DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 3

Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(c)**

We have, $z^2 + z|z| + |z|^2 = 0$

$\Rightarrow \left(\frac{z}{|z|}\right)^2 + \frac{z}{|z|} + 1 = 0$

This is a quadratic equation in $\frac{z}{|z|}$, therefore roots are $\frac{z}{|z|} = \omega, \omega^2 \Rightarrow z = \omega|z|$ or $z = \omega^2|z|$

Let $z = x + iy$

$\Rightarrow x + iy = |z|\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)$

or $x + iy = |z|\left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right)$

$\Rightarrow x = -\frac{1}{2}|z|, y = |z|\frac{\sqrt{3}}{2}$

or $x = -\frac{|z|}{2}, y = -\frac{|z|\sqrt{3}}{2}$

$\Rightarrow y + \sqrt{3}x = 0$

or $y - \sqrt{3}x = 0$

$\Rightarrow y^2 - 3x^2 = 0$

$\Rightarrow$ It represents a pair of straight lines

2 **(c)**

Clearly, $|z - i| = 1$ represeents a circle having centre C at $(0, 1)$ and radius 1. Let $P(z)$ be a point on the circle such that $z = r(\cos\theta + i\sin\theta)$

$\therefore \cot\theta - \frac{2}{z} = \cot\theta - \frac{2}{r}(\cot\theta - i\sin\theta)$

$\Rightarrow \cot\theta - \frac{2}{z} = \cot\theta - \frac{2}{r}\cos\theta + \left(\frac{2}{r}\sin\theta\right)i$

$\Rightarrow \cot\theta - \frac{2}{z} = \cot\theta - \cot\theta + i \qquad \left[\because \sin\theta = \frac{r}{2}\right]$

$\Rightarrow \cot\theta - \frac{2}{z} = i$

3 **(c)**

We have,

$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$,

Where $\theta_1 = \arg(z_1)$ and $\theta_2 = \arg(z_2)$
$\therefore \arg(z_1 - z_2) = 0$
$\Rightarrow |z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$
$\Rightarrow |z_1 - z_2|^2 = (|z_1| - |z_2|)^2$
$\Rightarrow |z_1 - z_2| = ||z_1| - |z_2||$
4 **(a)**
We have,
$x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$
$= x^2 + (2y)^2 + (3z)^2 - (2y)(3z) - (3z)x - x(2y)$
$= \frac{1}{2}\{(x - 2y)^2 + (2y - 3z)^2 + (3z - x)^2\} \geq 0$
Hence, the given expression is always non-negative
5 **(b)**
Let A,B be the centres of circles $|z - z_1| = a$ and $|z - z_2| = b$ respectively. Let $P(\alpha)$ be the centre of the variable circle $|z - \alpha| = r$ which touches the given circles externally. Then,
$AP = r + a$ and $PB = r + b$
$\Rightarrow AP - BP = (r + a) - (r + b)$
$\Rightarrow AP - BP = a - b$
$\Rightarrow$ Locus of P is a hyperbola having its foci at A and B respectively
6 **(b)**
Let $z = (1 + i\sqrt{3})^8$
$= (-2)^8\left(\frac{1 + i\sqrt{3}}{-2}\right) = (-2)^8(\omega^2)^8 \quad [\because \omega^3 = 1]$
$= 2^8\omega^{16} = 2^8\omega$
$= 2^8\left(\frac{-1 + i\sqrt{3}}{2}\right)$
$= 2^8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$
$\therefore$ Modulus $= 2^8 = 256$ and amplitude $= \frac{2\pi}{3}$
7 **(c)**
We have,
$x^2 + (a + b)x + ab < 0$
$\Rightarrow (x + a)(x + b) < 0 \Rightarrow -b < x < -a \Rightarrow x \in (-b, -a)$
8 **(a)**
$x^2 + y^2 + z^2 = (a + b)^2 + \omega^2(a + b\omega)^2 + (a\omega^2 + b\omega)^2$
$= a^2 + b^2 + 2ab + a^2\omega^2 + b^2\omega^4 + 2ab\omega^3 + a^2\omega^4 + b^2\omega^2 + 2ab\omega^3$
$= a^2(1 + \omega + \omega^2) + b^2(1 + \omega + \omega^2) + 6ab \quad [\because \omega^4 = \omega]$
$= 6ab \quad [\because 1 + \omega + \omega^2 = 0]$
9 **(c)**
$\sqrt{-7 - 24\sqrt{-1}} = \sqrt{-1}\sqrt{7 + 24i}$
We know

$\sqrt{a+ib} = \pm\left[\sqrt{\frac{1}{2}(\sqrt{a^2+b^2}+a)} + i\sqrt{\frac{1}{2}(\sqrt{a^2+b^2}-a)}\right]$

$\therefore i\sqrt{7+24i}$

$= i\left[\pm\left\{\sqrt{\frac{1}{2}(\sqrt{49+576}+7)} + i\sqrt{\frac{1}{2}(\sqrt{49+576}-7)}\right\}\right]$

$= i\left[\pm\left\{\sqrt{\frac{1}{2}(32)} + i\sqrt{\frac{1}{2}(18)}\right\}\right]$

$= \pm(3-4\sqrt{-1})$

10 **(c)**

Given, $\alpha - i\beta = \left(\frac{3+i(-4x)}{3+i(4x)}\right)$

$\Rightarrow \quad |\alpha + i(-\beta)| = \left|\frac{3+i(-4x)}{3+i(4x)}\right|$

$= \frac{|3+i(-4x)|}{|3+i(4x)|}$

$\Rightarrow \quad \alpha^2+\beta^2 = \frac{9+16x^2}{9+16x^2}$

$\Rightarrow \quad \alpha^2+\beta^2 = 1$

11 **(b)**

$(1+\omega)^7 = (1+\omega)(1+\omega)^6$

$= (1+\omega)(-\omega^2)^6 = (1+\omega)$

$\Rightarrow A + B\omega = 1+\omega$

$\Rightarrow A = 1, B = 1$

12 **(a)**

Given equation is $x^2 + 9y^2 - 4x + 3 = 0$ ...(i)

or $x^2 - 4x + 9y^2 + 3 = 0$

Since x is real.

$\therefore (-4)^2 - 4(9y^2+3) \geq 0$

$\Rightarrow 16 - 4(9y^2+3) \geq 0$

$\Rightarrow 4 - 9y^2 - 3 \geq 0$

$\Rightarrow 9y^2 - 1 \leq 0$

$\Rightarrow (3y-1)(3y+1) \leq 0$

$\Rightarrow \frac{-1}{3} \leq y \leq \frac{1}{3}$

Eq. (i) can also be written as

$9y^2 + 0y + x^2 - 4x + 3 = 0$

Since y is real.

$\therefore 0^2 - 4.9(x^2 - 4x + 3)) \geq 0$

$\Rightarrow x^2 - 4x + 3 \leq 0$

$\Rightarrow (x-1)(x-3)) \leq 0$

$\Rightarrow 1 \leq x \leq 3$

13 **(c)**

Let α,β be the roots of the equation

$(a+1)x^2+(2a+3)x+(3a+4)=0$. Then,

$$\alpha+\beta=-1 \Rightarrow -\left(\frac{2a+3}{a+1}\right)=-1 \Rightarrow a=-2$$

$\therefore$ Product of the roots $=\frac{3a+4}{a+1}=\frac{-6+4}{-2+1}=2$

14 **(d)**

We have, $2^{x+2}3^{3x/(x-1)}=9$

Taking log on both sides, we get

$$(x+2)\log 2+\left(\frac{3x}{x-1}\right)\log 3=2\log 3$$

$$\Rightarrow (x+2)\left(\log 2+\frac{1}{x-1}\log 3\right)=0$$

$$\Rightarrow x=-2 \text{ or } \frac{1}{1-x}=\frac{\log 2}{\log 3}$$

$$\Rightarrow 1-x=\frac{\log 3}{\log 2}$$

$$\Rightarrow x=1-\frac{\log 3}{\log 2}$$

15 **(a)**

Using $a+b+c=0$, the given equation reduces to $ax^2+bx+c=0$

Clearly, $x=1$ is a root of this equation

Let D be its discriminant. Then,

$D=b^2-4ac=(-a-c)^2-4ac=(a-c)^2>0 \quad [\because a\neq c]$

Hence, the roots are real and unequal

16 **(b)**

We have, $\alpha+\beta=-\sqrt{\alpha}$ and $\alpha\,\beta=\beta$

Now,

$\alpha\,\beta=\beta \Rightarrow \alpha=1$

$\therefore \alpha+\beta=-\sqrt{\alpha} \Rightarrow \beta=-2$

17 **(c)**

We have,

$(x-a)(x-b)-1=0$

$\Rightarrow x^2-x(a+b)+ab-1=0$

Let α,β be the roots of this equation. Then,

$\alpha+\beta=a+b$ and $\alpha\,\beta=ab-1$

$\Rightarrow$ If one root is less than a, then the other root is greater than b

$\Rightarrow$ One root lies in $(-\infty,a)$ and the other is in (b,∞)

ALITER Clearly, a and b are the roots of the equation $(x-a)(x-b)=0$

Therefore, the curve $y=(x-a)(x-b)$ opens upward and cuts x-axis at $(a,0)$ and $(b,0)$

The curve $y=(x-a)(x-b)-1$ is obtained by translating $y=(x-a)(x-b)$ through one unit in vertically downward direction. So, it will cross x-axis at two points one lying on the left of $(a,0)$ and other one the right of $(b,0)$

Hence, one of the roots lies in $(-\infty,a)$ and other in (b,∞)

18 **(c)**

$$\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\left(\cos\frac{\pi}{8}+\ i\sin\frac{\pi}{8}\right)\ldots\infty$$

$$=\cos\left(\frac{\pi}{2}+\frac{\pi}{4}+\frac{\pi}{8}+\ldots\infty\right)+i\sin\left(\frac{\pi}{2}+\frac{\pi}{4}+\frac{\pi}{8}+\ldots\infty\right)$$

$$=\cos\left(\frac{\frac{\pi}{2}}{1-\frac{1}{2}}\right)+i\sin\left(\frac{\frac{\pi}{2}}{1-\frac{1}{2}}\right)$$

$\cos\pi+i\sin\pi=-1$

19 **(c)**

$$2\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+3\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\ldots+(n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$$

$$=2(1+\omega)(1+\omega^2)+3(2+\omega)(2+\omega^2)+\ldots+(n+1)(n+\omega)(n+\omega^2)$$

$$=\sum_{r=1}^{n}(r+1)(r+\omega)(r+\omega^2)$$

$$=\sum_{r=1}^{n}(r+1)[r^2+(\omega+\omega^2)r+\omega^3]$$

$$=\sum_{r=1}^{n}(r+1)(r^2-r+1)$$

$$=\sum_{r=1}^{n}(r^3+1)$$

$$=\left[\frac{n(n+1)}{2}\right]^2+n$$

20 **(d)**

Let $\sqrt{6+4\sqrt{3}}=\sqrt{x}+\sqrt{y}$

$\Rightarrow 6+4\sqrt{3}=x+y+2\sqrt{xy}$

$\Rightarrow\ x+y=6,\ \ \sqrt{xy}=2\sqrt{3}$

Now, $(x-y)^2=(x+y)^2-4xy$

$=36-4(4\times3)$

$=-12<0$

It is not possible

Hence, square root is not possible

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	C	A	B	B	C	A	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	C	D	A	B	C	C	C	D