DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 10

Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(c)**

Let α,β be the roots of the equation $2x(2x+1)=1$. Then,

$\alpha+\beta=-\frac{1}{2}$ and $\alpha\,\beta=-\frac{1}{4}$

$\Rightarrow 4\alpha^2+2\,\alpha-1=0$ ...(i)

Again,

$\alpha+\beta=-\frac{1}{2}$

$\Rightarrow \beta=-\frac{1}{2}-\alpha$

$\Rightarrow \beta=-\frac{1+2\,\alpha}{2}$

$\Rightarrow \beta=-\frac{4\,\alpha^2+2\,\alpha+2\,\alpha}{2}$ [Using (i)]

$\Rightarrow \beta=-2\,\alpha\,(\alpha+1)$

$\Rightarrow \beta=-2\,\alpha^2-2\,\alpha$

$\Rightarrow \beta=-2\,\alpha\times\alpha-2\,\alpha$

$\Rightarrow \beta=\alpha(4\,\alpha^2-1)-2\,\alpha$ [Using (i)]

$\Rightarrow \beta=4\,\alpha^3-3\,\alpha$

2 **(a)**

Let two consecutive integers n and $(n+1)$ be the roots of $x^2-bx+c=0$. Then, $n+(n+1)=b$ and $n(n+1)=c$

$\therefore\ b^2-4c=(2n+1)^2-4n(n+1)=1$

3 **(b)**

Given, $a^x=b^y=c^z=m(\text{say})$

$\Rightarrow\ x=\log_a m,\ \ y=\log_b m,\ \ z=\log_c m$

Again as, x, y, z are in GP, so

$\frac{y}{x}=\frac{z}{y}$

$\Rightarrow \frac{\log_b m}{\log_a m}=\frac{\log_c m}{\log_b m}$

$\Rightarrow \dfrac{\log_m a}{\log_m b} = \dfrac{\log_m b}{\log_m c}$

$\Rightarrow \log_b a = \log_c b$

4 **(b)**

Let O,A (z_1) and $B(z_2)$ be the vertices of the triangle. The triangle is an equilateral triangle

$\therefore z_2 = z_1\, e^{\pm\, i\,\pi/3}$

$\Rightarrow 1 + i\,b = (a + i)(\cos\pi/3 \pm \sin\pi/3)$

$\Rightarrow 1 + ib = (a + i)(1/2 \pm i\sqrt{3}/2)$

$\Rightarrow 1 + i\,b = \left(\dfrac{a}{2} \pm \dfrac{\sqrt{3}}{2}\right) + i\left(\dfrac{1}{2} \pm a\dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow \dfrac{a}{2} \pm \dfrac{\sqrt{3}}{2} = 1$ and $b = \dfrac{1}{2} \pm \dfrac{1}{2}a\sqrt{3}$

$\Rightarrow (a = 2 - \sqrt{3}$ or $a = 2 + \sqrt{3})$ and $b = \dfrac{1}{2} \pm \dfrac{a}{2}\sqrt{3}$

$\Rightarrow a = 2 - \sqrt{3}$ and $b = 2 - \sqrt{3}$ $\quad [\because 0 < a,b < 1]$

5 **(d)**

We have,

$$\sum_{r=0}^{n} (-1)^r\ {}^nC_r\{i^{5r} + i^{6r} + i^{7r} + i^{8r}\}$$

$$= \sum_{r=0}^{n} (-1)^r\,{}^nC_r\{i^r + i^{2r} + i^{3r} + 1\}$$

$$= \sum_{r=0}^{n} (-1)^r\,{}^nC_r\, i^r + \sum_{r=0}^{n} (-1)^r\ {}^nC_r(i^2)^r + \sum_{r=0}^{n} (-1)^r\ {}^nC_r(i^3)^r + \sum_{r=0}^{n} (-1)^r\ {}^nC_r$$

$$= (1 - i)^n + (1 - i^2)^n + (1 - i^3)^n + (1 - 1)^n$$

$$= (1 - i)^n + 2^n + (1 + i)^n$$

$$= 2^n + 2^{n/2}\left\{\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right\}^n + 2^{n/2}\left\{\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right\}^n$$

$$= 2^n + 2^{n/2+1}\cos\frac{n\,\pi}{4}$$

6 **(c)**

Since, $b = \frac{a+c}{2}$ ...(i)

Now, discriminant, $D = B^2 - 4AC$

$= 4b^2 - 4ac$

$= 4\left(\frac{a+c}{2}\right)^2 - 4ac$ [from Eq. (i)]

$= (a - c)^2 \geq 0$

$\therefore$ Roots of the given equation are rational and distinct

7 **(a)**

We have,

$\log_{1/2}|z - 2| > \log_{1/2}|z|$

$\Rightarrow |z - 2| < |z|$

$\Rightarrow z$ lies on the right side of the perpendicular bisector of the segment joining (0, 0) and (2, 0)
$\Rightarrow \text{Re}(z) > 1$

9 **(d)**
Since, $x^2 - 3|x| + 2 = 0$
$\Rightarrow (|x| - 2)(|x| - 1) = 0$
$\Rightarrow |x| = 2$ or $|x| = 1$
$\Rightarrow x = \pm 2$ or $x = \pm 1$
$\therefore$ The given equation has four real roots
10 **(d)**
Let 4 and α be roots of given equation
$\therefore 4\alpha = 12 \quad \Rightarrow \alpha = 3$
And $4 + 3 = -p \quad \Rightarrow \quad p = -7$
$\therefore$ Equation $x^2 + px + q = 0$ will reduce to $x^2 - 7x + q = 0$
Let this equation have β, β as its roots
$\therefore \; 2\beta = 7 \;\; \Rightarrow \;\; \beta = \frac{7}{2}$ and $\beta^2 = q$
$\Rightarrow \quad q = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$
11 **(b)**
$[x]^2 - [x] - 2 = 0$
$\Rightarrow ([x] - 2)([x] + 1) = 0$
$\Rightarrow [x] = 2, \, -1$
$\Rightarrow x \in [-1, 0] \cup [2, 0]$
12 **(d)**
We have,
$\alpha + \beta = -b/a$ and $\alpha\,\beta = c/a$
Now,
Sum of the roots $= 2 + \alpha + 2 + \beta = 4 + (\alpha + \beta) = 4 - b/a$
Product of the roots $= (2 + \alpha)(2 + \beta)$
$= 4 + \alpha\,\beta + 2(\alpha + \beta)$
$= 4 + \frac{c}{a} - \frac{2\,b}{a} = \frac{4\,a + c - 2\,b}{a}$
Hence, required equation is
$x^2 - x\left(4 - \frac{b}{a}\right) + \frac{4\,a + c - 2\,b}{a} = 0$
or, $ax^2 + (b - 4a)x + 4a - 2b + c = 0$
ALITER Required equation can be obtained by replacing x by $x + 2$ in the given equation
13 **(c)**
Given, $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$ ...(i)
$\therefore \quad \tan(\alpha + \beta + \gamma)$
$= \frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha\tan\beta\tan\gamma}{1 - \tan\alpha\tan\beta - \tan\beta\tan\gamma - \tan\gamma\tan\alpha}$
$\Rightarrow \quad \tan(\alpha + \beta + \gamma)$

$$= \frac{0}{1 - \tan\alpha\tan\beta - \tan\beta\tan\gamma - \tan\gamma\tan\alpha}$$

[From Eq. (i)]

$\Rightarrow \tan(\alpha + \beta + \gamma) = 0$

$\Rightarrow \alpha + \beta + \gamma = 0^{\circ}$ or π

$\therefore xyz = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)(\cos\gamma + i\sin\gamma)$

$= \cos(\alpha + \beta + \gamma) + i\sin(\alpha + \beta + \gamma)$

$= \cos 0° + i\sin 0° = 1$

or $\quad xyz = \cos\pi + i\sin\pi = -1$

14 **(c)**

We have,

$\arg(z_1 z_2) = 0$

$\Rightarrow \arg(z_1) + \arg(z_2) = 0$

$\Rightarrow \arg(z_1) = -\arg(z_2)$

$\Rightarrow \arg(z_1) = \arg(\bar{z}_2)$

Since, $|z_1| = |z_2| = 1$. Therefore, $|z_1| = |\bar{z}_2| = 1$

Hence, $z_1 = \bar{z}_2$

15 **(c)**

Let α be a common root of the two equations. Then,

$2\alpha^2 - 7\alpha + 1 = 0$

$a\alpha^2 + b\alpha + 2 = 0$

$\Rightarrow \frac{\alpha^2}{-14 - b} = \frac{\alpha}{a - 4} = \frac{1}{2b + 7a}$

$\Rightarrow \frac{a - 4}{2b + 7a} = \frac{b + 14}{4 - a}$

$\Rightarrow (7a + 2b)(b + 14) + (a - 4)^2 = 0$

Clearly, $a = 4, b = -14$ satisfy this equation

16 **(b)**

We know that ω and ω^2 are roots of $x^2 + x + 1 = 0$. Therefore, $x^{3m} + 3^{3n+1} + x^{3k+2}$ will be exactly divisible by $x^2 + x + 1$, if ω and ω^2 are its roots

For $x = \omega$, we have

$x^{3m} + x^{3n+1} + x^{3k+2} = \omega^{3m} + \omega^{3n+1} + \omega^{3k+2} = 1 + \omega + \omega^2 = 0$ provided that m,n,k are integers

Similarly, $x = \omega^2$ will be a root of $x^{3m} + x^{3n+1} + x^{3k+2}$ if m,n,k are integers

17 **(d)**

$$\log_{10}\left(\frac{a + 10b + 10^2c}{10^{-4}a + 10^{-3}b + 10^{-2}c}\right)$$

$$= \log_{10}\left(\frac{a + 10b + 10^2c}{\frac{1}{10^4}(a + 10b + 10^2c)}\right)$$

$$= \log_{10} 10^4 = 4$$

18 **(a)**

Since, tan 30° and tan 15° are the roots of equation

$$x^2 + px + q = 0$$

$$\therefore \quad \tan 30° + \tan 15° = -p$$

And $\tan 30° \tan 15° = q$

Now, $2 + q - p = 2 + \tan 30° + \tan 15° + (\tan 30° + \tan 15°)$

$$= 2 + \tan 30° + \tan 15° + 1 - \tan 30° \tan 15°$$

$$\left(\because \tan 45° = \frac{\tan 30° + \tan 15°}{1 - \tan 30° \tan 15°}\right)$$

$\Rightarrow\ 2 + q - p = 3$

19 **(d)**

Given, $z^{1/3} = p + iq$

$\Rightarrow\quad (x - iy) = (p + iq)^3 \quad$ [put $z = x - iy$]

$\Rightarrow\quad (x - iy) = p^3 — iq^3 + 3p^2qi - 3pq^2$

$\Rightarrow\ (x - iy) = (p^3 - 3pq^2) + i(3p^2q - q^3)$

$\Rightarrow\ x = (p^3 - 3pq^2)$ and $-y = 3p^2q - q^3$

$\Rightarrow \frac{x}{p} = (p^2 - 3q^2)$ and $\frac{y}{q} = (q^2 - 3p^2)$

$\therefore\ \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2$

$\Rightarrow \frac{\frac{x}{p} + \frac{y}{q}}{(p^2 + q^2)} = -2$

20 **(c)**

Here, $\sec\alpha + \operatorname{cosec}\alpha = p$ and $\sec\alpha.\operatorname{cosec}\alpha = q$

$\Rightarrow \frac{\sin\alpha + \cos\alpha}{\sin\alpha\cos\alpha} = p$ and $\sin\alpha\cos\alpha = \frac{1}{q}$

$\Rightarrow (\sin\alpha + \cos\alpha)^2 = \left(\frac{p}{q}\right)^2$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \frac{p^2}{q^2}$

$\Rightarrow q^2\left(1 + \frac{2}{q}\right) = p^2$

$\Rightarrow q(q + 2) = p^2$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	B	B	D	C	A	B	D	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	C	C	C	B	D	A	D	C