DPP
DAILY PRACTICE PRODUCTS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 4

Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(d)**

We have, $|x| - 1 < 1 - x$

Two cases arise

<u>CASE I</u> $When\ x \geq 0$

In this case, we have $|x| = x$

$\therefore |x| - 1 < 1 - x \Rightarrow x - 1 < 1 - x \Rightarrow 2(x - 1) < 0 \Rightarrow x < 1$

But, $x \geq 0$. Therefore, $x \in [0, 1)$

<u>CASE II</u> $When\ x < 0$

In this case, we have $|x| = -x$

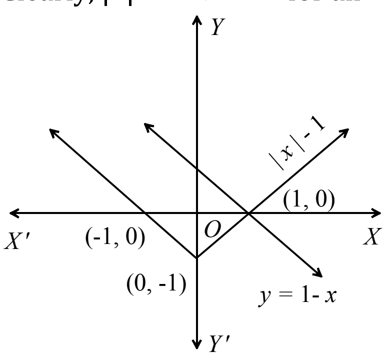
$\therefore |x| - 1 < 1 - x \Rightarrow -x - 1 < 1 - x \Rightarrow -1 < 1$

This is true for all $x < 0$

Hence, $x \in (-\infty, 0) \cup [0,1)$ i.e. $x \in (-\infty, 1)$

<u>ALITER</u> Draw the graphs of $y = |x| - 1$ and $y = 1 - x$

Clearly, $|x| - 1 < 1 - x$ for all $x \in (-\infty, 1)$



2 **(c)**

We have,

$(\sqrt{3} + i)^{10} = a + ib$

$\Rightarrow i^{10}(1 - i\sqrt{3})^{10} = a + i\,b$

$\Rightarrow -(-2\omega)^{10} = a + i\,b \qquad \left[\because \omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$

$\Rightarrow -2^{10}\omega^{10} = a + i\,b$

$\Rightarrow -2^{10}\omega = a + i\,b$

$\Rightarrow -2^{10}\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)=a+i\,b$

$\Rightarrow 2^9-2^9\sqrt{3}\,i=a+i\,b \Rightarrow a=2^9$ and $b=-2^9\sqrt{3}$

3 **(b)**

We have,

$(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{5})^{x^2-3}=(5+2\sqrt{6})+(5-2\sqrt{6})$

$\Rightarrow x^2-3=\pm\ 1 \Rightarrow x=\pm\ 2, \pm\ \sqrt{2}$

4 **(d)**

If $x\neq 1$, multiplying each term by $(x-1)$the given equation reduces to $x(x-1)=(x-1)$ or $(x-1)^2=0$ or $x=1$, which is not possible as considering $x\neq 1$, thus given equation has no roots

5 **(b)**

Given, $(1+i)^{2n}=(1-i)^{2n}$

$\Rightarrow\ 2^n i^n=2^n(-1)^n i^n\ \Rightarrow\ 1=(-1)^n$

$\therefore$ The smallest value of n is 2

6 **(a)**

Since, $\frac{z-1}{z+1}$ is purely imaginary

$\therefore\ \frac{z-1}{z+1}=-\overline{\left(\frac{z-1}{z+1}\right)}$

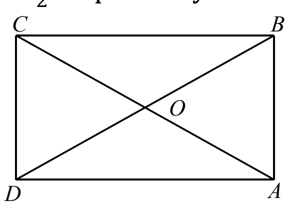
$\Rightarrow\ \frac{z-1}{z+1}=\frac{\bar{z}-1}{\bar{z}+1}$

$\Rightarrow\ \frac{2z}{-2}=\frac{2}{-2\bar{z}}\ \Rightarrow\ z\bar{z}=1$

$\Rightarrow\ |z|^2=1\ \Rightarrow\ |z|=1$

7 **(a)**

Let the vertex A be $3(\cos\theta+i\sin\theta)$, then OB and OD can be obtained by rotating OA through $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ respectively



Thus, $\overrightarrow{OB}=(\overrightarrow{OA})e^{i\frac{\pi}{2}}$ and, $\overrightarrow{OD}=\overrightarrow{OA}\,e^{-i\frac{\pi}{2}}$

$\Rightarrow\overrightarrow{OB}=3(\cos\theta+i\sin\theta)\,i$ and, $\overrightarrow{OD}=3(\cos\theta+i\sin)(-i)$

$\Rightarrow\overrightarrow{OB}=3(-\sin\theta+i\cos\theta)$ and, $\overrightarrow{OD}=3(\sin\theta-i\cos\theta)$

Thus, vertices B and D are represented by $\pm 3(\sin\theta-i\cos\theta)$

8 **(a)**

Let α,β be the roots of the given quadratic equation. Then,

$\alpha+\beta=-b/a, \alpha\beta=c/a$

It is given that

$\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{b^2}$

$\Rightarrow \alpha^2+\beta^2=(\alpha+\beta)\alpha^2\beta^2$

$\Rightarrow (\alpha+\beta)^2-2\,\alpha\,\beta=(\alpha+\beta)(\alpha\,\beta)^2$

$\Rightarrow \frac{b^2}{a^2}-\frac{2c}{a}=\frac{-bc^2}{a^3}$

$\Rightarrow \frac{2c}{a}=\frac{b^2}{a^2}+\frac{bc^2}{a^3}$

$\Rightarrow 2a^2c=ab^2+bc^2 \Rightarrow c^2b, a^2c, b^2a$ are in A.P.

Dividing both sides of $2a^2c-ab^2+bc^2$ by abc, we get

$2\frac{a}{b}=\frac{b}{c}+\frac{c}{a}\Rightarrow \frac{b}{c},\frac{a}{b},\frac{c}{a}$ are in A.P.

9 **(c)**

Clearly, angle between z and iz is a right angle

$\therefore \angle OPQ=\frac{\pi}{2}$

10 **(d)**

We have,

$\frac{2^n}{(1-i)^{2n}}+\frac{(1+i)^{2n}}{2^n}$

$=\frac{2^n}{\{(1-i)^2\}n}+\frac{\{(1+i)^2\}^n}{2^n}$

$=\frac{2^n}{(1-2i+i^2)^n}+\frac{(1+2i+i^2)^n}{2^n}$

$=\frac{2^n}{(-2i)^n}+\frac{(2i)^n}{2^n}=\left(-\frac{1}{i}\right)^n+i^n=i^n+i^n=2i^n$

11 **(d)**

Since, the equation $x^2-px+r=0$ has roots (α,β) and the equation $x^2-qx+r=0$ has roots $\left(\frac{\alpha}{2},2\beta\right)$

$\therefore\ \alpha+\beta=p$ and $r=\alpha\beta$ and $\frac{\alpha}{2}+2\beta=q$

$\Rightarrow \beta=\frac{2q-p}{3}$ and $\alpha=\frac{2(2p-q)}{3}$

$\therefore\ \alpha\beta=r=\frac{2}{9}(2q-p)(2p-q)$

12 **(d)**

We have, $(1+\omega-\omega^2)^7=(-\omega^2-\omega^2)^7$

$=(-2)^7(\omega^2)^7=-128\,\omega^2$

13 **(d)**

We have,

$z+z^{-1}=1\Rightarrow z^2-z+1=0\Rightarrow z=-\omega$ or $-\omega^2$

For $z=-\omega$, we have

$z^{100}+z^{-100}=(-\omega)^{100}+(-\omega)^{-100}=\omega+\frac{1}{\omega}=\omega+\omega^2=-1$

For $z=-\omega^2$, we have

$z^{100}+z^{-100}=(-\omega^2)^{100}+(-\omega^2)^{-100}=\omega^{200}+\frac{1}{\omega^{200}}$

$\Rightarrow z^{100}+z^{-100}=\omega^2+\frac{1}{\omega^2}=\omega^2+\omega=-1$

14 **(c)**

Let $z=\frac{3+2i\sin\theta}{1-2i\sin\theta}$

$\Rightarrow \quad z=\frac{3+2i\sin\theta}{1-2i\sin\theta}\times\frac{(1+2i\sin\theta)}{1+2i\sin\theta}$

$=\frac{3-4\sin^2\theta+8i\sin\theta}{1+4\sin^2\theta}$

For purely imaginary of z, put $\text{Re}(z)=0$

ie, $\quad \frac{3-4\sin^2\theta}{1+4\sin^2\theta}=0$

$\Rightarrow \quad \sin\theta=\pm\frac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta=n\pi+(-1)^n\left(+\frac{\pi}{3}\right)=n\pi\pm\frac{\pi}{3}$

15 **(a)**

We have,

$x^2+2ax+10-3a>0$ for all $x\in R$

$\Rightarrow 4a^2-40+12a<0$ [Using: discriminant <0]

$\Rightarrow a^2+3a-10<0$

$\Rightarrow (a+5)(a-2)<0 \Rightarrow -5<a<2$

16 **(b)**

Let $a_1=a+ib$, $z_2=c+id$. Then,

z_1+z_2 is real

$\Rightarrow (a+c)+i(b+d)$ is real

$\Rightarrow b+d=0 \Rightarrow d=-b$ ...(i)

z_1z_2 is real

$\Rightarrow (ac-bd)+i(ad+bc)$ is real

$\Rightarrow ad+bc=0$

$\Rightarrow a(-b)+bc=0$ Using (i)

$\Rightarrow a=c$

$\therefore z_1=a+ib=c-id=\bar{z}_2$ [$\because a=c$ and $b=-d$]

17 **(a)**

Let $z=z+i\,y$. Then,

$\frac{2\,z+1}{i\,z+1}=\frac{(2x+1)+2\,i\,y}{(1-y)+ix}=\frac{(1-y+2x)+i\,(2\,y-2y^2-2x^2-x)}{(1-y)^2+x^2}$

$\text{Im}\left(\frac{2\,z+1}{i\,z+1}\right)=3$

$\Rightarrow \frac{2\,y-2\,y^2-2\,x^2-x}{(1-y)^2+x^2}=3$

$\Rightarrow 2y - 2y^2 - 2x^2 - x = 3x^2 + 3(1-y)^2$

$\Rightarrow 5x^2 + 5y^2 - 8y + x + 3 = 0$, which is a circle

18 **(a)**

$z_2 + ax + a^2 = 0 \Rightarrow z = a\omega, a\omega^2$

(where $'\omega'$ is a non-real root of unity)

$\Rightarrow$ Locus of z is a pair of straight lines

and $\arg(z) = \arg(a) + \arg(\omega)$

or $\arg(z) = \arg(a) + \arg(\omega^2)$

$\Rightarrow \arg(z) = \pm \frac{2\pi}{3}$

Also, $|z| = |a||\omega|$ or $|z| = |a||\omega^2|$

$\Rightarrow |z| = |a|$

19 **(b)**

Diagonals of parallelogram $ABCD$ are bisected each other at a point ie,

$\frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2}$

$\Rightarrow \quad z_1 + z_2 = z_2 + z_4$

20 **(a)**

Now, $\frac{1}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$

Where $Bx + C = f(x)$

$\Rightarrow 1 = A(1+x^2) + (Bx+C)(1+x)$

On comparing the coefficient of x^2, x and constant terms, we get

$0 = A + B,\ 0 = B + C$ and $1 = A + C$

$\Rightarrow A = C = \frac{1}{2}$ and $B = -\frac{1}{2}$

$\therefore \quad \frac{1}{(1+x)(1+x^2)} = \frac{1}{2(1+x)} + \frac{-\frac{x}{2} + \frac{1}{2}}{1+x^2}$

$\therefore \quad f(x) = -\frac{x}{2} + \frac{1}{2} = \frac{1-x}{2}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	B	D	B	A	A	A	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	D	D	C	A	B	A	A	B	A