DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 6

Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(b)**

Since, α and β be the roots of the equation $x^2+\sqrt{\alpha}x+\beta=0$, therefore

$\alpha+\beta=-\sqrt{\alpha}$ and $\alpha\beta=\beta$

From second relation $\beta\neq 0$

$\therefore\ \alpha=1$

$\therefore\ 1+\beta=-1\Rightarrow\beta=-2$

Hence, $\alpha=1$ and $\beta=-2$

2 **(d)**

The equation has no real root, because LHS is always positive while RHS is zero

3 **(a)**

Let $z=x+iy$. Then,

$$\frac{z-1}{z+1}=\frac{(x^2+y^2-1)+2\,iy}{(x+1)^2+y^2}$$

Since $\frac{z-1}{z+1}$ is purely imaginary. Therefore,

$$\text{Re}\left(\frac{z-1}{z+1}\right)=0$$

$$\Rightarrow\frac{x^2+y^2-1}{(x+1)^2+y^2}=0$$

$\Rightarrow x^2+y^2=1\Rightarrow|z|^2=1\Rightarrow|z|=1$

ALITER We have,

$\left(\frac{z-1}{z+1}\right)$ is purely imaginary

$$\Rightarrow\arg\left(\frac{z-1}{z+1}\right)=\pm\frac{\pi}{2}$$

$\Rightarrow z$ lies on the circle $|z|=1$

4 **(a)**

Let z be the fourth vertex of parallelogram, then

$$\frac{z_1+z_3}{2}=\frac{z_2+z}{2}\ \Rightarrow\ z=z_1+z_3-z_2$$

5 **(a)**

Let $z=x+iy$

$\Rightarrow\ zz = (x + iy)(x + iy)$
$= x^2 - y^2 + 2ixy$
$= 0 + 2ixy \quad [\because\ \text{Re}\,(z) = \text{Im}\,(z)\ \Rightarrow\ x = y]$
$\Rightarrow\ \text{Re}(z^2) = 0$

6 **(c)**

Let $x = \sqrt{-1 - \sqrt{-1 - \sqrt{-1 - \ldots\infty}}}$

Then, $x = \sqrt{-1 - x}$ or $x^2 = -1 - x$

or $x^2 + x + 1 = 0$

$$\therefore\ x = \frac{-1 \pm \sqrt{1 - 4.1.1}}{2.1} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{3}i}{2} = \omega \text{ or } \omega^2$$

7 **(c)**

We have, $z_k = 1 + a + a^2 + \ldots + a^{k-1} = \frac{1 - a^k}{1 - a}$

$$\Rightarrow z_k - \frac{1}{1 - a} = \frac{-a^k}{1 - a}$$

$$\Rightarrow \left|z_k - \frac{1}{1 - a}\right| = \frac{|a^k|}{|1 - a|}$$

$$= \frac{|a^k|}{|1 - a|} < \frac{1}{|1 - a|} \quad (\because |a| < 1)$$

$\Rightarrow z_k$ lies within a circle $\left|z - \frac{1}{1 - a}\right| = \frac{1}{|1 - a|}$

8 **(b)**

Here, $\sum \alpha = 0, \sum \alpha\beta = -7, \alpha\beta\gamma = -7$

$$\therefore \frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} = \frac{\alpha^4\beta^4 + \beta^4\gamma^4 + \gamma^4\alpha^4}{\alpha^4\beta^4\gamma^4}$$

$= \frac{\sum \alpha^4\beta^4}{\alpha^4\beta^4\gamma^4} \quad$ ...(i)

Now, $\sum \alpha\beta \sum \alpha\beta \sum \alpha\beta \sum \alpha\beta = (\sum \alpha\beta)^2\ (\sum \alpha\beta)^2$

$\Rightarrow (-7)^4 = [\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)]$
$[\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)]$
$= (\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2)(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2) \quad [\therefore \sum \alpha = \alpha + \beta + \gamma = 0]$
$= \alpha^4\beta^4 + \beta^4\gamma^4 + \gamma^4\alpha^4 + 2\alpha^4\beta^2\gamma^2 + 2\alpha^2\beta^4\gamma^2 + 2\alpha^2\beta^2\gamma^4$

$$= \sum \alpha^4\beta^4 + 2\alpha^2\beta^2\gamma^2(\alpha^2 + \beta^2 + \gamma^4)$$

$$= \sum \alpha^4\beta^4 + 2\alpha^2\beta^2\gamma^2\left[\left(\sum a\right)^2 - 2\sum \alpha\beta\right]$$

$$= \sum \alpha^4\beta^4 + 2\alpha^2\beta^2\gamma^2[\,0 - 2 \times (-7)]$$

$$= \sum \alpha^4\beta^4 + 2(-7)^2(2 \times 7)$$

$$\Rightarrow \sum \alpha^4\beta^4 = (-7)^4 + 4(-7)^3$$

$\Rightarrow \sum \alpha^4\beta^4 = (-7)^3(-7+4) = -3(-7)^3$

On putting this value in Eq. (i) , we get

$$\frac{1}{\alpha^4}+\frac{1}{\beta^4}+\frac{1}{\gamma^4}=\frac{-3(-7)^3}{(-7)^4}=\frac{-3}{-7}=\frac{3}{7}$$

9 **(b)**

Given, $\sin\theta + \cos\theta = h$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = h^2$ [squaring]

$\Rightarrow \sin\theta\cos\theta = \frac{h^2-1}{2}$

The quadratic equation having the roots $\sin\theta$ and $\cos\theta$ is

$x^2 - (\sin\theta + \cos\theta)x + \sin\theta\cos\theta = 0$

$\therefore \quad 2x^2 - 2hx + (h^2-1) = 0$

10 **(a)**

Replacing x by $\frac{1-bx}{ax}$ we get the required equation

$a\left(\frac{1-bx}{ax}\right)^2 + b\left(\frac{1-bx}{ax}\right) + c = 0$

$\Rightarrow a(1+b^2x^2-2bx) + ax(b-b^2x) + ca^2x^2 = 0$

$\Rightarrow a + ab^2x^2 - 2abx + abx - ab^2x^2 + a^2cx^2 = 0$

$\Rightarrow \quad acx^2 - bx + 1 = 0$

11 **(d)**

$\sqrt{i} = \sqrt{\frac{2i}{2}} = \frac{1}{\sqrt{2}}\sqrt{2i+1+i^2}$

$= \frac{1}{\sqrt{2}}\sqrt{(1+i)^2} = \pm\frac{1}{\sqrt{2}}(1+i)$

12 **(b)**

Let α and α^n be the roots of the equation, then

$\alpha + \alpha^n = -\frac{b}{a}$ and $\alpha.\alpha^n = \frac{c}{a} \Rightarrow \alpha^{n+1} = \frac{c}{a}$

On eliminating α, we get

$\left(\frac{c}{a}\right)^{\frac{1}{n+1}} + \left(\frac{c}{a}\right)^{\frac{1}{n+1}} = -\frac{b}{a}$

$\Rightarrow a.a^{-\frac{1}{n+1}}c^{\frac{1}{n+1}} + a.a^{-\frac{n}{n+1}}c^{\frac{n}{n+1}} = -b$

$\Rightarrow (a^nc)^{\frac{1}{n+1}} + (ac^n)^{\frac{1}{n+1}} = -b$

13 **(d)**

Let $z = x + iy$

$\therefore \quad |z+3-i| = |(x+3)+i(y-1)| = 1$

$\Rightarrow \quad \sqrt{(x+3)^2+(y-1)^2} = 1$ ...(i)

$\because \quad \arg z = \pi \quad \Rightarrow \quad \tan^{-1}\frac{y}{x} = \pi$

$\Rightarrow \quad \frac{y}{x} = \tan\pi = 0 \quad \Rightarrow y = 0$ ...(ii) from Eqs.(i)and (ii)we get

$x = -3, \quad y = 0$

$\therefore \quad z = -3$

$\Rightarrow \quad |z| = |-3| = 3$

14 **(a)**

Let $x = (-1)^{1/3}$

$x = (\cos \pi + i \sin \pi)^{1/3}$

$x = \left[\cos\left(\frac{2n+1}{3}\right)\pi + i \sin\left(\frac{2n+1}{3}\right)\pi\right] = e^{i(2n+1)\pi/3}$

Put $n = 0, 1, 2$ we get

$x = e^{i\pi/3}, e^{i\pi}, e^{5i\pi/3}$

$\therefore$ Products of roots $= e^{i\pi/3}, e^{\pi i}.e^{5\pi i/3} = e^{3\pi i}$

$= (\cos 3\pi + i \sin 3\pi) = -1$

Alternate Method

We know that the cube roots of -1 are -1, -ω, $-\omega^2$

$\therefore$ Their product $= (-1)(-\omega)(-\omega^2) = -1$

15 **(c)**

Sum of the roots

$= -\frac{b}{a} = -\frac{(-3)}{1} = 3$

From the given options only (c) $ie, -2, 1, 4$ satisfies this condition

16 **(c)**

If $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in x, then

$a^2 - 3a + 2 = 0, a^2 - 5a + 6 = 0$ and $a^2 - 4 = 0$ must holdgood simultaneously.

Clearly, $a = 2$ is the value of $'a'$ which satisfies these equations

17 **(a)**

Since z_2 and z_3 can be obtained by rotating vector representing z_1 through $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively

$\therefore z_2 = z_1\,\omega$ and $z_3 = z_1\,\omega^2$

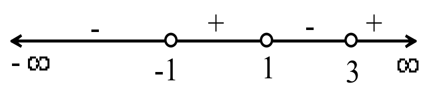
$\Rightarrow z_2 = (1 + i\sqrt{3})\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ and, $z_3 = (1 + i\sqrt{3})\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$\Rightarrow z_2 = -2 + 0\,i$ and $z_3 = 1 - i\sqrt{3}$

18 **(b)**

We have,

$\frac{x^2 - 3x + 4}{x + 1} > 1$



$\Rightarrow \frac{x^2 - 4x + 3}{x + 1} > 0$

$\Rightarrow \frac{(x-1)(x-3)}{x+1} > 0 \Rightarrow x \in (-1,1) \cup (3,\infty)$

19 **(a)**

$\left(\frac{9}{10}\right)^x = -3 + x - x^2$

$\Rightarrow \left(\frac{9}{10}\right)^x = -\left\{\left(x - \frac{1}{2}\right)^2 + \frac{11}{4}\right\}$

⇒ LHS is always positive while RHS is always negative. Hence, the given equation has no solution.

20 **(a)**

Let root of $3ax^2 + 3bx + c = 0$ be α, then

$$3a\alpha^2 + 3b\alpha + c = 0$$

According to the given condition,

$\Rightarrow \quad x = 3\alpha$

$\Rightarrow \quad \alpha = \frac{x}{3}$

$\therefore \quad 3a\frac{x^2}{9} + 3b\frac{x}{3} + c = 0$

$\Rightarrow \quad ax^2 + 3bx + 3c = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	A	A	A	C	C	B	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	B	D	A	C	C	A	B	A	A