DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 8

Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(b)**

Given, $(\sqrt{5}+\sqrt{3}i)^{33} = 2^{49}z$

Let $\sqrt{5} = r\cos\theta, \quad \sqrt{3} = r\sin\theta$

$\therefore \quad r^2 = 5+3 \Rightarrow r = 2\sqrt{2}$

$\therefore \quad (r\cos\theta + ir\sin\theta)^{33} = 2^{49}z$

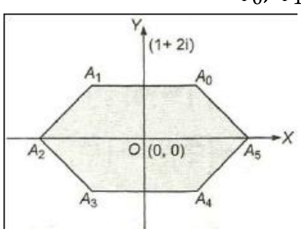
$\Rightarrow \quad |r^{33}(\cos 33\theta + i\sin 33\theta)| = |2^{49}z|$

$\Rightarrow \quad (2\sqrt{2})^{33}|\cos 33\theta + i\sin 33\theta| = 2^{49}|z|$

$\Rightarrow \quad 2^{\frac{99}{2}}(1) = 2^{49}|z| \quad \Rightarrow \quad |z| = \sqrt{2}$

2 **(d)**

Let the vertices be $z_0, z_1, \dots, z_5$ w.r.t. centre O at origin and $|z_0| = \sqrt{5}$



$\Rightarrow A_0A_1 = |z_1 - z_0|$

$= |z_0e^{i\theta} - z_0|$

$= |z_0||\cos\theta + i\sin\theta - 1|$

$= \sqrt{5}\sqrt{(\cos\theta-1)^2 + \sin^2\theta}$

$= \sqrt{5}\sqrt{2(1-\cos\theta)}$

$= \sqrt{5}.2\sin\frac{\theta}{2}$

$\Rightarrow A_0A_1 = \sqrt{5}\, 2\sin\left(\frac{\pi}{6}\right) = \sqrt{5} \ \left(\because \theta = \frac{2\pi}{6} = \frac{\pi}{3}\right) \dots \text{(i)}$

Similarly, $A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_0 = \sqrt{5}$

Hence, the perimeter of regular hexagon

$= A_0A_1 + A_1A_2 + A_2A_3 + A_3A_4 + A_4A_5 + A_5A_0$

$= 6\sqrt{5}$

3 **(d)**

Let $z=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$, then by using De Moivre's theorem

$\therefore\ z^k=\cos\frac{2\pi k}{7}+i\sin\frac{2\pi k}{7}$ ...(i)

Let $S=\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$

$=\sum_{k=1}^{6}\left[(-i)\left(\cos\frac{2\pi k}{7}+i\sin\frac{2\pi k}{7}\right)\right]$

$$=(-i)\sum_{k=1}^{6}\left(\cos\frac{2\pi k}{7}+i\sin\frac{2\pi k}{7}\right)$$

$=(-i)\sum_{k=1}^{6}z^k$ [from Eq.(i)]

$=(-i)[z+z^2+z^3+...+z^6]$

It is GP of which the first term is z, number of terms is 6 and the common ratio is

$$z=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}\neq 1$$

$$\therefore\ S=(-i)\frac{z(1-z^6)}{1-z}$$

$$=(-i)\frac{z-z^7}{1-z}$$

$$=(-i)\frac{z-z^7}{1-z}=i\quad\left[\begin{array}{l}\because\ z^7=\left(\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}\right)^7\\ \quad=\cos 2\pi+i\sin 2\pi=1\end{array}\right]$$

4 **(d)**

Let $\alpha,\beta\ and\ \gamma$ be the roots of the given equation

$\therefore\ \alpha+\beta+\gamma=-2, \alpha\beta+\beta\gamma+\gamma\alpha=-4$

And $\alpha\beta\gamma=-1$

Let the required cubic equation has the roots 3α, 3β and 3γ.

$\therefore 3\alpha+3\beta+3\gamma=-6$

$3\alpha.3\beta+3\beta.3\gamma+3\gamma.3\alpha=-36$

And $3\alpha.3\beta.3\gamma=-27$

$\therefore$ Required equation is

$x^3-(-6)x^2+(-36)x-(-27)=0$

$\Rightarrow x^3+6x^2-36x+27=0$

5 **(a)**

Since, $D>0,\sin^2 a-4\sin a(1-\cos a)>0$

$\Rightarrow\sin a>0$ or $(\sin a-4+4\cos a)>0$

$\Rightarrow a\in(0,\pi)$ or $\frac{1-\cos a}{\sin a}<\frac{1}{4}$

$\Rightarrow a\in(0,\pi)$ or $a\in\left(0,2\tan^{-1}\left(\frac{1}{4}\right)\right)$

$$\Rightarrow a\in\left(0,2\tan^{-1}\left(\frac{1}{4}\right)\right)$$

6 **(b)**

Since, α , β are the roots of equation $x^2+bx+c=0$.

Here, $D = b^2 - 4c > 0$ because $c < 0 < b$. So, roots are real and unequal.
Now, $\alpha + \beta = -b < 0$ and $\alpha\,\beta = c < 0$
$\therefore$ One root is positive and the other is negative, the negative root being numerically bigger. As $\alpha < \beta$, α is the negative root while β is the positive root. So, $|\alpha| > \beta$ and $\alpha < 0 < \beta$.

7 **(d)**

Given, $x^2 - \sqrt{3}x + 1 = 0$

$$\Rightarrow \quad x = \frac{\sqrt{3} \pm \sqrt{3-4}}{2} = \frac{\sqrt{3} \pm i}{2} = \cos\frac{\pi}{6} \pm i \sin\frac{\pi}{6}$$

$$\Rightarrow \quad x^n = \cos\frac{n\pi}{6} \pm i \sin\frac{n\pi}{6}$$

And $\frac{1}{x^n} = \cos\frac{n\pi}{6} \pm i \sin\frac{n\pi}{6}$

$$\therefore \quad x^n - \frac{1}{xn} = \pm\, 2i \sin\frac{n\pi}{6}$$

$$\Rightarrow \quad \left(x^n - \frac{1}{x^n}\right)^2 = -4\sin^2\frac{n\pi}{6}$$

Hence, $\sum_{n=1}^{24}\left(x^n - \frac{1}{x^n}\right)^2$

$$= -4\left[\sin^2\frac{\pi}{6} + \sin^2\frac{2\pi}{6} + \dots + \sin^2\frac{24\pi}{6}\right]$$

$$= -4(12) = -48$$

8 **(d)**

We have,

$$|x^2 - x - 6| = \begin{cases} x^2 - x - 6, \text{ if } x \le -2 \text{ or } x \ge 3 \\ -(x^2 - x - 6), \text{ if } -2 < x < 3 \end{cases}$$

<u>CASE I</u> *When* $x \le -2$ or, $x \ge 3$
In this case, we have $|x^2 - x - 6| = x^2 - x - 6$
$\therefore |x^2 - x - 6| = x + 2$
$\Rightarrow x^2 - x - 6 = x + 2$
$\Rightarrow x^2 - 2x - 8 = 0$
$\Rightarrow (x-4)(x+2) = 0$
$\Rightarrow x = -2, 4$
CASE II *When* $-2 < x < 3$
In this case, we have $|x^2 - x - 6| = -(x^2 - x - 6)$
$|x^2 - x - 6| = x + 2$
$\Rightarrow -(x^2 - x - 6) = x + 2$
$\Rightarrow x^2 - 4 = 0$
$\Rightarrow x = \pm\, 2$
$\Rightarrow x = 2 \qquad [\because 2 \in (-2, 3)]$
Hence, the roots are $-2, 2, 4$

9 **(d)**

We have,

$$\begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$$

$$= \begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2$$

Now,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & \alpha-1 & \beta-1 \\ 0 & \alpha^2-1 & \beta^2-1 \end{vmatrix} \quad \begin{bmatrix} \text{Applying } R_2 \to R_2 - R_1, \\ R_3 \to R_3 - R_1 \end{bmatrix}$$

$$= (\alpha-1)(\beta^2-1) - (\beta-1)(\alpha^2-1)$$
$$= \alpha\beta^2 - \alpha - \beta^2 - \alpha^2\beta + \beta + \alpha^2$$
$$= (\alpha^2-\beta^2) - (\alpha-\beta) - \alpha\beta(\alpha-\beta)$$
$$= (\alpha-\beta)[\alpha+\beta-1-\alpha\beta]$$
$$= \sqrt{(\alpha+\beta)^2 - 4\,\alpha\beta}\,\{\alpha+\beta-1-\alpha\,\beta\}$$
$$= \sqrt{\frac{b^2-4ac}{a^2}}\left\{-\frac{b}{a}-1-\frac{c}{a}\right\} = -\sqrt{\frac{b^2-4ac}{a^2}}\left(\frac{a+b+c}{a}\right)$$

Hence,

$$\begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$$

$$= \left\{-\sqrt{\frac{b^2-4ac}{a^2}}\left(\frac{a+b+c}{a}\right)\right\}^2 = \frac{(b^2-4ac)(a+b+c)^2}{a^4}$$

10 **(d)**

We have,

$z_k = e^{\frac{i\,2\,\pi\,k}{n}}, \qquad k = 0,1,2,\ldots,n-1$

$\therefore |z_k| = \left|e^{\frac{i\,2\,\pi\,k}{n}}\right| = 1 \quad \text{for all} = 0,1,2,\ldots n-1$

$\Rightarrow |z_k| = |z_{k+1}| \qquad \text{for all } k = 0,1,2,\ldots,n-1$

11 **(a)**

Here, $\alpha+\beta = 1+n^2$ and $\alpha\beta = \frac{1+n^2+n^4}{2}$

Now, $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (1+n^2)^2 - (1+n^2+n^4) = n^2$

12 **(b)**

Since, 4is a root of $x^2 + ax + 12 = 0$

$\therefore 16+4a+12=0 \Rightarrow a=-7$
Let the roots of the equation $x^2 + ax + b = 0$ be α and α
$\therefore 2\alpha = -a$
$\Rightarrow \alpha = \frac{7}{2}$
And $\alpha.\alpha = b$
$\Rightarrow \left(\frac{7}{2}\right)^2 = b$
$\Rightarrow b = \frac{49}{4}$
13 **(d)**
$\log_{140} 63 = \log_{2^2\times5\times7}(3\times3\times7)$
$= \frac{\log_2(3\times3\times7)}{\log_2(2^2\times5\times7)}$
$= \frac{2\log_2 3 + \log_2 7}{2\log_2 2 + \log_2 5 + \log_2 7}$
$= \frac{2a+\frac{1}{c}}{2+b+\frac{1}{c}} = \frac{2ac+1}{2c+bc+1}$
14 **(d)**
We have,
$(1-i)^n = 2^n$
$\Rightarrow |1-i|^n = |2|^n$
$\Rightarrow (\sqrt{2})^n = 2^n \Rightarrow 2^{n/2} = 2^n \Rightarrow 2^{n/2} = 1 \Rightarrow n = 0$
So, there is no non-zero integral solution of the given equation
15 **(a)**
We have the following cases:
<u>CASE I</u> $When\ x<0$
In this case, we have $Sgn\ x = -1$
$\therefore x^2 - 5x - (Sgn\ x)6 = 0$
$\Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2,3$
But, $x < 0$. So, the equation has no solution in this case.
<u>CASE II</u> $When\ x > 0$
In this case, we have $Sgn\ x = 1$
$\therefore x^2 - 5x - (Sgn\ x)6 = 0$
$\Rightarrow x^2 - 5x - 6 = 0$
$\Rightarrow (x-6)(x+1) = 0 \Rightarrow x = -1,6 \Rightarrow x = 6\ [\because x > 0]$
Hence, the given equation has only one solution
16 **(a)**
We have,
$z^n = (1+z)^n$
$\Rightarrow |z^n| = |(1+z)^n|$
$\Rightarrow |z|^n = |1+z|^n$

$\Rightarrow |z| = |1 + z|$

$\Rightarrow |z - 0| = ||z - (-1)|$

$\Rightarrow z$ lies on the perpendicular bisector of the segment joing $(0,0)$ and $(0, -1)$

$\Rightarrow z = -\frac{1}{2} \Rightarrow \text{Re}\,(z) < 0$

17 **(a)**

Given, $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$

$= (1+\omega)(-\omega)(1+\omega)(1+\omega^2)$

$[\because 1+\omega+\omega^2 = 0 \text{ and } \omega^4 = \omega]$

$= (1+\omega)^2(-\omega-\omega^3)$

$= (1+\omega^2+2\omega)(-\omega-1)$

$= (\omega)(\omega^2) = 1$

18 **(d)**

We have,

$$\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = \begin{vmatrix} 6i & 0 & 1 \\ 4 & 0 & -1 \\ 20 & 0 & i \end{vmatrix} \text{Applying } C_2 \rightarrow C_2 + 3i\, C_3$$

$= 0 = 0 + 0i$

$\therefore x = 0, y = 0$

19 **(c)**

Since z_1, z_2, z_3 are vertices of an equilateral triangle

$\therefore z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

$\Rightarrow (z_1 + z_2 + z_3)^2 = 3(z_1^2 + z_2^2 + z_3^2)$

$\Rightarrow (3\, z_0)^2 = 3(z_1^2 + z_2^2 + z_3^2) \quad \left[\because \frac{z_1 + z_2 + z_3}{3} = z_0\right]$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 = 3\, z_0^2$

20 **(b)**

As we know, $ax^2 + bx + c > 0$ for all $x \in R$, iff $a > 0$ and $D < 0$

$\therefore x^2 + 2ax + (10 - 3a) > 0, \forall x \in R$

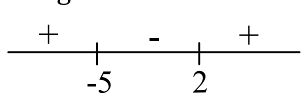
$\Rightarrow D < 0$

$\Rightarrow 4a^2 - 4(10 - 3a) < 0$

$\Rightarrow 4(a^2 + 3a - 10) < 0$

$\Rightarrow (a + 5)(a - 2) < 0$

Using number line rule



$a \in (-5, 2)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	D	D	A	B	D	D	D	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	D	D	A	A	A	D	C	B