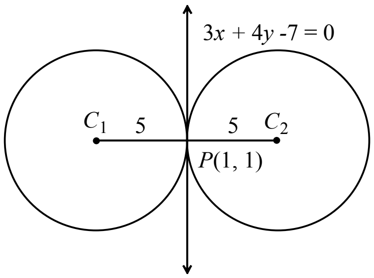
The equation of line $C_1C_2$ is

$$\frac{x-1}{3/5}=\frac{y-1}{4/5}$$

So, the coordinates of $C_1$ and $C_2$ are given by

$$\frac{x-1}{3/5}=\frac{y-1}{4/5}=\pm 5 \Rightarrow x=1\pm 3, y=1\pm 4$$

Thus, the coordinates of the centres are $(4,5),(-2,-3)$




6 **(d)**

The tangent at (1, 7) to the parabola $x^2=y-6$ is

$$x=\frac{1}{2}(y+7)-6$$

$\Rightarrow 2x=y+7-12$

$\Rightarrow y=2x+5$

Which is also tangent to the circle

$x^2+y^2+16x+12y+c=0$

$\therefore x^2+(2x+5)^2+16x+12(2x+5)+c=0$

Or $5x^2+60x+85+c=0$

Must have equal roots

Let α and β are the roots of the equation

$\Rightarrow \alpha+\beta=-12 \Rightarrow \alpha=-6 \quad (\because \alpha=\beta)$

$\therefore x=-6$ and $y=2x+5=-7$

$\Rightarrow$ point of contact is $(-6,-7)$

7 **(c)**

Let $C(0,0)$ be the centre and $L(ae,b^2/a)$ and $L'(-ae,b^2/a)$ be the vertices of latusrectum $LL'$. Then,

$$m_1=\text{Slope of } CL=\frac{b^2/a-0}{ae-0}=\frac{b^2}{a^2e}$$

$$m_2=\text{Slope of } CL'=\frac{b^2/a-0}{-ae-0}=\frac{-b^2}{a^2e}$$

It is given that $\angle LCL'=\pi/2$

$\therefore m_1m_2=-1$

$$\Rightarrow \frac{b^2}{a^2e}\times\frac{-b^2}{a^2e}=-1$$

$\Rightarrow (e^2-1)^2=e^2$

$\Rightarrow e^2 - 1 = e \Rightarrow e^2 - e - 1 = 0 \Rightarrow e = \dfrac{1+\sqrt{5}}{2}$

8 **(c)**

Given, ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$

$\therefore\ e_1 = \sqrt{1 - \dfrac{7}{16}} = \dfrac{3}{4}$

and hyperbola $\frac{x^2}{9} - \frac{y^2}{7} = 1$

$\therefore\ e_2 = \sqrt{1 + \dfrac{7}{9}} = \dfrac{4}{3}$

Now, $e_1 + e_2 = \frac{3}{4} + \frac{4}{3} = \frac{25}{12}$

9 **(c)**

The equation of normal to the given ellipse at $P(a\cos\theta, b\sin\theta)$ is

$ax\sec\theta - by\,\text{cosec}\,\theta - a^2 = b^2$

$\Rightarrow y = \left(\frac{a}{b}\tan\theta\right)x - \frac{a^2 - b^2}{b}\sin\theta$ ...(i)

Let $\frac{a}{b}\tan\theta = m$, then $\sin\theta = \frac{bm}{\sqrt{a^2 + b^2m^2}}$

$\therefore$ From Eq. (i), we get

$y = mx - \dfrac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}$

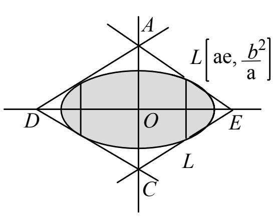
$\because \frac{a}{b}\tan\theta \in R \Rightarrow m \in R$

10 **(d)**

Given, $\frac{x^2}{9} + \frac{y^2}{5} = 1$

Latusrectum of an ellipse be

$ae = \sqrt{a^2 - b^2} = \sqrt{4} = 2$




By symmetry the quadrilateral is rhombus

$\Rightarrow$ Equation of tangent at $\left(ae, \frac{b^2}{a}\right) = \left(2, \frac{5}{3}\right)$

$ie, \frac{2}{9}\, x + \frac{5}{3} . \frac{y}{5} = 1$

$\Rightarrow \frac{x}{9/2} + \frac{y}{3} = 1$

$\therefore$ Area of quadrilateral $ABCD = 4$(area of $\Delta\, AOB$ )

$= 4.\left\{\frac{1}{2} . \frac{9}{2} .3\right\}$

$= 27$ sq units

11 **(c)**

The equation of the tangent at $P(a\sec\theta, b\tan\theta)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

This cuts the line $\frac{x}{a} - \frac{y}{b} = 0$ and $\frac{x}{a} + \frac{y}{b} = 0$ at $Q$ and $R$

The coordinates of $Q$ and $R$are

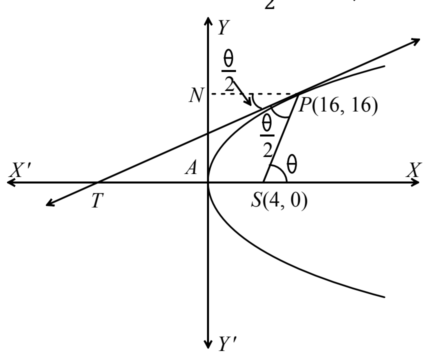
$Q\left(\frac{a}{\sec\theta - \tan\theta}, \frac{b}{\sec\theta - \tan\theta}\right), R\left(\frac{a}{\sec\theta + \tan\theta}, \frac{-b}{\sec\theta + \tan\theta}\right)$

$\therefore CQ \cdot CR = \frac{\sqrt{a^2 + b^2}}{(\sec\theta - \tan\theta)} \times \frac{\sqrt{a^2 + b^2}}{(\sec\theta + \tan\theta)} = a^2 + b^2$

12 **(c)**

We know that $PT$ bisects $\angle NPS$

Let $\angle NPT = \angle TPS = \frac{\theta}{2}$. Then,




$\angle PSX = \theta$

$\Rightarrow \tan\theta = \frac{16 - 0}{16 - 4}$

$\Rightarrow \tan\theta = \frac{4}{3}$

$\Rightarrow \frac{2\tan\theta/2}{1 - \tan^2\theta/2} = \frac{4}{3}$

$\Rightarrow 3\tan\frac{\theta}{2} = 2 - 2\tan^2\frac{\theta}{2}$

$\Rightarrow 2\tan^2\frac{\theta}{2} + 3\tan\frac{\theta}{2} - 2 = 0$

$\Rightarrow \left(2\tan\frac{\theta}{2} - 1\right)\left(\tan\frac{\theta}{2} + 2\right) = 0$

$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{2}$ $\left[\because \frac{\theta}{2} \text{ is acute}\right]$

$\Rightarrow \frac{\theta}{2} = \tan^{-1}\left(\frac{1}{2}\right) \Rightarrow \angle TPS = \tan^{-1}\left(\frac{1}{2}\right)$

13 **(d)**

The centres and radii of gives circles are $C_1(0, 0)$, $C_2(4, 0)$ and $r_1 = 2,\ r_2 = 2$

Now, $C_1C_2 = \sqrt{(4-0)^2 + 0} = 4$

and $r_1 + r_2 = 2 + 2 = 4$

$\therefore\ C_1C_2 = r_1 + r_2$

Hence, three common tangents are possible

14 **(b)**

Given, circle cuts the parabola

$\therefore x^2 + \left(\frac{x^2}{4a}\right)^2 + 2gx + 2f\left(\frac{x^2}{4a}\right) + c = 0$

$\Rightarrow x^4 + 16a^2x^2 + 8afx^2 + 32gxa^2 + 16a^2c = 0$

$\sum x_i = 0$ ...(i)

$\sum x_1x_2 = 16a^2 + 8af$ ...(ii)

Now, $\sum y_i = \frac{1}{4a}\sum x_i^2$

$= \frac{1}{4a}[(x_1 + x_2 + x_3 + x_4)^2 - 2\sum x_1x_2]$

$= -\frac{1}{2a}(16a^2 + 8af) = -4(f + 2a)$

15 **(b)**

Let the coordinates of $A$ and $B$ be $(a,0)$ and $(0,b)$ respectively. then,

$a^2 + b^2 = 9^2$ ...(i)

Let $P(h,k)$ be the centroid of $\Delta\ OAB$. Then,

$h = \frac{a}{3}$ and $k = \frac{b}{3} \Rightarrow a - 3h$ and $b = 3k$

Substituting the values of $a$ and $b$ in (i), we get

$9h^2 + 9k^2 = 9^2 \Rightarrow h^2 + k^2 = 9$

Hence, the locus of $(h,k)$ is $x^2 + y^2 = 9$

16 **(a)**

Given focal chord of parabola $y^2 = ax$ is $2x - y - 8 = 0$

Since, this chord passes through focus $\left(\frac{a}{4},0\right)$

$\therefore\ 2.\frac{a}{4}-0-8=0 \Rightarrow a=16$

Hence, directrix is $x=-4 \Rightarrow x+4=0$

17 **(b)**

Let one of the points be $P(r\cos\theta, r\sin\theta)$. Then, the other point is $Q(r\cos(\pi/2+\theta)), (r\sin(\pi/2+\theta))$ i.e. $Q(-r\sin\theta, r\cos\theta)$. The equations of tangents at $P$ and $Q$ are

$x\cos\theta + y\sin\theta = r$ and $-x\sin\theta + y\cos\theta = r$

The locus of the point of intersection of these two is obtained by eliminating $\theta$ from these two equations. Squaring and adding the two equations, we get

$(x\cos\theta + y\sin\theta)^2 + (-x\sin\theta + y\cos\theta)^2 = r^2 + r^2$

or, $x^2+y^2=2\,r^2$, which is the required locus

18 **(a)**

The coordinates of a point dividing $PQ$ internally in the ratio $1:\lambda$ are

$\left(\frac{1+\lambda}{\lambda+1}, \frac{1+3\,\lambda}{\lambda+1}\right)$

This point is an interior point of the parabola $y^2=4x$

$\therefore \left(\frac{1+3\,\lambda}{\lambda+1}\right)^2 - 4\left(\frac{1+\lambda}{\lambda+1}\right) < 0$

$\Rightarrow (3\,\lambda+1)^2 - 4(\lambda+1)^2 < 0$

$\Rightarrow 5\,\lambda^2 - 2\,\lambda - 3 < 0$

$\Rightarrow (5\,\lambda+3)(\lambda-1) < 0$

$\Rightarrow \lambda - 1 < 0 \qquad [\because \lambda > 0]$

$\Rightarrow 0 < \lambda < 1 \Rightarrow \lambda \in (0,1)$

19 **(b)**

Given that, $y=2x+c$ ...(i)

And $x^2+y^2=16$ ...(ii)

We know that, if $y=mx+c$ is tangent to the circle

$x^2+y^2=a^2$, then $c=\pm\, a\sqrt{1+m^2}$, here, $m=2, a=4$

$\therefore\ c=\pm\, 4\sqrt{1+2^2}=\pm\, 4\sqrt{5}$

20 **(a)**

Given, $x^2+y^2=6x$ ...(i)

and $x^2+y^2+6x+2y+1=0$ ...(ii)

From Eq. (i), $x^2-6x+y^2=0$

$\Rightarrow (x-3)^2+y^2=3^2$

$\therefore$ Centre $(3, 0), r=3$

From Eq. (ii),

$x^2+6x+y^2+2y+1+3^2=3^2$

$\Rightarrow\ (x+3)^2+(y+1)^2=3^2$

$\therefore$ Centre $(-3, -1)$, radius=3

Now, distance between centres

$= \sqrt{(3+3)^2 + 1}$

$= \sqrt{37} > r_1 + r_2 = 6$

$\therefore$ Circles do not cut each other

$\Rightarrow$ 4 tangents (two direct and two transversal) are possible

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | C | D | C | B | C | D | C | C | C | D |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | C | C | D | B | B | A | B | A | B | A |
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