DPP
DAILY PRACTICE PRODUCTS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 1

Topic :- COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1 **(c)**

Let $z=\frac{1-i}{3+i}+\frac{4i}{5}$

$=\frac{5-5i+12i-4}{5(3+i)}=\frac{1+7i}{5(3+i)}$

$=\frac{(1+7i)(3-i)}{5(9+1)}=\frac{10+20i}{50}=\frac{1+2i}{5}$

$\therefore \quad |z|=\sqrt{\left(\frac{1}{5}\right)^2+\left(\frac{2}{5}\right)^2}=\frac{1}{5}\sqrt{1+4}=\frac{\sqrt{5}}{5}$

2 **(b)**

Let each ratio be k and let $A=xyz$,

Then $\log x=k(a-b),\ \log y=k(b-c)$

And $\log z=k(c-a)$

$\therefore \quad \log A = \log x+\log y+\log z$

$=k(a-b)+k(b-c)+k(c-a)$

$=k[a-b+b-c+c-a]$

$=k[0]$

$\therefore \quad \log A= \log(xyz)=0 \quad [\because A=xyz]$

$\Rightarrow xyz=e^0=1$

3 **(c)**

Let $z=x+iy$. Then, coordinates of the vertices of the triangle are $(-x,-y),(-y,x)$ and $(x+y,y-x)$

$\therefore$ Area of the triangle

$=\frac{1}{2}\begin{vmatrix}-x & -y & 1\\ -y & x & 1\\ x+y & y-x & 1\end{vmatrix}$

$=\frac{1}{2}\begin{vmatrix}-x & -y & 1\\ x-y & x+y & 0\\ 2x+y & 2y-x & 0\end{vmatrix}$ Applying $R_2\to R_2\to R_1$, $R_3\to R_3\to R_1$

$=-\frac{3}{2}(x^2+y^2)=-\frac{3}{2}|z|^2$

Hence, Area $=\frac{3}{2}|z|^2$

4 **(c)**

Given, $\frac{(1+i)^2}{2-i} = x + iy$

$\Rightarrow \quad \frac{2i}{2-i} \times \frac{2+i}{2+i} = x + iy$

$\Rightarrow \quad \frac{4i-2}{5} = x + iy$

$\Rightarrow \ x + iy = -\frac{2}{5} + \frac{4}{5} i$

$\therefore \ \ x + y = -\frac{2}{5} + \frac{4}{5} = \frac{2}{5}$

5 **(c)**

We have,

$|z - 3 + i| = |z - 2 - i|$

$\Rightarrow |z - (3 - i)| = |z - (2 + i)|$

$\Rightarrow AP = BP$

$\Rightarrow$ locus of P is the perpendicular bisector of AB

7 **(a)**

We have, $z = \frac{1+ir}{1+p} \quad \therefore iz = \frac{-r+iq}{1+p}$

By componendo and dividendo

$\frac{1+iz}{1-iz} - \frac{1+p-r+iq}{1+p+r-iq}$

$\therefore \ \frac{p+iq}{1+r} = \frac{1+iz}{1-iz}$ if $\frac{p+iq}{1+r} = \frac{1+p-r+iq}{1+p+r-iq}$

or $p(1+p+r) + q^2 + i\{q(1+p+r) - pq\}$

$= (1+r)(1+p-r) + iq(1+r)$

$\Rightarrow \ p(1+p+r) + q^2 = (1+r)(1+p-r)$

and $q(1+p+r) - pq = q(1+r)$

[this is obviously true]

$\therefore$ The condition is

$p(1+p+r) + q^2 = (1+r)(1+p-r)$

or $p + p^2 + pr + q^2 = 1 + p - r + r + pr - r^2$

or $p^2 + q^2 + r^2 = 1$

9 **(b)**

Since, $2q = p + r$

Given that, $px^2 + qx + r = 0$ has complex roots

$\therefore \quad D < 0$

$\Rightarrow q^2 - 4pr < 0$

$\Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr < 0$

$\Rightarrow \ p^2 + r^2 - 14pr < 0$

$\Rightarrow \ \frac{p^2}{r^2} + 1 - \frac{14p}{r} < 0$

$\Rightarrow \left(\frac{p^2}{r^2} - \frac{14\,p}{r} + 49\right) - 48 < 0$

$\Rightarrow \left(\frac{p}{r} - 7\right)^2 < 48 \ \Rightarrow \ \left|\frac{p}{r} - 7\right| < 4\sqrt{3}$

10 **(b)**

Given, $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$

$\Rightarrow r(2x + p + q) = [x^2 + (p+q)x + pq]$

$\Rightarrow x^2 + (p + q - 2r)x + pq - r(p+q) = 0$

As we know, if roots are equal in magnitude but opposite in sign, then coefficient of x will be zero

$\therefore \ \ p + q - 2r = 0 \ \Rightarrow p + q = 2r$

11 **(b)**

We have, $|2x - 3| < |x + 2|$

Following cases arise:

CASE I *When* $x < -2$

In this case, we have

$|2x - 3| = -(2x - 3)$ and $|x + 2| = -(x + 2)$

$\therefore |2x - 3| < |x + 2|$

$\Rightarrow -(2x - 3) < -(x + 2)$

$\Rightarrow 2x - 3 > x + 2 \Rightarrow x - 5 > 0 \Rightarrow x > 5$

But, $x < -2$. So, there is no solution in this case

CASE II *When* $-2 \le x < \frac{3}{2}$

In this case, we have

$|x + 2| = x + 2$ and $|2x - 3| = -(2x - 3)$

$\therefore |2x - 3| < |x + 2|$

$\Rightarrow -(2x - 3) < x + 2 \Rightarrow 3x - 1 > 0 \Rightarrow x > \frac{1}{3}$

But, $-2 \le x < \frac{3}{2}$. Therefore, $x \in \left(\frac{1}{3}, \frac{3}{2}\right)$

CASE III *When* $x \ge \frac{3}{2}$

In this case, we have

$|x + 2| = x + 2$ and $|2x - 3| = 2x - 3$

$\therefore |2x - 3| < |x + 2| \Rightarrow 2x - 3 < x + 2 \Rightarrow x < 5$

But, $x \ge \frac{3}{2}$. Therefore, $x \in [3/2, 5)$

Hence, the solution set is $x \in (1/3, 5)$

12 **(b)**

Let the correct equation is

$ax^2 + bx + c = 0,$

Then $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

When b is written incorrectly, then the roots are equal.

Let these are γ and γ.

$\therefore\ \gamma.\gamma=\frac{c}{a}\Rightarrow \gamma^2=\alpha\beta$ ...(i)

When c is written icorrectly, then the roots are γ and 2γ.

$\therefore\ \gamma+2\gamma=-\dfrac{b}{a}\Rightarrow 3\gamma=\alpha+\beta$

$\Rightarrow 9\gamma^2=(\alpha+\beta)^2 \Rightarrow 9\alpha\beta=(\alpha-\beta)^2+4\alpha\beta$

[from Eq. (i)]

$\therefore (\alpha-\beta)^2=5\alpha\beta$

13 **(c)**

Let $y=\frac{x^2+34x-71}{x^2+2x-7}$

$\Rightarrow\ x^2(y-1)+x(2y-34)+71-7y=0$

Since, x is complex number

$\therefore\ \ D<0$

$\Rightarrow (2y-34)^2-4(y-1)(71-7y)<0$

$\Rightarrow\ (y-17)^2-(71y-7y^2-71+7y)<0$

$\Rightarrow 8y^2-112y+360<0$

$\Rightarrow y^2-14y+45<0$

$\Rightarrow (y-9)(y-5)<0$

$\Rightarrow\ 5<y<9$

$\therefore\ \ a=5, b=9$

14 **(a)**

Given, a, b, c are real, $ax^2+bx+c=0$ has two real roots α and β, where $\alpha<-2$ and $\beta>2$

$\Rightarrow\ f(-2)<0$ and $f(2)>0$

$\Rightarrow\ \ 4a-2b+c<0$ and $4a+2b+c>0$

$\Rightarrow\ \ 4-\frac{2b}{a}+\frac{c}{a}<0$ and $4+\frac{2b}{a}+\frac{c}{a}>0$

15 **(d)**

Let the correct equation be $ax^2+bx+c=0$ and the correct roots are α and β. Taking c wrong, the roots are 3 and 2.

$\therefore\ \ \alpha+\ \beta=3+2=5$ ...(i)

Also, $a=1$ and $c=-6$

$\therefore\ \alpha\beta=\frac{c}{a}=-6$ ...(ii)

On solving Eqs.(i) and (ii), the correct roots are 6 and -1.

16 **(a)**

Since, 1 is root of $ax^2+bx+c=0$

$\Rightarrow\ \ a+b+c=0$

$\therefore E_1: a+b+c=0$ is true

Since, $\cos\theta, \sin\theta$ are the roots of $ax^2+bx+c=0$

$\therefore\ \ \sin\theta+\cos\theta=-\dfrac{b}{a}$

And $\ \sin\theta\cos\theta=\frac{c}{a}$

$\Rightarrow\ \ (\sin\theta+\cos\theta)^2=\dfrac{b^2}{a^2}$

$\Rightarrow\ \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\dfrac{b^2}{a^2}$

$\Rightarrow\ \ 1+2\left(\dfrac{c}{a}\right)=\dfrac{b^2}{a^2}$

$\Rightarrow b^2-a^2=2ac$

Hence, E_1 and E_2 both are true

17 **(c)**

$(3+\omega+3\omega^2)^4=[3+(1+\omega^2)+\omega]^4$

$=[-3\omega+\omega]^4$

$=(-2\omega)^4$

$=16\omega$

18 **(a)**

$z^3+\dfrac{1}{i}z^2-\dfrac{z}{i}+1=0$

$\Rightarrow\ z^3-iz^2+iz+1=0$

$\Rightarrow\ z^2(z-i)+i(z-i)=0$

$\Rightarrow\ (z-i)(z^2+i)\ \Rightarrow |z|=1$

19 **(a)**

Given equation $x^2+ax+1=0$.

Since, roots are $\tan\theta$ and $\cot\theta$.

$\therefore\ $ Product of roots, $\tan\theta\cdot\cot\theta=a\Rightarrow a=1$

Again, since roots are real.

$\therefore a^2-4\geq 0\Rightarrow |a|\geq 2$

Thus, the least value of $|a|$ is 2.

20 **(c)**

If 1,2, 3, 4 are the roots of given equation, then

$(x-1)(x-2)(x-3)(x-4)=x^4+ax^3+bx^2+cx+d$

$\Rightarrow(x^2-3x+2)(x^2-7x+12)=x^4+ax^3+bx^2+cx+d$

$\Rightarrow x^4-10x^3+35x^2-50x+24$

$=x^4+ax^3+bx^2+cx+d$

$\Rightarrow a=-10, b=35, c=-50, d=24$

$\therefore\ a+2b+c=-10+2\times35-50=10$

Alternate

Since, 1, 2, 3 and 4 are the roots of the equation

$x^4+ax^3+bx^2+cx+d=0$, then

$1+a+b+c+d=0$ ...(i)

$16+8a+4b+2c+d=0$ ...(ii)

$81+27a+9b+3c+d=0$ ...(iii)

And $256+64a+16b+4c+d=0$ ...(iv)

On solving Eqs. (i), (ii), (iii) and (iv), we get

$a=-10, b=35, c=-50, \quad d=24$

Now, $a+2b+c=-10+2\times35+(-50)$

$=-10+70-50=10$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	C	C	C	A	B	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	C	A	D	A	C	A	A	C