DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 5

Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(c)**
We have, $a+b+c=0$ ...(i)
Let $D=B^2+4AC$
$=9b^2-4(4a)(2c)=9b^2-32ac$
$=9(a+c)^2-32ac$ [from Eq. (i)]
$=9(a-c)^2+4ac$
Hence, roots are real.
2 **(a)**
Given, $x^2(1+2k)+x(1-2k)+1(1-2k)=0$ ...(i)
Given, $D=0,\ b^2-4ac=0$
$\Rightarrow (1-2k)^2-4(1+2k)(1-2k)=0$
$\Rightarrow 20k^2-4k-3=0$
$\Rightarrow k=\frac{1}{2},\frac{3}{10}$
3 **(d)**
We have, $\frac{\log 5+\log(x^2+1)}{\log(x-2)}=2$
$\Rightarrow \log\{5(x^2+1)\}=\log(x-2)^2$
$\Rightarrow 5(x^2+1)=(x-2)^2$
$\Rightarrow 4x^2+4x+1=0$
$\Rightarrow x=-\frac{1}{2}$
But for $x=-\frac{1}{2}$,$\log(x-2)$ is not meaningful.
$\therefore$ It has no root.
4 **(a)**
We have,
$|x|^2-3|x|+2=0$
$\Rightarrow(|x|-1)(|x|-2)=0$
$\Rightarrow|x|=1,2\Rightarrow x=\pm 1,\pm 2$
5 **(a)**
Let α_1,β_1 be the roots of $x^2+ax+b=0$ and α_2,β_2 be the roots of $x^2+bx+a=0$. Then,
$\alpha_1+\beta_1=-a,\ \alpha_1\beta_1=b;\ \alpha_2+\beta_2=-b,\ \alpha_2\beta_2=a$

It is given that

$|\alpha_1-\beta_1|=|\alpha_2-\beta_2|$

$\Rightarrow(\alpha_1-\beta_1)^2=(\alpha_2-\beta_2)^2$

$\Rightarrow(\alpha_1+\beta_1)^2-4\,\alpha_1\beta_1=(\alpha_2+\beta_2)^2-4\alpha_2\beta_2$

$\Rightarrow a^2-4b=b^2-4a$

$\Rightarrow(a^2-b^2)+4(a-b)=0\Rightarrow a+b+4=0\quad[\because a\neq b]$

6 **(b)**

$\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}=\log_x\sqrt{2}$

$\Rightarrow\ \frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}=\frac{1}{2\log_2 x}$

$\Rightarrow\ 3(\log_2 x)^3+4(\log_2 x)^2-5(\log_2 x)-2=0$

Put $\log_2 x=y$

$\therefore\ \ 3y^2+4y^2-5y-2=0$

$\Rightarrow\ \ (y-1)(y+2)(3y+1)=0$

$\Rightarrow\ \ y=1,\ -2,\ \frac{-1}{3}$

$\Rightarrow\ \ \log_2 x=1,\ -2,\ -\frac{1}{3}$

$\Rightarrow\ x=2,\frac{1}{2^{1/3}},\frac{1}{4}$

7 **(d)**

Since $|z+a|\leq a$ implies z lies on or inside a circle with centre $(-a,0)$ and radius a, we have $|z_1|+|z_2|+|z_3|\leq 14$

8 **(b)**

$\log_{\sqrt{3}}300=\log_{\sqrt{3}}3+\log_{\sqrt{3}}100$

$=2\log_{\sqrt{3}}\sqrt{3}+2\log_{\sqrt{3}}5+2\log_{\sqrt{3}}2$

$=2(1+a+b)\qquad[\because \log_{\sqrt{b}}5=a,\log_{\sqrt{b}}2=b]$

9 **(a)**

We have,

$p+q<r+s$ ...(i)

$q+r<s+t$ ...(ii)

$r+s<t+p$ ...(iii)

and, $s+t<p+q$ ...(iv)

From (i) and (iii), we have

$p+q<r+s<t+p\Rightarrow q<t$

From (ii) and (iv), we have

$q+r<s+t<p+q\Rightarrow r<p$

From (i) and (iv), we have

$s+t<p+q<r+s\Rightarrow t<r$

$\therefore q<t<r<p$

From (i), we have $p+q<r+s$

Also, $r<p$

$\therefore p+q+r<r+s+p \Rightarrow q<s$
From (iv), we have $s+t<p+q$
Also, $q<t$
$\therefore s+t+q<p+q+t \Rightarrow s<p$
$\therefore q<s<p$
Hence, the largest and the smallest numbers are p and q respectively
10 **(c)**
We have,
$\frac{x+2}{x^2+1}>\frac{1}{2}$
$\Rightarrow 2x+4>x^2+1$
$\Rightarrow x^2-2x-3<0$
$\Rightarrow (x-3)(x+1)<0$
$\Rightarrow -1<x<3 \Rightarrow x=0,1,2$ [$\because x$ is an integer]
11 **(a)**
Let r be the common ratio of the GP. Since $\alpha, \beta, \gamma, \delta$ are in GP, then $\beta=\alpha r, \gamma=\alpha r^2$ and $\delta=\alpha r^3$.
For equations, $x^2-x+p=0$
$\therefore \alpha+\beta=1$
$\Rightarrow \alpha+\alpha r=1$
$\Rightarrow \alpha(1+r)=1$ ....(i)
and $\alpha\beta=p \Rightarrow \alpha(\alpha r)=p$
$\Rightarrow \alpha^2 r=p$ ...(ii)
For equation, $x^2-x+q=0$
$\gamma+\delta=4$
$\Rightarrow \alpha r^2+\alpha r^3=4$
$\Rightarrow \alpha r^2(1+r)=4$ ...(iii)
and $\gamma\delta=q \Rightarrow \alpha r^3 \cdot \alpha r^2=q$
$\Rightarrow \alpha^2 r^5=q$ ...(iv)
On dividing Eq. (iii) by Eq. (i),we get
$r^2=4 \Rightarrow r=\pm 2$
If we take $r=2$, then α is not integral, so we take $r=-2$.
Substituting $r=-2$ in Eq. (i), we get
$\alpha=-1$
Now, from Eq. (ii), we have
$p=\alpha^2 r=(-1)^2(-2)=-2$
and from Eq. (iv), we have
$q=\alpha^2 r^5=(-1)^2(-2)^5=-32$
$\Rightarrow (p,q)=(-2,-32)$
12 **(a)**
Let the vertices of triangle be $A(z_1)$, $B(z_2)$ and $C(z_3)$
Given, $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$

$\Rightarrow \quad \left|\frac{z_1 - z_3}{z_2 - z_3}\right| = \frac{|2|}{|2|} = 1$

$\therefore \quad |z_1 - z_3| = |z_2 - z_3|$

$\Rightarrow \quad |AC| = |BC|$

Now, $\frac{z_1 - z_3}{z_2 - z_3} = e^{-i\pi/3}$

$\Rightarrow \quad \arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right) = -\frac{\pi}{3}$

$\therefore \quad \angle BCA = \frac{\pi}{3}$

$\Rightarrow \quad |AC| = |BC|$ and $\angle BCA = 60°$

$\Rightarrow \quad |AB| = |BC| = |CA|$

$\Rightarrow \quad \Delta ABC$ is an equilateral triangle.

13 **(d)**

We have, $225 + (3\omega + 8\omega^2)^2 + (3\omega^2 + 8\omega)^2$

$= 225 + 9\omega^2 + 64\omega^4 + 48\omega^3 + 9\omega^4 + 64\omega^2 + 48\omega^3$

$= 225 + 9\omega^2 + 64\omega + 48 + 9\omega + 64\omega^2 + 48$

$= 225 + 73(\omega^2 + \omega) + 96 = 225 - 73 + 96 = 248$

14 **(c)**

Let $z = x + iy$

Given, $\left|\frac{z + 2i}{2z + i}\right| < 1$

$\Rightarrow \quad \frac{\sqrt{(x)^2 + (y + 2)^2}}{(2x)^2 + (2y + 1)^2} < 1$

$\Rightarrow \quad x^2 + y^2 + 4 + 4y < 4x^2 + 4y^2 + 1 + 4y$

$\Rightarrow \quad 3x^2 + 3y^2 > 3$

$\Rightarrow \quad x^2 + y^2 > 1$

15 **(c)**

Let $a - d, a, a + d$ be the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$. Then,

$a - d + a + a + d = 12$ and, $(a - d)(a + d) = 28$

$\Rightarrow 3a = 12$ and $a(a^2 - d^2) = 28$

$\Rightarrow a = 4$ and $d = \pm\ 3$

16 **(b)**

We have,

$\frac{2}{|x - 4|} > 1$

$\Rightarrow 2 > |x - 4|$

$\Rightarrow |x - 4| < 2 \Rightarrow -2 < x - 4 < 2 \Rightarrow 2 < x < 6$

But $\frac{2}{|x - 4|} > 1$ is not defined at $x = 4$

$\therefore x \in (2,4) \cup (4,6)$

17 **(b)**

As sum of any four consecutive powers of iota is zero

$$\therefore \sum_{n=1}^{13}\left(i^n + i^{n+1}\right) = \left(i + i^2 + \dots + i^{13}\right) + \left(i^2 + i^3 + \dots + i^{14}\right)$$

$= i + i^2 = i - 1$

18 **(b)**

The complex cube roots of unity are 1, ω, ω^2

Let $\alpha = \omega$, $\beta = \omega^2$

Then, $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = \omega^4 + \left(\omega^2\right)^4 + (\omega)^{-1}\left(\omega^2\right)^{-1}$

$= \omega + \omega^2 + 1 = 0$

19 **(b)**

Since a,b,c are in H.P.

$\therefore b = \dfrac{2\,ac}{a + c}$

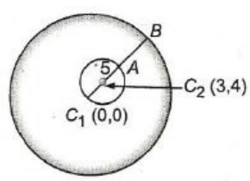
Now,

$$\text{Disc} = 4\,b^2 - 4\,ac = 4\left\{\frac{4\,a^2c^2}{(a+c)^2} - ac\right\} = -4\,ac\,\frac{(a-c)^2}{(a+c)^2} < 0$$

Hence, roots of the given equation are imaginary

20 **(d)**

The two circle whose centre and radius are $C_1(0, 0)$, r_1=12, $C_2(3, 4)$, $r_2 = 5$ and it passes through origin *ie*, the centre of C_1



Now, $C_1C_2 = \sqrt{3^2 + 4^2} = 5$

And $r_1 - r_2 = 12 - 5 = 7$

$\therefore \quad C_1C_2 < r_1 - r_2$

Hence, circle C_2 lies inside the circle C_1

From figure the minimum distance between them, is

$AB = C_1B - C_1A$

$= r_1 - (C_1C_2 + C_2A)$

$= 12 - 10 = 2$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	D	A	A	B	D	B	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	D	C	C	B	B	B	B	D