DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 1

Topic :- CONIC SECTION

1 **(d)**

The circle $x^2+y^2+2gx+2fy+c=0$ cuts an intercept of length $2\sqrt{f^2-c}$ on y-axis.

For the circle $x^2+y^2+4x-7y+12=0$, we have

$g=2, f=-7/2$ and $c=12$

$\therefore y-\text{intercept}=2\sqrt{f^2-c}=2\sqrt{\frac{49}{4}-12}=1$

2 **(a)**

$\because$ Eccentricity of ellipse $=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

$\because$ Eccentricity of hyperbola $=2$

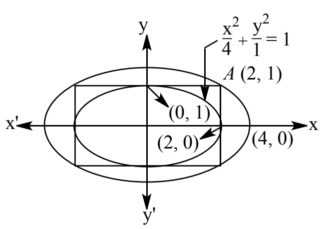
$\Rightarrow \sqrt{1+\frac{b^2}{64}} \Rightarrow 2$

$\Rightarrow 4=1+\frac{b^2}{64} \Rightarrow 192=b^2$

3 **(a)**

Let the equation of the required ellipse be $\frac{x^2}{16}+\frac{y^2}{b^2}=1$

But the ellipse passes through the point (2,1)



$\Rightarrow \frac{1}{4}+\frac{1}{b^2}=1$

$\Rightarrow \ \dfrac{1}{b^2} = \dfrac{3}{4} \Rightarrow b^2 = \dfrac{4}{3}$

Hence, equation is

$\dfrac{x^2}{16} + \dfrac{3y^2}{4} = 1$

$\Rightarrow x^2 + 12y^2 = 16$

4 **(b)**
We have,
$x = 2t + 1, y = t^2 + 2$
$\Rightarrow y = \left(\dfrac{x-1}{2}\right)^2 + 2$
$\Rightarrow (x-1)^2 = 4(y-2)$
The equation of the directrix of this parabola is
$y - 2 = -1$ or, $y = 1$ [Using $y = -a$]
5 **(c)**
Given equation can be rewritten as
$y^2 = \dfrac{4k}{4}\left(x - \dfrac{8}{k}\right)$
The standard equation of parabola is
$Y^2 = 4AX$, where $A = \frac{k}{4}$
$\therefore$ Equation of directrix is $X + \frac{k}{4} = 0$
$\Rightarrow x - \dfrac{8}{k} + \dfrac{k}{4} = 0$
But the given equation of directrix is $x - 1 = 0$
Since, both equations are same
$\therefore \ \dfrac{8}{k} - \dfrac{k}{4} = 1$
$\Rightarrow 32 - k^2 = 4k \Rightarrow k = -8, 4$
6 **(d)**
The equation of the ellipse is
$3(x+1)^2 + 4(y-1)^2 = 12$ or, $\frac{(x+1)^2}{2^2} + \frac{(y-1)^2}{(\sqrt{3})^2} = 1$
The equations of its major and minor axes are $y - 1 = 0$ and $x + 1 = 0$ respectively
7 **(a)**
Let mid point of the chord be (h, k), then equation of the chords be
$\dfrac{hx^2}{a^2} + \dfrac{ky^2}{b^2} - 1 = \dfrac{h^2}{a^2} + \dfrac{k^2}{b^2} - 1$
$\Rightarrow y = -\frac{b^2}{a^2} \cdot \frac{h}{k} x + \left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)\frac{b^2}{k}$ ...(i)
Since, line (i) is touching the circle $x^2 + y^2 = c^2$

$\therefore \left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)\frac{b^4}{k^2}=c^2\left(1+\frac{b^4h^2}{a^4k^2}\right)$

Hence, locus is $(b^2x^2+a^2y^2)^2=c^2(b^4x^2+a^4y^2)$

8 **(c)**

Given curve is $y^2=4x$ ...(i)

Let the equation of line be $y=mx+c$

Since, $\frac{dy}{dx}=m=1$ and above line is passing through the point (0, 1)

$1=1(0)+c \Rightarrow c=1$

$y=x+1$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x=1$ and $y=2$

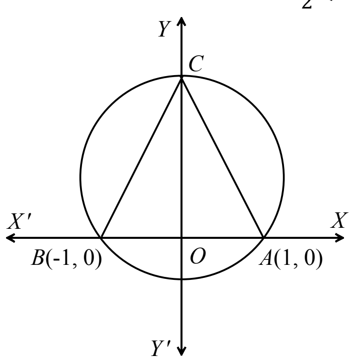
This shows that line touch the curve at one point. So, length of intercept is zero.

9 **(c)**

We have, $AB=2$

Since $\Delta\ ABC$ is equilateral. Therefore,

$AC=BC=2$ and $OC=\frac{\sqrt{3}}{2}(\text{Side})=\sqrt{3}$



Thus, the coordinates of C are $(0,\sqrt{3})$

Let the circumcircle of $\Delta\ ABC$ be

$x^2+y^2+2gx+2fy+c=0$

It passes through $(1,0),(-1,0)$ and $(0,\sqrt{3})$

$\therefore 1+2g+c=0, 1-2g+c=0$ and $3+2\sqrt{3}\,f+c=0$

Solving these three equations, we get

$g=0, c=-1$ and $f=-\frac{1}{\sqrt{3}}$

Thus, the equation of the circumcircle is

$x^2+y^2-\frac{2}{\sqrt{3}}y-1=0$

10 **(c)**

The coordinates of P be (h,k)
Let the equation of a tangent from $P(h,k)$ to the circle
$x^2+y^2=a^2$ be $y=mx+a\sqrt{1+m^2}$
Since $P(h,k)$ lies on $y=mx+a\sqrt{1+m^2}$
$\therefore k=mh+a\sqrt{1+m^2}$
$\Rightarrow (k=mh)^2=a(1+m^2)$
$\Rightarrow m^2(h^2-a^2)-2\,mkh+k^2-a^2=0$
This is a quadric in m. Let the two roots be m_1 and m_2. Then,
$$m_1+m_2=\frac{2\,hk}{h^2-a^2}$$
But, $\tan\alpha=m_1$, $\tan\beta=m_2$ and it is given that
$\cot\alpha+\cot\beta=0$
$$\Rightarrow\frac{1}{m_1}+\frac{1}{m_2}=0\Rightarrow m_1+m_2=0\Rightarrow\frac{2\,hk}{k^2-a^2}=0\Rightarrow hk=0$$
Hence, the locus of (h,k) is $xy=0$

11 **(b)**

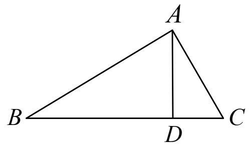
We have,
$x=2+t^2, y=2\,t+1$
$\Rightarrow x-2=t^2$ and $y-1=2t$
$\Rightarrow (y-1)^2=4t^2$ and $x-2=t^2$
$\Rightarrow (y-1)^2=4(x-2)$,
Which is a parabola with vertex at $(2,1)$

12 **(b)**

Given equation of ellipse is
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a<b)$$
It is a vertical ellipse with foci $(0,\ \pm be)$
Equation of any tangent line to the above ellipse is
$y=mx+\sqrt{a^2m^2+b^2}$
$\therefore$ Required product
$$=\left|\frac{-be+\sqrt{a^2m^2+b^2}}{\sqrt{m^2+1}}\right|\left|\frac{be+\sqrt{a^2m^2+b^2}}{\sqrt{m^2+1}}\right|$$
$$=\left|\frac{a^2m^2+b^2-b^2e^2}{m^2+1}\right|$$
$$=\left|\frac{a^2m^2+b^2(1-e^2)}{m^2+1}\right|$$
$$=\left|\frac{a^2m^2+a^2}{m^2+1}\right|\quad[\because a^2=b^2(1-e^2)]$$
$$=a^2$$

13 **(b)**

Since, $\angle ADB = \angle ADC = 90°$, circle on AB and AC as dismeters pass through D and therefore the altitude AD is the common chord. Similarly, the other two common chords are the other two altitudes and hence they concur at the orthocenter



14 **(b)**

Given equation of ellipse can be rewritten as

$$\frac{(x-2)^2}{25}+\frac{(y+3)^2}{16}=1 \Rightarrow \frac{X^2}{25}+\frac{Y^2}{16}=1$$

Where $X = x-2, Y = y+3$

Here, $a > b$

$$\therefore \ e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$

$\therefore$ Focus $(\pm ae, 0) = (\pm 3,0)$

$\Rightarrow \ x-2 = \pm 3, y+3 = 0$

$\Rightarrow \ x = 5, \ = -1, y = -3$

$\therefore$ Foci are $(-1,-3)$and $(-1, \ -3)$

Distance between (2,-3) and (-1,-3)

$= \sqrt{(2+1)^2+(-3+3)^2} = 3$

and distance between $(2, \ -3)$ and $(5, \ -3)$

$= \sqrt{(2-5)^2+(-3+3)^2} = 3$

Hence, sum of the distance of point $(2, \ -3)$from the foci

$= \ 3+3 = 6$

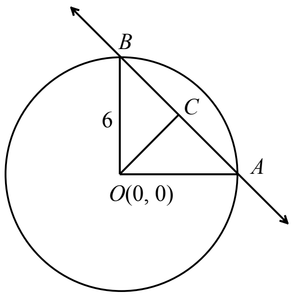
15 **(c)**

We have,

$OC =$ Length of the perpendicular from (0,0) on the line $3\,x+4\,y-15=0$

$\Rightarrow OC = \dfrac{15}{\sqrt{3^2+4^2}} = 3$

$\therefore AB = 2\,AC = 2\sqrt{OA^2-OC^2} = 2\sqrt{36-9} = 6\sqrt{3}$



16 **(a)**

We know that the normal at $(at_1^2, 2at_1)$ meets the parabola at $\left(at_2^2, 2\,at_2\right)$, if $t_2 = -\,t_1 - \frac{2}{t_1}$

Here, the normal is drawn at (x_1, x_1)

$\therefore at_1^2 = 2\,at_1 \Rightarrow t_1 = 2 \Rightarrow t_2 = -2 - \dfrac{2}{2} = -3$

The coordinates of the end points of the normal chord are $P(4a, 4a)$ and $Q(9a, -6a)$

Clearly, PQ makes a right angle at the focus $(a, 0)$

17 **(a)**

The equation of the family of circles touching $2x - y - 1 = 0$ at $(3,5)$ is

$(x-3)^2 + (y-5)^2 + \lambda(2x - y - 1) = 0$ ...(i)

It has its centre $\left(-\lambda + 3, \frac{\lambda + 10}{2}\right)$ on the line $x + y = 5$

$\therefore -\lambda + 3 + \dfrac{\lambda + 10}{2} = 5 \Rightarrow \lambda = 6$

Putting $\lambda = 6$ in (i), we get

$x^2 + y^2 + 6x - 16y + 28 = 0$

As the equation of the required circle

18 **(d)**

Given that equation of parabola is $y^2 = 9x$

On comparing with $y^2 = 4ax$, we get $a = \frac{9}{4}$

Now, equation of tangent to the parabola $y^2 = 9x$ is

$y = mx + \frac{9/4}{m}$ ...(i)

If this tangent passing through the point (4, 10), then

$10 = 4m + \dfrac{9}{4m}$

$\Rightarrow 16m^2 - 40m + 9 = 0$

$\Rightarrow (4m - 9)(4m - 1) = 0$

$\Rightarrow m = \dfrac{1}{4}, \dfrac{9}{4}$

On putting the values of m in Eq. (i)

$4y = x + 36$ and $4y = 9x + 4$

$\Rightarrow x - 4y + 36 = 0$ and $9x - 4y + 4 = 0$

19 **(a)**

Required length $= y$-intercept $= 2\sqrt{\frac{9}{4} - 2} = 1$

20 **(b)**

Given equation is $xy = a$

On differentiating, we get

$$x\frac{dy}{dx} + y = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(a,1)} = -\frac{1}{a}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	A	A	B	C	D	A	C	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	B	B	C	A	A	D	A	B