# DPP
DAILY PRACTICE PRACTICE PROBLEMS

CLASS : XIth
DATE :

**Solutions**

SUBJECT : MATHS
DPP NO. : 2

## Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(b)**

We have,

$2(x+2) > x^2 + 1$

$\Rightarrow x^2 - 2x - 3 < 0 \Rightarrow (x-3)(x+1) < 0 \Rightarrow -1 < x < 3$

So, there are three integral values viz. 0,1, 2

2 **(a)**

Let the roots be $\alpha$ and 2 $\alpha$. Then,

$\alpha + 2\,\alpha = -\dfrac{a}{a-b}$ and $2\,\alpha^2 = \dfrac{1}{a-b}$

$\Rightarrow \alpha = -\dfrac{a}{3(a-b)}$ and $\alpha^2 = \dfrac{1}{2(a-b)}$

$\Rightarrow \dfrac{a^2}{9(a-b)^2} = \dfrac{1}{2(a-b)}$

$\Rightarrow 2a^2 = 9a - 9b$

$\Rightarrow 2a^2 - 9a + 9b = 0$

$\Rightarrow 81 - 72b \geq 0$ $\qquad [\because a \in R]$

$\Rightarrow b \leq 9/8$

Hence, the greatest value of $b$ is $\frac{9}{8}$

3 **(d)**

Let $z = \frac{1-i\sqrt{3}}{1+i\sqrt{3}} = \frac{1-i\sqrt{3}}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}} = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$

$\Rightarrow \ \arg(z) = \theta = \tan^{-1}\left(\dfrac{\sqrt{3}/2}{1/2}\right) = \tan^{-1}(\sqrt{3})$

$\Rightarrow \quad \theta = 60°$

Since, given number lies in IIIrd quadrant

$\therefore \quad \theta = 180° + 60° = 240°$

4 **(c)**

Let $z = x + iy$

Then, $z + iz = (x+iy) + i(x+iy) = (x-y) + i(x+y)$

and $iz = i(x+iy) = -y + ix$

If $\Delta$ be the area of the triangle formed by $z$, $z + iz$ and $iz$, then

$$\Delta = \frac{1}{2}\left|\begin{vmatrix} x & y & 1 \\ x-y & x+y & 1 \\ -y & x & 1 \end{vmatrix}\right|$$

Applying $R_2 \to R_2 - (R_1 + R_3)$

Then $\Delta = \frac{1}{2}\left\|\begin{matrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 1 \end{matrix}\right\| = \frac{1}{2}(x^2 + y^2)$

$= \frac{1}{2}|z|^2 = 200$ (given)

$\Rightarrow |z|^2 = 400$

$\Rightarrow |z| = 20$

$\therefore |3z| = 3|z| = 60$

25 **(c)**

Given $bx + cx + a = 0$ has imaginary roots

$\Rightarrow c^2 - 4ab < 0$

$\Rightarrow c^2 < 4ab$

$\Rightarrow -c^2 > -4ab$ ...(i)

Let $f(x) = 3b^2x^2 + 6bcx + 2c^2$

Here, $3b^2 > 0$

So, the given expression has a minimum value

$\therefore$ Minimum value $= \frac{-D}{4a}$

$$= \frac{4ac - b^2}{4a}$$

$$= \frac{4(3b^2)(2c^2) - 36b^2c^2}{4(3b^2)}$$

$$= -\frac{12b^2c^2}{12b^2} = -c^2 > -4ab$$

[from Eq. (i)]

26 **(b)**

Given, $(ax^2 + c)y + (a' x^2 + c') = 0$

or $x^2(ay + a') + (cy + c') = 0$

Since, $x$ is rational, then the discriminant of the above equation must be a perfect square.

$\therefore 0 - 4\,(ay + a') + (cy + c') = 0$

$\Rightarrow -acy^2 - (ac' + a'c)y - a'c'$

Must be a perfect square

$\Rightarrow (ac' - a'c)^2 - 4aca'c' = 0$

$\Rightarrow (ac' - a'c)^2 = 0$

$\Rightarrow ac' = a'c$

$$\Rightarrow \frac{a}{a'} = \frac{c}{c'}$$

27 **(c)**

$(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n$

$$= 2^n\left[\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^n + \left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^n\right]$$

$$= 2^n\left[\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^n + \left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)^n\right]$$

$$= 2^n\left[2\cos\frac{n\pi}{3}\right] = 2^{n+1}\cos\frac{n\pi}{3}$$

8 **(c)**

Let $z_1 = 1 + i$, $z_2 = -2 + 3i$ and $z_3 = 0 + \frac{5}{3}i$

Then, $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ -2 & 3 & 1 \\ 0 & \frac{5}{3} & 1 \end{vmatrix}$

$$= 1\left(3-\frac{5}{3}\right) + 1(2) + 1\left(\frac{-10}{3}\right)$$

$$= \frac{4}{3} + 2 - \frac{10}{3}$$

$$= \frac{4+6-10}{3} = 0$$

Hence, area of triangle is zero, therefore points are collinear

9 **(d)**

We have, $z - 2 - 3i = x + iy - 2 - 3i = (x-2) + i(y-3)$

Given, $\tan^{-1}\left(\frac{y-3}{x-2}\right) = \frac{\pi}{4}$

$\Rightarrow \quad y - 3 = x - 2$

$\Rightarrow \quad x - y + 1 = 0$

10 **(b)**

$$\frac{[(\cos 20° + i\sin 20°)(\cos 75° + i\sin 75°)(\cos 10° + i\sin 10°)]}{\sin 15° - i\cos 15°}$$

$$= \frac{e^{i20°}e^{i75°}.e^{i10°}}{-i(\cos 15° + i\sin 15°)}$$

$$= -\frac{e^{i105°}}{ie^{i15°}}$$

$$= -\frac{e^{i90°}}{i} = -1$$

11 **(c)**

Since $\alpha, \beta$ are roots of $x^2 + b\,x + 1 = 0$

$\therefore \alpha + \beta = -b, \alpha\beta = 1$

We have,

$$\left(-\alpha - \frac{1}{\beta}\right) + \left(-\beta - \frac{1}{\alpha}\right)$$

$$= -(\alpha+\beta) - \left(\frac{1}{\beta}+\frac{1}{\alpha}\right) = -(\alpha+\beta) - \frac{(\alpha+\beta)}{\alpha\,\beta} = b + b = 2b$$

and, $\left(-\alpha - \frac{1}{\beta}\right)\left(-\beta - \frac{1}{\alpha}\right) = \alpha\,\beta + 2 + \frac{1}{\alpha\,\beta} = 1 + 2 + 1 = 4$

Thus, the equation whose roots are $-\alpha-\frac{1}{\beta}$ and $-\beta-\frac{1}{\alpha}$ is
$x^2-2bx+4=0$
12 **(a)**
The required vector is given by
$\frac{3}{2}(z)e^{i\pi}=\frac{3}{2}(-4+5i)(-1+0i)=6-\frac{15}{2}i$
13 **(a)**
Given, $\frac{z}{\bar{z}}=\frac{3-i}{3+i}$ [let $z=x+iy$]
$\Rightarrow \frac{x+iy}{x-iy}=\frac{3-i}{3+i} \Rightarrow x=3a, \quad y=-a$
$\Rightarrow z=a(3-i)$, where $a\in R$
14 **(b)**
Let $m=\frac{(x-b)(x-c)}{x-a}$
$\Rightarrow x^2-(b+c+m)x+(bc+am)=0$
Since $x$ is real, we must have
$(b+c+m)^2-4(bc+am)\geq 0$
$\Rightarrow m^2+2(b+c-2a)m+(b-c)^2\geq 0$ for all $m$
$\Rightarrow 4(b+c-2a)^2-4(b-c)^2\leq 0$
$\Rightarrow (b+c-2a)^2-(b-c)^2\leq 0$
$\Rightarrow (b+c-2a+b-c)(b+c-2a-b+c)\leq 0$
$\Rightarrow 2(b-a)2(c-a)\leq 0$
$\Rightarrow (a-b)(a-c)\leq 0$
$\Rightarrow b\leq a\leq c$ or, $c\leq a\leq b$
15 **(b)**
Let $f(x)=x^4+ax^3+bx^2+cx-1$
Since $(x-1)^3$ is a factor of $f(x)$. Therefore, $(x-1)^2$ is a factor of $f'(x)$ and $(x-1)$ is a factor of $f''(x)$
$\therefore f(1)=0, f'(1)=0$ and $f''(1)=0$
$\Rightarrow a+b+c=0, 3a+2b+c=-4$ and $6a+2b=-12$
$\Rightarrow a=-2, b=0, c=2$
$\therefore f(x)=x^4-2x^3+2x-1=(x^4-1)-2x(x^2-1)$
$\Rightarrow f(x)=(x^2-1)(x^2+1-2x)=(x+1)(x-1)^3$
Hence, $(x+1)$ is the other factor of $f(x)$
16 **(a)**
Required vertices are given by
$z=(1+i)\,e^{\pm i\pi/2}=(1+i)(\pm i)=\pm(-1+i)$
17 **(b)**
Let all four roots are imaginary. Then roots of both equation $P(x)=0$ and $Q(x)=0$ are imaginary.
Thus, $b^2-4ac<0; d^2-4ac<0$, so $b^2+d^2<0$ which is impossible unless $b=0, d=0$.
So, if $b\neq 0$ or $d\neq 0$ at least two roots must be real, if $b=0$, $d=0$ we have the equations
$P(x)=ax^2+c=0$

and $Q(x) = -ax^2 + c = 0$
or $x^2 = -\frac{c}{a}; x^2 = \frac{c}{a}$ as one of $\frac{c}{a}$ and $-\frac{c}{a}$ must be positive so two roots must be real.

18 **(c)**

$$\frac{1+a}{1-a} = \frac{e^{\frac{-i\theta}{2}}(1+e^{i\theta})}{e^{\frac{-i\theta}{2}}(1-e^{i\theta})} = \frac{e^{-i\left(\frac{\theta}{2}\right)} + e^{i\frac{\theta}{2}}}{e^{-i\left(\frac{\theta}{2}\right)} - e^{-i\frac{\theta}{2}}}$$

$$= \frac{2\cos\frac{\theta}{2}}{-2i\sin\frac{\theta}{2}} = i\cot\frac{\theta}{2}$$

19 **(a)**

Let, $f(x) = x^2 + 2ax + b$
$= (x+a)^2 + b - a^2$
So, minimum value of $f(x) = b - a^2$.
Since, $f(x) \geq c, \forall x \in R$ hence $b - a^2 \geq c$
$ie, b - c \geq a^2$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | B | A | D | C | C | B | C | C | D | B |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | C | A | A | B | B | A | B | C | A | B |
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