DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 4

Topic :- CONIC SECTION

1 **(b)**

Given, $y = mx + 2$

and $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Condition of tangency, $c = \pm\sqrt{a^2m^2 - b^2}$

$2 = \pm\sqrt{9m^2 - 4} \Rightarrow m = \pm\frac{2\sqrt{2}}{3}$

2 **(c)**

Let any point $P(x_1, y_1)$ outside the circle. Then, equation of tangent to the circle $x^2 + y^2$ $+6x + 6y = 2$ at the point P is

$xx_1 + yy_1 3(x + x_1) + 3(y + y_1) - 2 = 0$ ...(i)

The Eq. (i) and the line $5x - 2y + 6 = 0$ intersect at a point Q on y-axis ie, $x = 0$

$\Rightarrow 5(0) - 2y + 6 = 0 \Rightarrow y = 3$

$\therefore$ Coordinates of Q are $(0, 3)$

Point Q satisfies Eq. (i)

$\therefore 3x_1 + 6y_1 + 7 = 0$ ...(ii)

Distance between P and Q is given by

$PQ^2 = x_1^2 + (y_1 - 3)^2$

$= x_1^2 + y_1^2 - 6y_1 + 9$

$= 11 - 6x_1 - 12y_1 \left(\because x_1^2 + y_1^2 + 6x_1 + 6y_1 - 2 = 0\right)$

$= 11 - 2(3x_1 - 6y_1)$

$= 11 - 2(-7) = 25$ [from Eq. (ii)]

$\therefore PQ = 5$

3 **(b)**

Equation of circle which touches x-axis and coordinates of centre are (h, k) is

$(x - h)^2 + (y - k)^2 = k^2$

Since, it is passing through $(-1, 1)$, then

$(-1 - h)^2 + (1 - k)^2 = k^2$

$\Rightarrow h^2 + 2h - 2k + 2 = 0$

For real circles, $D \geq 0$,

$\Rightarrow \quad (2)^2-4(-2k+2)\geq 0 \Rightarrow \; k\geq \frac{1}{2}$

4 **(b)**

The required equation of circle is

$\left(x^2+y^2-6\right)+\lambda\left(x^2+y^2-6y+8\right)=0$ ...(i)

It passes through (1, 1)

$\therefore \quad (1+1-6)+\lambda(1+1-6+8)=0$

$\Rightarrow -4+4\lambda=0$

$\Rightarrow \lambda=1$

$\therefore$ required equation of circle is

$x^2+y^2-6+x^2+y^2-6y+8=0$

$\Rightarrow \; 2x^2+2y^2-6y+2=0$

$\Rightarrow \; x^2+y^2-3y+1$

5 **(a)**

The equation of a normal to $y^2=4x$ is $y=mx-2m-m^3$.

If it passes through (11/4,1/4), then

$\frac{1}{4}=\frac{11\,m}{4}-2m-m^3$

$\Rightarrow 1=11\,m-8\,m-4\,m^3$

$\Rightarrow 4\,m^3-3m+1=0 \Rightarrow m=\frac{1}{2},\frac{-1\pm\sqrt{3}}{2}$

Hence, three normals can be drawn from (11/4,1/4) to $y^2=4x$

6 **(d)**

Here, $a^2=\cos^2\alpha$ and $b^2=\sin^2\alpha$

Now, $e=\sqrt{1+\frac{b^2}{a^2}} \;\Rightarrow\; e=\sqrt{1+\frac{\sin^2\alpha}{\cos^2\alpha}}$

$\Rightarrow \quad e=\sqrt{1+\tan^2\alpha} \;\Rightarrow e=sec\alpha$

Coordinates of foci are $(\pm ae,0) ie, (\pm 1,0)$

Hence, abscissae of foci remain constant when α varies.

8 **(c)**

It is a known result

$t_1t_2=-1$

9 **(a)**

Here, $g_1=-1, f_1=11, c_1=5$

and $g_2=7, f_2=3, c_2=k$

$\Rightarrow \; 2(-1.7+11.3)=5+k \;\Rightarrow k=47$

10 **(c)**

If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $y=ma+c$ intersect in real points, then the quadratic equation $\frac{x^2}{a^2}+\frac{(mx+c)^2}{b^2}=1$ must have real roots.

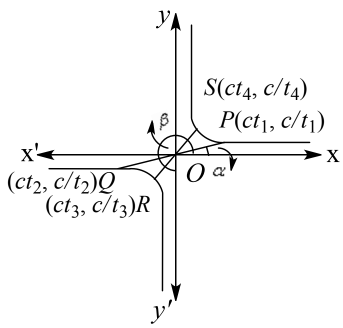
$\therefore$ Discriminant $\geq 0 \Rightarrow c^2 \leq a^2m^2+b^2$

11 **(a)**

Let the equation of rectangular hyperbola is $xy=c^2$.

Take any four points on the hyperbola

$P\left(ct_1,\frac{c}{t_1}\right)$, $Q\left(ct_2,\frac{c}{t_2}\right)$, $R\left(ct_3,\frac{c}{t_3}\right)$ and $S\left(ct_4,\frac{c}{t_4}\right)$ Such that PQ is perpendicular toRS.



Since, OP makes angle α withOX.

Therefore, $\tan\alpha=\frac{\frac{c}{t_1}}{ct_1}=\frac{1}{t_1^2}$

Similarly, $\tan\beta=\frac{1}{t_2^2}$, $\tan\gamma=\frac{1}{t_3^2}$ and $\tan\delta=\frac{1}{t_4^2}$

$\therefore\ \tan\alpha\tan\beta\tan\gamma\tan\delta=\frac{1}{t_1^2t_2^2t_3^2t_4^2}$ ...(i)

Now, PQ is perpendicular to RS.

$$\therefore\ \frac{\frac{c}{t_2}-\frac{c}{t_1}}{ct_2-c_1}\times\frac{\frac{c}{t_4}-\frac{c}{t_3}}{ct_4-ct_3}=-1$$

$\Rightarrow -\frac{1}{t_1t_2}\times\left(-\frac{1}{t_3t_4}\right)=-1$

$\Rightarrow \frac{1}{t_1t_2t_3t_4}=-1$

$\Rightarrow t_1t_2t_3t_4=-1$

From Eq.(i),

$\tan\alpha\tan\beta\tan\gamma\tan\delta=1$

12 **(a)**

Equation of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Distance between foci of hyperbola $=2ae$

and its distance between directrices $=\frac{2a}{e}$

According to the question,

$$\frac{2ae}{2a/e}=\frac{3}{2}$$

$$\Rightarrow e^2=\frac{3}{2}$$

Using , $b^2 = a^2(e^2 - 1) \Rightarrow \frac{b^2}{a^2}=\frac{3}{2}-1$

$$\Rightarrow \frac{a}{b}=\frac{\sqrt{2}}{1}$$

13 **(b)**

Equation of pair of tangents is

$SS_1=T^2$

$\Rightarrow (x^2+y^2-4)(9+4-4)=(3x+2y-4)^2$

$\Rightarrow 5y^2+16y-12xy+24x-50=0$

$$\therefore\ m_1+m_2=-\frac{2h}{b}=\frac{12}{5}$$

and $m_1m_2=0$

Now, $m_1-m_2=\sqrt{(m_1+m_2)^2-4m_1m_2}$

$$=\sqrt{\left(\frac{12}{5}\right)^2-0}=\frac{12}{5}$$

14 **(c)**

Given equation is $9x^2+4y^2-6x+4y+1=0$

$\Rightarrow 9\left(x^2-\frac{2}{3}x+\frac{1}{3^2}\right)+4\left(y^2+y+\frac{1}{4}\right)+1-1-1=0$

$\Rightarrow \frac{\left(x-\frac{1}{3}\right)^2}{\left(\frac{1}{3}\right)^2}+\frac{\left(y+\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^2}=1$ (here, $a<b$)

Length of major axis $=2b=2\left(\frac{1}{2}\right)=1$

Length of minor axis $=2a=2\left(\frac{1}{3}\right)=\frac{2}{3}$

15 **(a)**

Equation of two straight lines are

$\sqrt{3}\,x-y=4\sqrt{3}\alpha$

and $\sqrt{3}\,x + y = \frac{4\sqrt{3}}{\alpha}$

Solving above equations, we get

$$3x^2 - y^2 = 48 \Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$$

Which is a hyperbola

Whose eccentricity

$$e = \sqrt{\frac{48+16}{16}} = \sqrt{4} = 2$$

16 **(c)**

Given equation of circle can be rewritten as

$$x^2 + y^2 - 2x + 4y + \frac{k}{4} = 0$$

$\therefore$ Radius of circle $= \sqrt{1 + 4 - \frac{k}{4}} = \sqrt{5 - \frac{k}{4}}$

Area of circle $= 9\pi$ (given)

$$\Rightarrow \pi\left(5 - \frac{k}{4}\right) = 9\pi$$

$$\Rightarrow 5 - 9 = \frac{k}{4} \Rightarrow k = -16$$

17 **(c)**

The circle passes through $(0,0),(a,0),(0,a)$ and (a,a)

Hence, the required equation is $x^2 + y^2 - ax - ay = 0$

18 **(c)**

It is given that the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of the circle $x^2 + y^2 + 2g'x + 2f'y + c' = 0$. Therefore, the common chord of these two circles passes through the centre $(-g', -f')$ of $x^2 + y^2 + 2g'x + 2f'y + c' = 0$

The equation of the common chord of the two given circles is

$$2x(g - g') + 2y(f - f') + c - c' = 0$$

This passes through $(-g', -f')$

$$\therefore -2g'(g - g') - 2f'(f - f') + c - c' = 0$$

$$\Rightarrow 2g'(g - g') + 2f'(f - f') = c - c'$$

19 **(a)**

The slope of the tangent to $y^2 = 4x$ at $(16,8)$ is given by

$$m_1 = \left(\frac{dy}{dx}\right)_{(16,8)} = \left(\frac{4}{2y}\right)_{(16,8)} = \frac{2}{8} = \frac{1}{4}$$

The slope of the tangent to $x^2 = 32y$ at $(16,8)$ is given by

$$m_2 = \left(\frac{dy}{dx}\right)_{(16,8)} = \left(\frac{2x}{32}\right)_{(16,8)} = 1$$

$$\therefore \tan\theta = \frac{1-\frac{1}{4}}{1+\frac{1}{4}} = \frac{3}{5} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{5}\right)$$

20 **(a)**

Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Given equation of circles are

$x^2 + y^2 - 2x + 3y - 7 = 0$ ...(ii)

$x^2 + y^2 + 5x - 5y + 9 = 0$ ...(iii)

and $x^2 + y^2 + 7x - 9y + 29 = 0$ ...(iv)

Since, the circle (i) cut all three circles orthogonally,

$\therefore\ 2g(-1) + 2f(3/2) = c - 7 \Rightarrow -2g + 3f - c = -7$ ...(v)

$2g(5/2) + 2f(-5/2) = c + 29 \Rightarrow 5g - 5f - c = 9$ ...(vi)

$2g\left(\frac{7}{2}\right) + 2f\left(-\frac{9}{2}\right) = c + 29 \Rightarrow 7g - 9f - c = 29$ ...(vii)

On solving Eqs. (v), (vi) and (vii), we get

$g = -8$, $f = -9$ and $c = -4$

On putting the values of g, f and c in Eq. (i), we get

$x^2 + y^2 - 16x - 18y - 4 = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	B	B	A	D	D	C	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	B	C	A	C	C	C	A	A