DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 5

Topic :- CONIC SECTION

1 **(a)**

Using $SS' = T^2$, the combined equation of the tangents drawn from (0,0) to $y^2 = 4\,a(x-a)$ is

$(y^2 - 4\,ax + 4\,a^2)(0 - 0 + 4\,a^2) = [y \cdot 0 - 2\,a(x + 0 - 2a)]^2$

$\Rightarrow (y^2 - 4\,ax + 4\,a^2)(4a^2) = 4\,a^2(x-2a)^2$

$\Rightarrow y^2 - 4\,ax + 4\,a^2 = (x-2a)^2$

$\Rightarrow x^2 - y^2 = 0$

Clearly, Coeff. of $x^2 +$ Coeff. of $y^2 = 0$. Therefore, the required angle is a right angle

ALITER The point (0,0) lies on the directrix $x = 0$ of the parabola $y^2 = 4\,a(x-a)$, therefore the tangents are at right angle

2 **(c)**

We know that length of latusrectum of an ellipse $= \frac{2b^2}{a}$ and length of its minor axis $= 2b$

Then, $\frac{2b^2}{a} = b \Rightarrow 2b = a$

$$\therefore\ e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{b^2}{4b^2}} = \frac{\sqrt{3}}{2}$$

3 **(b)**

The required point is the radical centre of the given circles

4 **(a)**

Equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a parabola, if $h^2 = ab$

5 **(d)**

Let e and e' be the eccentricities of the ellipse and hyperbola

$$\therefore e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{25-16}{25}} = \frac{3}{5}$$

and $e' = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{25+16}{25}} = \frac{\sqrt{41}}{5}$

1. Centre of ellipse is (0, 0) and centre of hyperbola is (0, 0)

2. Foci of ellipse are $(\pm ae, 0)$ or $(\pm 3, 0)$ foci of hyperbola are $(\pm ae', 0)$ or $(\pm\sqrt{41}, 0)$

3. Directrices of ellipse are $x = \pm\frac{a}{e} \Rightarrow x = \pm\frac{25}{3}$

Directrices of hyperbola are $x = \pm\frac{a}{e}$

$\Rightarrow x = \pm\dfrac{25}{\sqrt{41}}$

4. Vertices of ellipse are $(\pm a, 0)$ or $(\pm 5, 0)$

Vertices of hyperbola are $(\pm a, 0)$ or $(\pm 5, 0)$

From the above discussions, their are common in centre and vertices.

6 **(c)**

Given equation is $\frac{x^2}{16} - \frac{y^2}{25} = 1$

$$\therefore\ e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{25}{16}} = \frac{\sqrt{41}}{4}$$

7 **(d)**

Equation of tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$y = mx + \sqrt{a^2m^2 + b^2}$

And equation of tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2$ is

$y = mx + \sqrt{2a^2m^2 + 2b^2}$

For common tangent,

$a^2m^2 + b^2 = 2a^2m^2 - 2b^2$

$\Rightarrow a^2m^2 = 3b^2 \Rightarrow m = \pm\dfrac{\sqrt{3}b}{a}$

$\therefore$ Equation of common tangent is $y = \frac{\sqrt{3}b}{a}x + 2b$.

8 **(a)**

The equation of a tangent to $xy = c^2$ is

$\dfrac{x}{t} + yt = 2c$ (i)

If $lx + my + n = 0$ is a tangent to $xy = c^2$, then it should be of the form of equation (i).

$\therefore \dfrac{l}{1/t} = \dfrac{m}{t} = \dfrac{-n}{2c}$

$\Rightarrow lt = \dfrac{m}{t} = -\dfrac{n}{2c}$

$\Rightarrow lt = -\dfrac{n}{2c}$ and $\dfrac{m}{t} = -\dfrac{n}{2c}$

$\Rightarrow lm = \frac{n^2}{4c^2}$

$\Rightarrow lm > 0 \Rightarrow l$ and m are of the same sign

10 **(c)**

The equation of the tangent at $(4, -2)$ to $y^2 = x$ is

$-2y = \frac{1}{2}(x + 4) \Rightarrow x + 4y + 4 = 0$

Its slope is $-1/4$. Therefore, the slope of the perpendicular line is 4. Since the tangents at the end points of a focan chord of a parabola are at right angles. Therefore, the slope of the tangent at Q is 4

11 **(a)**

The equation of a normal to $y^2 = 4x$ is

$y + tx = 2t + t^3$ ...(i)

If it passes through (3,0), then

$3t = 2t + t^3 \Rightarrow t = 0, \pm 1$

Putting the values of t in (i), we get

$y = 0, y + x = 3$ and $y - x = -3$

As the equation of the normals

12 **(a)**

Let $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ be tangent at

$P(a\cos\theta, b\sin\theta)$.

Its cuts the coordinates axes at $P(a\sec\theta, 0)$ and $Q(0, b\,\text{cosec}\,\theta)$

$\therefore CP = a\sec\theta$ and $CQ = b\,\text{cosec}\,\theta$

$\Rightarrow \frac{a^2}{CP^2} + \frac{b^2}{CQ^2} = 1$

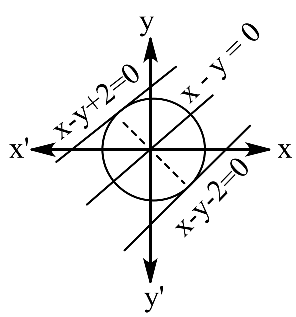
13 **(c)**

Since, the equation of tangents $x - y - 2 = 0$ and $x - y + 2 = 0$ are parallel.

$\therefore$ Distance between them=Diameter of the circle $= \frac{2-(-2)}{\sqrt{1^2+1^2}}$

$\left(\because \frac{c_2 - c_1}{\sqrt{a^2+b^2}}\right)$

$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$



$\therefore$ Radius $= \frac{1}{2}(2\sqrt{2}) = \sqrt{2}$

It is clear from the figure that centre lies on the origin.

$\therefore$ Equation of circle is

$$(x-0)^2+(y-0)^2=(\sqrt{2})^2$$

$\Rightarrow x^2+y^2=2$

14 **(b)**

Equation of family of concentric circles to the circle $x^2+y^2+6x+8y-5=0$ is $x^2+y^2+6x+8y+\lambda=0$ which is similar to $x^2+y^2+2gx+2fy+c=0$. Since, it is equation of concentric circle to the circle $x^2+y^2+6x+8y-5=0$. Thus, the point $(-3,2)$ lies on the circle x^2+y^2

$$+6x+8y+c=0$$

$\Rightarrow(-3)^2+(2)^2+6(-3)+8(2)+c=0$

$\Rightarrow 9+4-18+16+c=0$

$\Rightarrow c=-11$

15 **(d)**

On solving the given equations, we get $(0,0)$, $B(0,5/3)$, $C(5/2,0)$.

Let equation of circle be

$x^2+y^2+2gx+2fy+c=0$ ...(i)

Eq. (i) passes through $A(0,0)$, we get $c=0$

Similarly, Eq. (i) passes through $B(0,5/3)$ and $C(5/2,0)$, we get

$2f=-5/3$ and $2g=-5/2$

$\therefore$ Required equation of circle is

$x^2+y^2-\frac{5}{2}x-\frac{5}{3}y=0$

$\Rightarrow 6x^2+6y^2-15x-10y=0$

16 **(c)**

We have,

$OM=OA+AM=2+5/2=9/2$

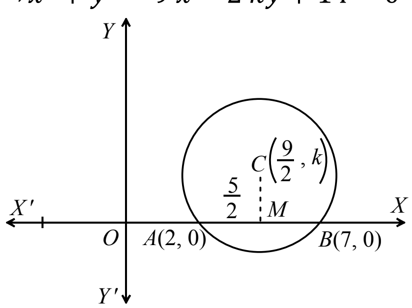
So, the x-coordinate of the centre is $9/2$

$\therefore$ Radius $=CA=\sqrt{(9/2-2)^2+(k-0)^2}$

Hence, the equation of the circle is

$(x-9/2)^2+(y-k)^2=\sqrt{(9/2-2)^2+k^2}$

$\Rightarrow x^2+y^2-9x-2ky+14=0$



18 **(b)**

Let $P(x_1, y_1)$ be a point on $x^2+y^2=a^2$. Then,

$x_1^2+y_1^2=a^2$ ...(i)

Let QR be the chord of contact of tangents drawn from $P(x_1,y_1)$ to the circle $x^2+y^2=b^2$. Then, the equation QR is

$xx_1 + yy_1 = b^2$ ...(ii)

This touches the circle $x^2+y^2=c^2$

$\therefore \left|\frac{0x_1+0y_1-b^2}{\sqrt{x_1^2+y_1^2}}\right| = c \Rightarrow b^2 = ac$ [Usin :(i)]

Let D be the discriminant of $ax^2 + 2bx + c = 0$. Then,

$D = 4(b^2 - ac) = 0$ [$\because b^2 = ac$]

Hence, the roots of the given equal are real and equal

19 **(c)**

The equation of the line joining (3,3) and $(-3,3)$ i.e. axis of the parabola is $y-3=0$.

Since the directrix is a line perpendicular to the axis. Therefore, its equation is $x+\lambda=0$.

The directrix intersects with the axis at $(-\lambda,3)$ and the vertex is the mid point of the line segment joining the focus and the point of intersection of the directrix and axis

$\therefore \frac{-\lambda-3}{2} = 3 \Rightarrow \lambda = -9$

So, the equation of the directrix is $x-9=0$

Let $P(x,y)$ be any point on the parabola. Then, by definition, we have

$(x+3)^2 + (y-3)^2 = (x-9)^2$

$\Rightarrow y^2 - 6\,y + 24\,x - 63 = 0$

20 **(d)**

Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

It is given that,

$$2\,a = 3(2\,b) \Rightarrow a^2 = 9\,b^2 = a^2 = 9\,a^2(1-e^2) \Rightarrow e = \frac{2\sqrt{2}}{3}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	A	D	C	D	A	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	C	B	D	C	A	B	C	D