DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 3

Topic :- CONIC SECTION

2 **(b)**

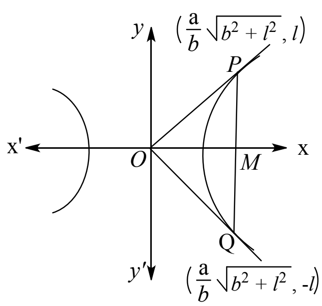
$\because PQ$ Is the double ordinate. Let $MP = MQ = l$.

Given that ΔOPQ is an equilateral, then $OP = OQ = PQ$

$\Rightarrow (OP)^2 = (OQ)^2 = (PQ)^2$

$\Rightarrow \frac{a^2}{b^2}(b^2 + l^2) + l^2 = \frac{a^2}{b^2}(b^2 + l^2) + l^2 = 4l^2$

$\Rightarrow \frac{a^2}{b^2}(b^2 + l^2) + 3l^2$



$\Rightarrow a^2 = l^2\left(3 - \frac{a^2}{b^2}\right)$

$\Rightarrow\ l^2 = \frac{a^2b^2}{(3b^2 - a^2)} > 0$

$\therefore\ 3b^2 - a^2 > 0$

$\Rightarrow\ 3b^2 > a^2$

$\Rightarrow 3a^2(e^2 - 1) > a^2$

$\Rightarrow e^2 > 4/3$

$\therefore\ e > \frac{2}{\sqrt{3}}$

3 **(b)**

Clearly, $x^2 - y^2 = c^2$ and $xy = c^2$ are rectangular hyperbolas each of eccentricity $\sqrt{2}$

$\therefore e = e_1 = \sqrt{2} \Rightarrow e^2 + e_1^2 = 4$

4 **(b)**

Since, both the given hyperbolas are rectangular hyperbolas

$\therefore\ e=\sqrt{2}, e_1=\sqrt{2}$

Hence, $e^2+e_1^2=2+2=4$

5 **(d)**

Since, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, passes through (3, 0)and $(3\sqrt{2},2)$

$$\therefore\ \frac{9}{a^2}=1$$

$$\Rightarrow\quad a^2=9$$

and $\frac{9\times 2}{9}-\frac{4}{b^2}=1 \Rightarrow b^2=4$

$$\therefore\ e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{4}{9}}=\frac{\sqrt{13}}{3}$$

6 **(d)**

Let the equation of circles are

$S_1\equiv x^2+y^2+2x-3y+6=0$ ...(i)

$S_2\equiv x^2+y^2+x-8y-13=0$ ...(ii)

$\therefore$ Equation of common chord is

$S_1-S_2=0$

$\Rightarrow(x^2+y^2+2x-3y+6)-(x^2+y^2+x-8y-13)=0$

$\Rightarrow\ x+5y+19=0$ ...(iii)

In the given option only the point $(1,\ -4)$ satisfied the Eq. (iii)

7 **(d)**

Let $P(h,k)$ be the given point. Then, the chord of contact of tangents drawn from P to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\frac{hx}{a^2}+\frac{ky}{b^2}=1 \qquad \text{...(i)}$$

This subtends a right angle at the centre $C(0,0)$ of the ellipse. The combined equation of the pair of straight lines joining C to the points of intersection of (i) and the ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\left(\frac{hx}{a^2}+\frac{ky}{b^2}\right)^2$$

This equation represents a pair of perpendicular straight lines.

$$\therefore \frac{1}{a^2}-\frac{h^2}{a^4}+\frac{1}{b^2}-\frac{k^2}{b^4}=0\Rightarrow\frac{h^2}{a^4}+\frac{k^2}{b^4}=\frac{1}{a^2}+\frac{1}{b^2}$$

Hence, the locus of (h,k) is $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}+\frac{1}{b^2}$

8 **(c)**

The locus is a hyperbola.

9 **(a)**

Given equation of ellipse can be rewritten as

$$\frac{(x-1)^2}{1/8}+\frac{\left(y+\frac{3}{4}\right)^2}{1/16}=1$$

$$\therefore \text{Eccentricity}=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{8}{16}}=\frac{1}{\sqrt{2}}$$

11 **(a)**

The equation of the tangent at $(-3,2)$ to the parabola $y^2+4x+4y=0$ is

$2y+2(x-3)+2(y+2)=0$

$\Rightarrow 2x+4y-2=0$

$\Rightarrow x+2y-1=0$

Since the tangent at one end of the focal chord is parallel to the normal at the other end. Therefore, the slope of the normal at the other end of the focal chord is $-1/2$

12 **(c)**

Solving the equations of lines in pairs, we obtain that the vertices of the ΔABC are $A(0,6)$,$B(-2\sqrt{3},0)$ and $C(2\sqrt{3},0)$

Clearly, $AB=BC=CA$

So, ΔABC is an equilateral triangle. Therefore, centroid of the triangle ABC coincides with the circumcentre. Co-ordinates of the circumcentre are $O'(0,2)$ and the radius $=O'A=4$.

Hence, the equation of the circumcircle is

$(x-0)^2+(y-2)^2=4^2$ or, $x^2+y^2-4y=12$

13 **(c)**

Given, $r^2-4r(\cos\theta+\sin\theta)-4=0$ ...(i)

Put $x=r\cos\theta$, $y=r\sin\theta$, then $r^2=x^2+y^2$

$\therefore$ From Eq. (i)

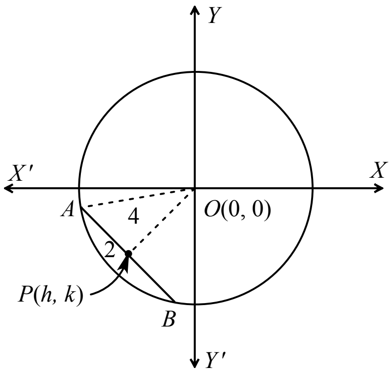
$x^2+y^2-4(x+y)-4=0$

$\Rightarrow x^2+y^2-4x-4y-4=0$

$\therefore$ Centre of circle is (2, 2)

14 **(c)**

Let $P(h,k)$ be the mid-point of a chord AB of length 4 units



In $\Delta\, OPA$, we have

$OA^2 = OP^2 + AP^2 \Rightarrow 4^2 = h^2 + k^2 + 2^2 \Rightarrow h^2 + k^2 = 12$

Hence, the locus of $P(h,k)$ is $x^2 + y^2 = 12$, which is a circle of radius $2\sqrt{3}$

15 **(b)**

Equation of normal at $P\ (a\sec\phi, b\tan\phi)$ is

$a\,x\cos\phi + by\cot\phi = a^2 + b^2$

Then, coordinates of L and M are

$\left(\frac{a^2+b^2}{a}.\sec\phi, 0\right)$ and $\left(0, \frac{a^2+b^2}{b}\tan\phi\right)$ respectively.

Let mid point of ML is $Q\ (h, k)$,

Then $h = \frac{(a^2+b^2)}{2a}\sec\phi$

$\therefore \sec\phi = \frac{2ah}{(a^2+b^2)}$ ...(i)

and $k = \frac{(a^2+b^2)}{2b}\tan\phi$

$\therefore \tan\phi = \frac{2bk}{(a^2+b^2)}$ ...(ii)

From Eqs.(i) and (ii), we get

$$\sec^2\phi - \tan^2\phi = \frac{4a^2h^2}{(a^2+b^2)^2} - \frac{4b^2k^2}{(a^2+b^2)^2}$$

Hence, required locus is

$$\frac{x^2}{\left(\frac{a^2+b^2}{2a}\right)} - \frac{y^2}{\left(\frac{a^2+b^2}{2b}\right)^2} = 1$$

Let eccentricity of this curve is e_1.

$\Rightarrow \left(\frac{a^2+b^2}{2a}\right)^2 = \left(\frac{a^2+b^2}{2a}\right)^2 (e_1^2 - 1)$

$\Rightarrow a^2 = b^2(e_1^2 - 1)$

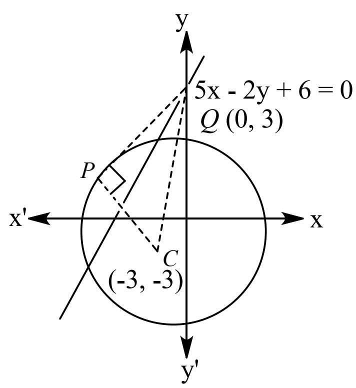
$\Rightarrow a^2 = a^2(e^2-1)(e_1^2-1)$ $[\because b^2 = a^2(e^2-1)]$

$\Rightarrow e^2e_1^2 - e^2 - e_1^2 + 1 = 1$

$\Rightarrow e_1^2(e^2-1) = e^2$

$\Rightarrow e_1 = \frac{e}{\sqrt{e^2-1}}$

16 **(b)**
Let (h,k) be the mid-point of a chord of the hyperbola $x^2-y^2=a^2$. Then, the equation of the chord is
$hx-ky=h^2-k^2$ [Using :$T=S'$]
$\Rightarrow y=\frac{h}{k}x+\frac{k^2-h^2}{k}$
This touches the parabola $y^2=4ax$
$\therefore \frac{k^2-h^2}{k}=\frac{a}{h/k}$ [Using:$c=a/m$]
$\Rightarrow h(k^2-h^2)=ak^2$
Hence, the locus of (h,k) is $x(y^2-x^2)=ay^2$ or, $y^2(x-a)=x^3$
17 **(c)**
$x^2+y^2+6x+6y-2=0$
Centre $(-3,-3)$, radius $=\sqrt{9+9+2}=\sqrt{20}$



Now, $QC=\sqrt{(-3)^2+6^2}=\sqrt{45}$
In right ΔCPQ
$PQ=\sqrt{45-20}=5$
18 **(d)**
We have, $2a=6, 2b=4$
$\therefore e=\sqrt{1-\frac{b^2}{a^2}} \Rightarrow e=\sqrt{\frac{5}{3}}$
So, distance between foci $=2ae=6\sqrt{\frac{5}{3}}=2\sqrt{5}$
and, length of the string $=2a+2ae=6+2\sqrt{5}$
19 **(b)**
The equation of a tangent to the given parabola is

$$y=mx+\frac{9}{4m}$$

If it passes through $(4,10)$, then

$10 = 4m + \frac{9}{4m}$

$\Rightarrow 16m^2 - 40m + 9 = 0$

$$\Rightarrow (4m-1)(4m-9) = 0 \Rightarrow m = \frac{1}{4}, \frac{9}{4}$$

20 **(b)**

We know that the area Δ of the triangle formed by the tangent drawn from (x_1, y_1) to the circle $x^2 + y^2 = a^2$ and their chord of contact is given by

$\Delta = \frac{a(x_1^2 + y_1^2 - a^2)^{3/2}}{x_1^2 + y_1^2}$

Here, the point is $P(4,3)$ and the circle is $x^2 + y^2 = 9$

$$\therefore \text{Required area} = \frac{3(4^2 + 3^2 - 9)^{3/2}}{4^2 + 3^2} \text{ sq. units} = \frac{192}{25} \text{ sq. units}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	B	B	B	D	D	D	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	C	B	B	C	D	B	B