DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 9

Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(b)**

Given that α_1,α_2 are the roots of the equation $ax^2 + bx + c = 0$, then

$\alpha_1 + \alpha_2 = -\frac{b}{a}$ and $\alpha_1\alpha_2 = \frac{c}{a}$ ...(i)

Now, β_1,β_2 are the roots of $px^2 + qx + r = 0$, then

$\beta_1 + \beta_2 = -\frac{q}{p}$ and $\beta_1\beta_2 = \frac{r}{p}$ ...(ii)

Given system is $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$.

$$\Rightarrow \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$$

Now, $\frac{\alpha_1\alpha_2}{\beta_1\beta_2} = \frac{\frac{c}{a}}{\frac{r}{p}}$

$\Rightarrow \frac{\alpha_1}{\beta_1} \cdot \frac{\alpha_2}{\beta_2} = \frac{cp}{ar}$ ...(iii)

Since, $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{\beta_1}{\beta_2} \Rightarrow \frac{\alpha_1^2}{\alpha_2^2} = \frac{\beta_1^2}{\beta_2^2}$

$\Rightarrow \frac{\alpha_1^2 + \alpha_2^2}{\alpha_2^2} = \frac{\beta_1^2 + \beta_2^2}{\beta_2^2}$ (on adding 1 on both sides)

$$\Rightarrow \frac{\alpha_2^2}{\beta_2^2} = \frac{\alpha_1^2 + \alpha_2^2}{\beta_1^2 + \beta_2^2}$$

$$= \frac{(\alpha_1 + \alpha_2)^2 - 2\alpha_1\alpha_2}{(\beta_1 + \beta_2)^2 - 2\beta_1\beta_2}$$

On substituting the values from Eqs. (i), (ii) and (iii), we get

$$\frac{cp}{ar} = \frac{\frac{b^2}{a^2} - 2\left(\frac{c}{a}\right)}{\frac{q^2}{p^2} - 2\left(\frac{r}{p}\right)} = \frac{(b^2 - 2ac)p^2}{(q^2 - 2pr)a^2}$$

$$\Rightarrow \frac{c}{r} = \frac{pb^2 - 2acp}{q^2 a - 2apr}$$

$$\Rightarrow b^2 rp - 2acpr = q^2 ac - pr2ac$$

$$\Rightarrow b^2 pr = q^2 ac$$

2 **(b)**

$(1-\omega+\omega^2)(1-\omega^2+\omega^3.\omega)$

$(1-\omega^3.\omega+\omega^6.\omega^2)(1-\omega^6.\omega^2+\omega^{15}.\omega)$... upto $2n$

$=(1\omega+\omega^2)(1-\omega^2+\omega)$

$(1-\omega+\omega^2)(1-\omega^2+\omega)$... upto $2n$

$=[(-2\omega)(-2\omega^2)]\times[(-2\omega)(-2\omega^2)]\times$ ... upto $2n$

$=(2^2\omega^3)\times(2^2\omega^3)\times$ .... upto n

$=[2^2\times 2^2\times 2^2\times$...upto n$]=2^{2n}$

3 **(d)**

Given, α and β are different complex numbers and

$|\beta|=1$

$$\therefore \quad \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=\frac{|\beta-\alpha|}{|\beta\bar{\beta}-\bar{\alpha}\beta|}=\frac{|\beta-\alpha|}{|\beta||\bar{\beta}-\bar{\alpha}|}=1$$

4 **(d)**

$$\frac{\log_{c+b} a+\log_{c-a} a}{2\log_{c+b} a\,.\log_{c-b} a}$$

$$=\frac{\frac{\log a}{\log(c+b)}+\frac{\log a}{\log(c-b)}}{2\frac{\log a}{\log(c+b)}\cdot\frac{\log a}{\log(c-b)}}$$

$$=\frac{\log a\{\log(c-b)+\log(c+b)\}}{2(\log a)^2}=\frac{\log(c^2-b^2)}{2\log a}$$

$$=\frac{\log a^2}{\log a^2} \qquad (\because a^2+b^2=c^2)$$

$=1$

5 **(b)**

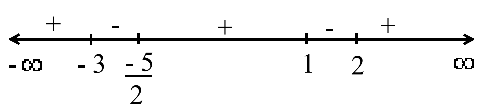
We have,

$$\frac{10x^2+17x-34}{x^2+2x-3}<8$$

$$\Rightarrow\frac{10x^2+17x-34-8x^2-16x+24}{x^2+2x-3}<0$$

$$\Rightarrow\frac{2x^2+x-10}{x^2+2x-3}<0$$

$$\Rightarrow\frac{(2x+5)(x-2)}{(x+3)(x-1)}<0\Rightarrow x\in(-3,-5/2)\cup(1,2)$$



6 **(b)**

$$\left(\frac{1+\cos\phi+i\sin\phi}{1+\cos\phi-i\sin\phi}\right)^n=u+iv$$

$$\Rightarrow \left(\frac{2\cos^2\frac{\phi}{2}+2i\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{2\cos^2\frac{\phi}{2}-2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}\right)^n = u+iv$$

$$\Rightarrow \left(\frac{e^{i\frac{\phi}{2}}}{e^{-i\frac{\phi}{2}}}\right)^n = u+iv \quad \Rightarrow \quad (e^{in\phi}) = u+iv$$

$\Rightarrow \cos n\phi + i\sin n\phi = u+iv$

$\Rightarrow u = \cos n\phi, \ v = \sin n\phi$

7 **(c)**

We have,

$2x^4 + 5x^2 + 3 > 0$ for all $x \in R$

So, $2x^4 + 5x^2 + 3 = 0$ has no real root

8 **(c)**

Given, α, β are the roots of $x^2 - 2x + 4 = 0$

$\therefore \alpha + \beta = 2$ ...(i)

And $\alpha\beta = 4$ ...(ii)

Now, $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{4 - 4 \times 4} = \sqrt{-12}$

$\Rightarrow \alpha - \beta = 2\sqrt{3}i$ ...(iii)

On solving Eqs. (i) and (ii0, we get

$$\alpha = \frac{2+2\sqrt{3}i}{2} = -2\left(\frac{-1-\sqrt{3}i}{2}\right) = -2\omega^2$$

And $\beta = \frac{2-2\sqrt{3}i}{2} = -2\left(\frac{-1+\sqrt{3}i}{2}\right) = -2\omega$

Now, $\alpha^6 + \beta^6 = (-2\omega^2)^6 + (-2\omega)^6$

$= 64(\omega^3)^4 + 64(\omega^3)^2$

$= 128 \quad [\because \omega^3 = 1]$

9 **(b)**

We have, $|z+4| \le 3 \Rightarrow -3 \le z + 4 \le 3$

$\Rightarrow -6 \le z + 1 \le 0 \Rightarrow 0 \le -(z+1) \le 6$

$\Rightarrow 0 \le |z+1| \le 6$

Hence, greatest and least values of $|z+1|$ are 6 and 0 respectively

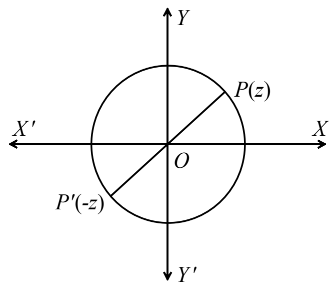
10 **(a)**

Let $P(z)$ be any point on the circle

$OP = OP''$

$\Rightarrow |z| = |z_1|$

$\Rightarrow |z|^2 = |z_1|^2 \Rightarrow z\bar{z} = z_1\overline{z_1} \Rightarrow \frac{z}{z_1} = \frac{\bar{z}_1}{z}$



11 **(c)**

It is given that $x+1$ be a factor of $f(x)$ given by

$f(x)=x^4+(p-3)x^3-(3p-5)x^2+(2p-9)x+6$

$\therefore f(-1)=0$

$\Rightarrow 1-p+3-3p+5-2p+9+6=0$

$\Rightarrow 6p=24 \Rightarrow p=4$

12 **(a)**

Let $\alpha \in A \cap B$. Then,

$\alpha \in A \cap B$

$\Rightarrow \alpha \in A$ and $\alpha \in B$

$\Rightarrow f(\alpha)=0$ and $g(\alpha)=0$

$\Rightarrow [f(\alpha)]^2+[g(\alpha)]^2=0$

$\Rightarrow \alpha$ is a root of $[f(x)]^2+[g(x)]^2=0$

13 **(d)**

Here, $\alpha+\beta=-p$ and $\alpha\beta=q$

Now, $(\alpha+\beta)x-\frac{\alpha^2+\beta^2}{2}x^2+\frac{\alpha^3+\beta^3}{3}x^3-\dots$

$=\left(\alpha x-\frac{\alpha^2x^2}{2}+\frac{\alpha^3x^3}{3}-\dots\right)+\left(\beta x-\frac{\beta^2x^2}{2}+\frac{\beta^3x^3}{3}-\dots\right)$

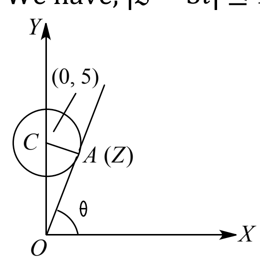
$=\log(1+\alpha x)+\log(1+\beta x)$

$=\log\{1+(\alpha+\beta)x+\alpha\beta x^2\}$

$=\log(1-px+qx^2)$

14 **(a)**

We have, $|z-5i|\le 1$



Let $\theta=\angle AOX=$ min.amp (z),

$\therefore \angle AOC=90°-\theta$

$\Rightarrow \sin(90°-\theta)=\frac{1}{5}$

$\Rightarrow \cos\theta = \frac{1}{5}$

$\therefore \; z = OA\cos\theta + \; i\, OA \sin\theta$

$\Rightarrow \; z = \sqrt{5^2 - 1}\left(\frac{1}{5}\right) + i\sqrt{5^2 - 1}\sqrt{1 - \frac{1}{5^2}}$

$= \frac{2\sqrt{6}}{5}(1 + i\, 2\sqrt{6})$

15 **(b)**

Since α,β are the roots of the equation $x^2 + px + 1$ and γ,δ are the roots of the equation $x^2 + qx + 1 = 0$

$\therefore \alpha^2 + p\,\alpha + 1 = 0, \beta^2 + p\,\beta + 1 = 0,$

$\gamma^2 + q\,\gamma + 1 = 0$ and $\delta^2 + q\,\delta + 1 = 0$ ...(i)

Also, $\alpha + \beta = -p, \alpha\,\beta = 1, \gamma + \delta = -q$ and $\gamma\,\delta = 1$

$\therefore (\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$

$= \{\alpha\,\beta - \gamma(\alpha + \beta) + \gamma^2\}\{\alpha\,\beta + \delta(\alpha + \beta) + \delta^2\}$

$= (\gamma^2 + p\,\gamma + 1)(\delta^2 - p\,\delta + 1)$

$= (p\gamma - q\gamma)(-q\,\delta - p\,\delta)$ [Using (i)]

$= (p + q)(q - p)\gamma\,\delta = (q^2 - p^2)$

16 **(a)**

Since, $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow \; |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$\Rightarrow \; \cos(\theta_1 - \theta_2) = 1 = \cos 0°$

$\Rightarrow \; \theta_1 - \theta_2 = 0 \Rightarrow \; \theta_1 = \theta_2$

$\Rightarrow \; \arg(z_1) = \arg(z_2)$

17 **(a)**

$\sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right\} = \sin\left\{(\omega + \omega^2)\pi - \frac{\pi}{4}\right\}$

$= \sin\left(-\pi - \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

18 **(d)**

Let $f(x) = x^3 - 3x + a$

If $f(x)$ has distinct roots between 0 and 1. Then,

$f'(x) = 0$ has a root between 0 and 1

But, $f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 \Rightarrow x = \pm\; 1$

Clearly, $f'(x) = 0$ does not have any root between 0 and 1.

So, $f(x)$ does not have distinct roots between 0 and 1 for any value of a

19 **(c)**

It is given that α,β are the roots of the equation

$375x^2 - 25x - 2 = 0$

$$\therefore \alpha+\beta=\frac{1}{15} \text{ and } \alpha\,\beta=\frac{-2}{375}$$

$$\therefore \lim_{n\to\infty}\sum_{r=1}^{n} S_r = \lim_{n\to\infty}\sum_{r=1}^{n}(\alpha^r+\beta^r)$$

$$\Rightarrow \lim_{n\to\infty}\sum_{r=1}^{n} S_r = (\alpha+\alpha^2+\alpha^3+\ldots\infty)+(\beta+\beta^2+\beta^3+\ldots\infty)$$

$$\Rightarrow \lim_{n\to\infty}\sum_{r=1}^{n} S_r = \frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta} \quad [\because |\alpha|<1, |\beta|<1]$$

$$\Rightarrow \lim_{n\to\infty}\sum_{r=1}^{n} S_r = \frac{\alpha+\beta-2\,\alpha\,\beta}{1-(\alpha+\beta)+\alpha\,\beta} = \frac{\frac{1}{15}+\frac{4}{375}}{1-\frac{1}{15}-\frac{2}{375}} = \frac{29}{348}$$

20 **(d)**

We have,

$y=\tan x \cot 3x$

$$\Rightarrow y=\frac{\tan x}{\tan 3x}$$

$$\Rightarrow y=\frac{\tan x(1-3\tan^2 x)}{3\tan x-\tan^3 x}$$

$$\Rightarrow y=\frac{1-3\tan^2 x}{3-\tan^2 x}$$

$$\Rightarrow \tan^2 x=\frac{3y-1}{y-3}$$

$$\Rightarrow \frac{3y-1}{y-3}\geq 0 \qquad [\because \tan^2 x\geq 0]$$

$$\Rightarrow y\leq\frac{1}{3} \text{ or } y>3$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	D	D	B	B	C	C	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	D	A	B	A	A	D	C	D