DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 7

Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(a)**

CASE I $When\ x^2 + 4x + 3 \geq 0\ i.e.\ x \leq -3$ or $x \geq -1$

In this case, we have

$|x^2 + 4x + 3| = x^2 + 4x + 3$

$\therefore |x^2 + 4x + 3| + (2x + 5) = 0$

$\Rightarrow x^2 + 4x + 3 + 2x + 5 = 0$

$\Rightarrow x = -2, -4 \Rightarrow x = -4 \qquad [\because x \leq -3 \text{ or, } x \geq -1]$

CASE II $When\ x^2 + 4x + 3 < 0$ i.e. $-3 < x < -1$

In this case, we have

$|x^2 + 4x + 3| = -(x^2 + 4x + 3)$

$\therefore |x^2 + 4x + 3| + (2x + 5) = 0$

$\Rightarrow - x^2 - 4x - 3 + 2x + 5 = 0$

$\Rightarrow - x^2 - 2x + 2 = 0$

$\Rightarrow x^2 + 2x - 2 = 0$

$$\Rightarrow x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}$$

$\Rightarrow x = -1 - \sqrt{3} \qquad [\because -3 < x < -1]$

2 **(d)**

Given, $x = \sqrt{\frac{2 + \sqrt{3}}{2 - \sqrt{3}}} = \sqrt{\frac{(2 + \sqrt{3})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}}$

$= 2 + \sqrt{3}$

$\therefore \quad x^2(x - 4)^4 = (2 + \sqrt{3})^2(2 + \sqrt{3} - 4)^2$

$= (\sqrt{3} + 2)^2(\sqrt{3} - 2)^2$

$= \left[(\sqrt{3})^2 - (2)^2\right]^2$

$= (-1)^2 = 1$

3 **(d)**

We have, $|\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a_n|$

$\leq |\lambda_1 a_1| + |\lambda_2 a_2| + \ldots + |\lambda_n a_n|$

$= |\lambda_1||a_1| + \ldots + |\lambda_n||a_n|$

$= \lambda_1|a_1| + \ldots + \lambda_n|a_n| \quad (\because \text{each } \lambda_k \geq 0)$

$< \lambda_1 + \ldots + \lambda_n$

$(\because |a_k| < 1$ and so $\lambda_k|a_k| < \lambda_k$ for all $k = 1, 2, \ldots n)$

Hence, $|\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a_n| < 1$

4 **(a)**

It is given that $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + px + q = 0$

$\therefore \tan\alpha + \tan\beta = -p$ and $\tan\alpha\tan\beta = q$

$$\Rightarrow \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{-p}{1-q} = \frac{p}{q-1}$$

The LHS of choice (a) can be written as

$$= \cos^2(\alpha+\beta)\{\tan^2(\alpha+\beta) + p\tan(\alpha+\beta) + q\}$$

$$= \frac{1}{1+\tan^2(\alpha+\beta)}\{\tan^2(\alpha+\beta) + p\tan(\alpha+\beta) + q\}$$

$$= \frac{1}{1+\frac{p^2}{(q-1)^2}}\left\{\frac{p^2}{(q-1)^2} + \frac{p^2}{q-1} + q\right\} = q$$

So, option (a) is correct

5 **(c)**

$$\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$$

$$= 2\sin\frac{\pi}{10}.\cos\frac{\pi}{10} + i\,2\sin^2\frac{\pi}{10}$$

$$=2\sin\tfrac{\pi}{10}\left(\cos\tfrac{\pi}{10} + i\sin\tfrac{\pi}{10}\right)$$

$$\therefore \quad \tan\theta = \frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}} = \tan\frac{\pi}{10} \quad \Rightarrow \quad \theta = \frac{\pi}{10}$$

6 **(b)**

We know that, sum of any four consecutive powers of i is zero

$$\therefore \quad \sum_{n=1}^{13}(i^n + i^{n+1}) = (i + i^2 + \ldots + i^{13}) + (i^2 + i^3 + \ldots + i^{14})$$

$$= i^{13} + i^{14}$$

$$= i - 1$$

7 **(a)**

$$\log_3 x + \log_3\sqrt{x} + \log_3\sqrt[4]{x} + \log_3\sqrt[8]{x} + \ldots = 4$$

$$\Rightarrow \log_3 x + \frac{1}{2} + \log_3 x + \frac{1}{4}\log_3 x + \frac{1}{8}\log_3 x + \ldots = 4$$

$$\Rightarrow \log_3 x\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right] = 4$$

$$\Rightarrow \log_3 x\left[\frac{1}{1-\frac{1}{2}}\right] = 4$$

$$\Rightarrow \log_3 x = 2$$

$$\Rightarrow x = 3^2 = 9$$

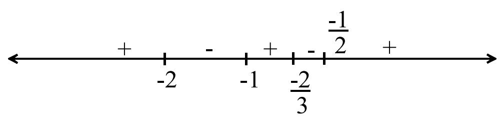
8 **(d)**

We have,

$$\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$$

$$\Rightarrow \frac{2x}{2x^2+5x+2} - \frac{1}{x+1} > 0$$

$$\Rightarrow \frac{2x^2+2x-2x^2-5x-2}{(x+1)(2x+1)(x+2)} > 0$$



$$\Rightarrow \frac{3x+2}{(x+1)(2x+1)(x+2)} < 0$$

$\Rightarrow x \in (-2,-1) \cup (-2/3, -1/2)$

9 **(c)**

Let α,β be the roots of the equation $x^2 + px + 8 = 0$

Then, $\alpha + \beta = -p$ and $\alpha\beta = 8$

Now,

$\alpha - \beta = 2$

$\Rightarrow (\alpha+\beta)^2 - 4\,\alpha\beta = (2)^2 \Rightarrow p^2 - 32 = 4 \Rightarrow p = \pm\ 6$

10 **(d)**

Let α be a common root of the equations $x^2 + ax + 10 = 0$ and $x^2 + bx - 10 = 0$. Then,

$\alpha^2 + a\,\alpha + 10 = 10$

and, $\alpha^2 + b\,\alpha - 10 = 0$

Adding and subtracting these two equations, we get

$2\,\alpha^2 + \alpha(a+b) = 0$ and $(a-b)\alpha + 20 = 0$

$$\Rightarrow \alpha = -\frac{a+b}{2} \text{ and } \alpha = -\frac{20}{a-b}$$

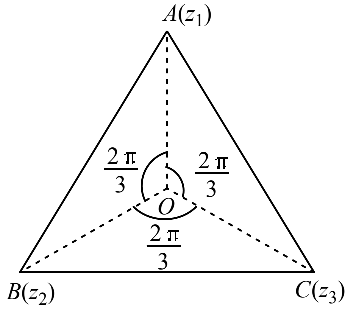
$$\Rightarrow -\frac{a+b}{2} = -\frac{20}{a-b} \Rightarrow a^2 - b^2 = 40$$

11 **(a)**

We have,

$|z_1| = |z_2| = |z_3|$

$\Rightarrow OA = OB = OC$, where O is the origin

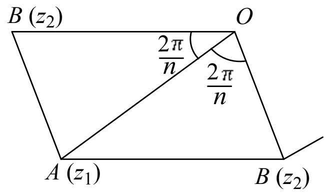


$\Rightarrow$ Circumcentre of $\Delta\, ABC$ is at the origin

But, the triangle is equilateral. Therefore , its centroid coincides with the circumcentre
Thus,
$\frac{z_1 + z_2 + z_3}{3} = 0 \Rightarrow z_1 + z_2 + z_3 = 0$
Clearly, $z_2 = z_1 e^{i\,2\pi/3} = z_1\,\omega$ and $z_3 = z_1\, e^{i\,4\pi/3} = z_1\,\omega^2$
Let OA be along x-axis such that $OA = 1$ unit. Then, $z_1 = 1$
$\therefore z_2 = \omega$ and $z_3 = \omega^2$
Hence, $z_1 z_2 z_3 = \omega^2 = 1$
Thus, we have
$z_1 + z_2 + z_3 = 0$ and $z_1 z_2 z_3 = 1$
12 **(c)**
We have,
$\sqrt{x + iy} = \pm\ (a + i\,b)$
$\Rightarrow x + iy = a^2 - b^2 + 2i\,ab$
$\Rightarrow x = a^2 - b^2, y = 2\,ab$
$\therefore \sqrt{-x - iy} = \sqrt{-(a^2 - b^2) - 2i\,ab}$
$\Rightarrow \sqrt{-x - iy} = \sqrt{b^2 - a^2 - 2i\,ab} = \sqrt{(b - ia)^2} = \pm\ (b - ia)$
13 **(c)**
Since, α , β are the roots of the equation $x^2 + px + q = 0$, then
$\alpha + \beta = p,\ \ \alpha\beta = q$ ...(i)
and α^4, β^4 are the roots of $x^2 - xr + s = 0$.
Then, $\alpha^4 + \beta^4 = r$ ...(ii)
and $\alpha^4\beta^4 = s$
If D is discriminant of the equation $x^2 - 4qx + 2q^2 - r = 0$,
Then $D = 16q^2 - 4(2q^2 - r) = 8q^2 + 4r$
$= 8\alpha^2\beta^2 + 4(\alpha^4 + \beta^4)$ [from Eqs. (i) and (ii)]
$= 4(\alpha^2\beta^2)^2 \geq 0$
Hence, the equation $x^2 - 4qx + 2q^2 - r = 0$ has always two real roots.
14 **(a)**
Since, a, b and c are the sides of a $\Delta\, ABC$, then
$|a - b| < |c| \Rightarrow a^2 + b^2 - 2ab < c^2$
Similarly, $b^2 + c^2 - 2bc < a^2$, $\ c^2 + a^2 - 2ca < b^2$
On adding, we get
$(a^2 + b^2 + c^2) < 2(ab + bc + ca)$
$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} < 2$ .....(i)
Also, $D \geq 0$, $(a + b + c)^2 - 3\lambda(ab + bc + ca) \geq 0$
$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} > 3\lambda - 2$ ....(ii)
From Eqs. (i) and (ii),
$3\lambda - 2 < 2 \Rightarrow \lambda < \frac{4}{3}$

15 **(a)**

Let A be the vertex with affix z_1. There are two possibilities of z_2 ie, z_2 can be obtained by rotating z_1 through $\frac{2\pi}{n}$ either in clockwise or in anti-clockwise direction.



$$\therefore \frac{z_2}{z_1} = \left|\frac{z_2}{z_1}\right| e^{\pm\frac{i2\pi}{n}}$$

$$\Rightarrow z_2 = z_1\left(\cos\frac{2\pi}{n} \pm i\sin\frac{2\pi}{n}\right) \quad (\because |z_2| = |z_1|)$$

16 **(d)**

Given, $z = \cos\theta + i\sin\theta = e^{i\theta}$

$$\therefore \sum_{m=1}^{15} \text{Im}\,(z^{2m-1}) = \sum_{m=1}^{15} \text{Im}\,(e^{i\theta})^{2m-1}$$

$$= \sum_{m=1}^{15} \text{Im}\, e^{i(2m-1)\theta}$$

$$= \sin\theta + \sin 3\theta + \sin 5\theta + \ldots + \sin 29\,\theta$$

$$= \frac{\sin\left(\frac{\theta+29\theta}{2}\right)\sin\left(\frac{15\times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$$

$$= \frac{\sin(15\,\theta)\,\sin(15\,\theta)}{\sin\theta} = \frac{1}{4\sin 2°}$$

17 **(d)**

We have,

$$2z^2 + 2z + a = 0 \Rightarrow z = \frac{-2 \pm \sqrt{4-8a}}{4} = \frac{-1 \pm \sqrt{1-2a}}{2}$$

For z to be non-real, we must have

$$4 - 8a < 0 \Rightarrow a > \frac{1}{2}$$

Let $z_1 = \frac{-1+\sqrt{1-2a}}{2}$ and $z_2 = \frac{-1-\sqrt{1-2a}}{2}$

Now, origin and points representing z_1 and z_2 will form an equilateral triangle in the argand plane, if

$z_1^2 + z_2^2 = z_1 z_2 \quad [\because z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1]$

$\Rightarrow (z_1 + z_2)^2 = 3\, z_1\, z_2$

$$\Rightarrow 1 = \frac{3a}{2} \Rightarrow a = \frac{2}{3}$$

Clearly, $a = 2/3$ satisfies the condition $a > 1/2$
Hence, $a = 2/3$

18 **(c)**

Let P, A, B represent complex numbers $z, 1 + 0i, -1 + 0i$ respectively, then
$|z - 1| + |z + 1| \leq 4 \Rightarrow \quad PA + PB \leq 4$
$\Rightarrow$ P moves in such a way that the sum of its distance from two fixed points is always less than or equal to 4
$\Rightarrow$ Locus of P is the interior and boundary of ellipse having foci at (1, 0) and (-1, 0)

19 **(b)**

On comparing the given circle with $\left|\frac{z-\alpha}{z-\beta}\right| = k$, we get
$\alpha = i, \ \beta = -i, \qquad k = 5$
$\therefore$ Radius $= \left|\frac{k(\alpha - \beta)}{1 - k^2}\right| = \left|\frac{5(i + i)}{1 - 25}\right| = \frac{5}{12}$

20 **(d)**

We have,
$(z + \alpha\beta)^3 = \alpha^3 \Rightarrow z = \alpha - \alpha\beta, z = \alpha\omega - \alpha\beta, z = \alpha\omega^2 - \alpha\beta$
Thus, the vertices A,B and C of $\Delta\, ABC$ are respectively, $\alpha - \alpha\,\beta, \alpha\omega - \alpha\,\beta$ and $\alpha\,\omega^2 - \alpha\,\beta$
Clearly, $AB = BC = AC = |\alpha|\,|1 - \omega| = \sqrt{3}\,|\alpha|$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	D	A	C	B	A	D	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	A	A	D	D	C	B	D