DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 10

Topic :- CONIC SECTION

1 **(b)**

Given that, $S_1 \equiv x^2 + y^2 + 4x + 22y + c = 0$, bisects the circumference of the circle

$S_2 \equiv x^2 + y^2 - 2x + 8y - d = 0$

The common chord of the given circle is

$S_1 - S_2 = 0$

ie, $6x + 14y + c + d = 0$ ...(i)

So, Eq. (i) passes through the centre of the second circle, *ie*, $(1, -4)$

$\therefore \quad 6 + 56 + c + d = 0$

$\Rightarrow \quad c + d = 50$

2 **(d)**

We have, $a^2 = 16, b^2 = 9$

$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{\sqrt{7}}{4}$

Coordinates of S are $(\sqrt{7}, 0)$. Therefore, $CS = \sqrt{7}$

$\therefore CS$:Major axis $= \sqrt{7} : 2a = \sqrt{7} : 8$

3 **(d)**

The given points are the ends of the latusrectum where the normals are always at right angle

4 **(a)**

Let (h, k) be the coordinates of the centre of circle C_2. Then its equation is

$(x - h)^2 + (y - k)^2 = 5^2$

The equation of C_1 is $x^2 - y^2 = 4^2$ and so the equation of the common chord of C_1 and $C_2$is

$2\,hx + 2\,ky = h^2 + k^2 - 9$ ...(i)

Let p be the length of the perpendicular from the centre $(0,0)$ of C_1 to (i). Then,

$p = \left| \frac{h^2 + k^2 - 9}{\sqrt{4\,h^2 + 4\,k^2}} \right|$

The length of the common chord is $2\sqrt{4^2 - p^2}$ which will be of maximum length, if

$p = 0 \Rightarrow h^2 + k^2 - 9 = 0$ ...(ii)

Now, Slope of common chord $= \frac{3}{4}$

$\therefore -\frac{h}{k} = \frac{3}{4} \Rightarrow k = -\frac{4\,h}{3}$ ...(iii)

Putting the value of k in (ii), we get

$h = \pm \frac{9}{5} \Rightarrow k = \mp \frac{12}{5}$ [From (iii)]

Hence, the centres of circle C_2 are $(9/5, -12/5)$ and $(-9/5, 12/5)$

5 **(a)**

Equation of the normal at point $(bt_1^2, 2bt_1)$ on parabola is

$y = -t_1 x + 2bt_1 + bt_1^3$

It is also passes through $(bt_2^2, 2bt_2)$, then

$2bt_2 = t_1.bt_2^2 + 2bt_1 + bt_1^3$

$\Rightarrow 2t_2 - 2t_1 = t_1(t_1^2 - t_1^2)$

$\Rightarrow \quad 2 = -t_1(t_2 + t_1)$

$\Rightarrow t_2 = -t_1 - \frac{2}{t_1}$

6 **(a)**

Let the equation of tangent which is perpendicular to the line $3x + 4y = 7$, is $4x - 3y = \lambda \Rightarrow y = \frac{4}{3} x - \frac{\lambda}{3}$

Since, it is a tangent to the ellipse

$\therefore \left(\frac{\lambda}{3}\right)^2 = 9 \times \left(\frac{4}{3}\right)^2 + 4$ $[\therefore a^2 = 9, b^2 = 4]$

$\Rightarrow \lambda^2 = 180 \Rightarrow \lambda = \pm 6\sqrt{5}$

$\therefore$ Equation is $4x - 3y = \pm 6\sqrt{5}$

7 **(d)**

Any point on the hyperbola

$\frac{(x+1)^2}{16} - \frac{(y-2)^2}{4} = 1$, is of the form

$(4 \sec \theta - 1, 2 \tan \theta + 2)$

8 **(c)**

In the given equation we observe that the denominator of y^2 is greater than that of x^2. So, the two foci lie on y-axis and their coordinates are $(0, \pm be)$, where

$b = 5$ and $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

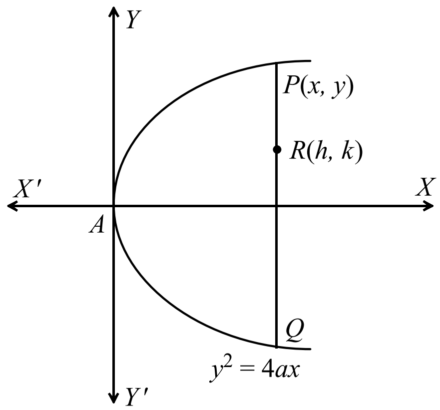
The focal distances of a point $P(x_1, y_1)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 > a^2$ are given by $b \pm ey_1$

Hence, required distances $= b \pm ey_1 = 5 \pm \frac{4}{5} y_1$

9 **(b)**

Let PQ be a double ordinate of $y^2 = 4\,ax$, and let $R(h,k)$ be a point of trisection. Let the coordinates of P be (x,y). Then,

$x = h$ and $y = 3\,k$



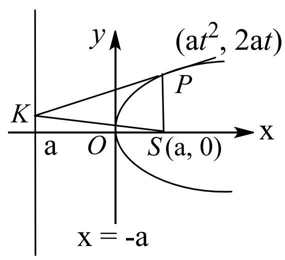
But, $(x - y)$ lies on $y^2 = 4\,ax$

$\therefore 9\,k^2 = 4\,ah$

Hence, the locus of (h,k) is $9y^2 = 4\,ax$

10 **(d)**

Let $P(at^2, 2at)$ any point on the parabola and focus is $(a, 0)$



The equation of tangent at P is $yt = x + at^2$

Since, it meets the directrix $x = -a$ at K

Then, the coordinate of K is $\left(-a, \frac{at^2 - a}{t}\right)$

Slope of $SP = m_1 = \frac{2at}{a(t^2 - 1)}$

Slope of $SK = m_2 = \frac{a(t^2 - 1)}{-2at}$

$$\therefore m_1 m_2 = \frac{2at}{a(t^2 - 1)} \cdot \frac{a(t^2 - 1)}{(-2at)} = -1$$

$\therefore \angle PSK = 90^\circ$

11 **(d)**

Since, $y = |x| + c$ and $x^2 + y^2 - 8|x| - 9 = 0$ both are symmetrical about y-axis for $x > 0$, $y = x + c$.

Equation of tangent to circle $x^2 + y^2 - 8x - 9 = 0$ which is parallel to $y = x + c$ is $y = (x - 4) + 5$

$\sqrt{1 + 1}$

$\Rightarrow y = x + (5\sqrt{2} - 4)$

For no solution $c > 5\sqrt{2} - 4$,

$\therefore c \in (5\sqrt{2} - 4, \infty)$

12 **(d)**

Centre is the point of intersection of two diameter, *ie*, the point of intersection of two diameters is $C(8, -2)$, therefore the distance from the centre to the point $P(6, 2)$ is

$r = CP = \sqrt{4 + 16} = \sqrt{20}$

13 **(a)**

Only the point (9,3) lies on the given circle

14 **(d)**

The equation of a tangent of slope m to the circle $x^2 + y^2 = a^2$ is $y = mx \pm a\sqrt{1 + m^2}$ and the coordinates of point of contact are

$$\left(\mp \frac{am}{\sqrt{1 + m^2}}, \pm \frac{a}{\sqrt{1 + m^2}}\right)$$

Here, $a = 5$ and $m = \tan 30 = 1/\sqrt{3}$

So, the coordinates of the points of contact are $\left(\mp \frac{5}{2}, \pm \frac{5\sqrt{3}}{2}\right)$

15 **(a)**

Given, $\frac{x^2}{32/5} + \frac{y^2}{32/9} = 1$

Let the equation of tangent be $y = mx + c$

$$y = mx \pm \sqrt{\frac{32}{5}m^2 + \frac{32}{9}} \quad ...(i)$$

$[\therefore c^2 = a^2m^2 + b^2 \text{for } a > b]$

Since, (2,3) lies on Eq.(i)

$$\Rightarrow 3 = m.2 \pm \sqrt{\frac{32}{5}m^2 + \frac{32}{9}}$$

$45(3 - 2m)^2 = 288m^2 + 160$

$\Rightarrow 108m^2 + 540m - 245 = 0$

$\therefore D = (540)^2 + 4.180.245 > 0 \Rightarrow D > 0$

$\Rightarrow$ Two values of m will exist

$\Rightarrow$ Two tangents will exist

Alternate

Let $S \equiv 5x^2 + 9y^2 - 32$

Now, $S(2,3) \equiv 20 + 81 - 32 > 0$

$\therefore$ Point (2,3) lies outside ellipse

Thus, two tangents can be drawn

16 **(d)**

As we know equation of tangent to the given hyperbola at (x_1, y_1) is $xx_1 - 2yy_1 = 4$ which is same as $2x + \sqrt{6}y = 2$

$\Rightarrow x_1 = 4$ and $y_1 = \sqrt{6}$

Thus, the point of contact is $(4, -\sqrt{6})$

17 **(b)**

Let (h,k) be the mid-point of a focal chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Then, its equation is

$$\frac{hx}{a^2} + \frac{ky}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$$

It passes through the focus $S(ae,0)$

$$\therefore \frac{he}{a} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$$

Hence, the locus of (h,k) is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{xe}{a}$

18 **(c)**

Given, $x = t^2 + 2t - 1$ ...(i)

and $y = 3t + 5 \Rightarrow t = \frac{y-5}{3}$ ...(ii)

On putting the value of t in Eq. (i), we get

$$x = \left(\frac{y-5}{3}\right)^2 + 2\left(\frac{y-5}{3}\right) - 1$$

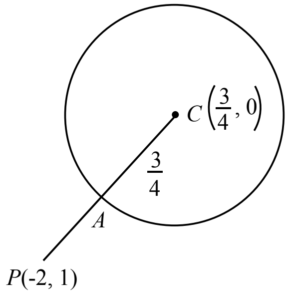
$$\Rightarrow x = \frac{1}{9}\{y^2 - 4y - 14\}$$

$$\Rightarrow (y-2)^2 = 9(x+2)$$

This is an equation of a parabola

19 **(b)**

We observe that the minimum distance between point P and the given circle is



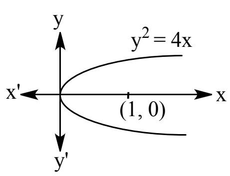
$PA = CP - CA = \frac{\sqrt{137}}{4} - \frac{3}{4} = \frac{\sqrt{137} - 3}{4} > 2$

So, there is no point on the circle whose distance from P is 2 units

20 **(b)**

Given curve is $y^2 = 4x$

Also, point (1, 0) is the focus of the parabola. It is clear from the graph that only normal is possible



ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	D	A	A	A	D	C	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	D	A	D	A	D	B	C	B	B