DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

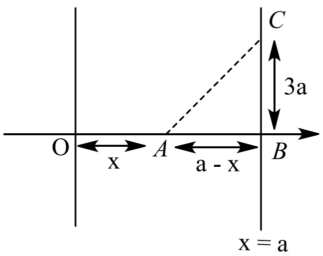
Solutions

SUBJECT : MATHS
DPP NO. : 9

Topic :- CO-ORDINATE GEOMETRY

1 **(d)**

Area of $\Delta ABC = a^2$



$\Rightarrow \frac{1}{2}(a-x)3a = a^2$

$\Rightarrow a - x = \frac{2}{3}a$

$\Rightarrow x = \frac{a}{3}$

Hence, one of the line on which third vertex lies is $x = \frac{a}{3}$

2 **(c)**

Draw BE perpendicular to CA produced, then

$BD = DC = \frac{a}{2}$ and $EA = AC = b$

In ΔAEB,

$\cos(\pi - A) = \frac{b}{c}$

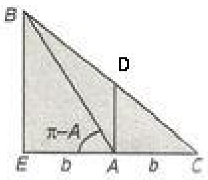
$\Rightarrow \cos A = -\frac{b}{c}$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc} = -\frac{b}{c}$

$\Rightarrow a^2 = 3b^2 + c^2$

$\therefore \cos B = \frac{c^2+a^2-b^2}{2ac}$

$= \frac{c^2+3b^2+c^2-b^2}{2ca} = \frac{b^2+c^2}{ca}$



4 **(d)**

Let the sides of ΔABC be $a = n, b = n + 1, c = n + 2$, where n is a natural number. Then, C is the greatest and A is the least angle

As given $C = 2A$

$\therefore \sin C = \sin 2A = 2\sin A \cos A$

$\therefore kc = 2ka\dfrac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow bc^2 = a(b^2 + c^2 - a^2)$

On substituting the values of a, b, c, we get

$(n + 1)(n + 2)^2 = n[(n + 1)^2 + (n + 2)^2 - n^2]$

$= n(n^2 + 6n + 5)$

$= n(n + 1)(n + 5)$

$\Rightarrow (n + 1)[(n + 2)^2 - n(n + 5)] = 0$

Since, $n \neq 1$

Thus, $(n + 2)^2 = n(n + 5)$

$\Rightarrow n^2 + 4n + 4 = n^2 + 5n$

$\Rightarrow n = 4$

Hence, the sides of the triangle are 4, 5 and 6

5 **(c)**

1. $\dfrac{b^2 - c^2}{a\sin(B - C)} = \dfrac{2R^2(\sin^2 B - \sin^2 C)}{2R\sin A\sin(B - C)}$

$= \dfrac{2R\sin(B + C)\sin(B - C)}{\sin(B + C)\sin(B - C)} = 2R$

2. $a\sin(B - C) + b\sin(C - A) + c\sin(A - B) = 0$

$= 2R[\sin A\sin(B - C) + \sin B\sin(C - A) + \sin C\sin(A - B)]$

$= 2R[\sin(B + C)\sin(B - C) + \sin(C + A)\sin(C - A) + \sin(A + B)\sin(A - B)]$

$= 2R[\sin^2 B - \sin^2 C + \sin^2 C - \sin^2 A + \sin^2 A - \sin^2 B]$

$= 2R(0) = 0$

Hence, both of statements are correct

6 **(b)**

Let the general equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

The equation of circle passing through (0, 0), (2, 0) and (0, − 2)

$c = 0$ ...(i)

$4 + 4g + c = 0$ ...(ii)
and $4 - 4f + c = 0$ ...(iii)
On solving Eqs. (i), (ii) and (iii), we get
$c = 0, \quad g = -1, \quad f = 1$
$\therefore$ The equation of circle becomes $x^2 + y^2 - 2x + 2y = 0$
Since, it is passes through $(k, -2)$, we get
$k^2 + 4 - 2k - 4 = 0 \Rightarrow k = 0, 2$
We have already take a point $(0, -2)$, so we take only $k = 2$

7 **(a)**

Let $X = x - h, \quad Y = y - k$
$\Rightarrow \quad 0 = 7 - h, \qquad 0 = -4 - k$
$\Rightarrow \quad h = 7, \qquad k = -4$
Hence, $X = x - 7$ and $Y = y + 4$, then the point $(4, 5)$ shifted to $(-3, 9)$

8 **(a)**

$$(a + b + c)\left(\tan\frac{A}{2} + \tan\frac{B}{2}\right) = 2\left(s\tan\frac{A}{2} + s\tan\frac{B}{2}\right)$$
$$= 2\left(\frac{\Delta}{s-a} + \frac{\Delta}{s-b}\right)$$
$$= 2\Delta\frac{2s - (a+b)}{(s-a)(s-b)}$$
$$= 2\Delta\left(\frac{c}{(s-a)(s-b)}\right)$$
$$= 2c\cot\frac{C}{2}$$

9 **(b)**

$$\because \frac{2}{1!9!} + \frac{2}{3!7!} + \frac{1}{5!5!} = \frac{8^a}{(2b)!}$$
$$\Rightarrow \frac{1}{1!9!} + \frac{1}{3!7!} + \frac{1}{5!5!} + \frac{1}{3!7!} + \frac{1}{9!1!} = \frac{8^a}{(2b)!}$$
$$\Rightarrow \frac{1}{10!}\left(\frac{10!}{1!9!} + \frac{10!}{3!7!} + \frac{10!}{5!5!} + \frac{10!}{7!3!} + \frac{10!}{9!1!}\right) = \frac{8^a}{(2b)!}$$
$$\Rightarrow \frac{1}{10!}\left({}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9\right) = \frac{8^a}{(2b)!}$$
$$\Rightarrow \frac{2^9}{10!} = \frac{8^a}{(2b)!} = \frac{2^{3a}}{(2b)!}$$
$\Rightarrow a = 3, \qquad b = 5$
Also, $2b = a + c \Rightarrow 10 = 3 + c \Rightarrow c = 7$
$\therefore a = 3, \qquad b = 5, \qquad c = 7$
$$\because \frac{\tan A + \tan B}{2} \geq \sqrt{\tan A \tan B} \quad \text{...(i)}$$
Also, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
$$= \frac{9 + 25 - 49}{30} = -\frac{1}{2}$$

$\Rightarrow \angle C = 120°$ and $A, B < 60°$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow \tan A + \tan B - \sqrt{3} = -\sqrt{3} \tan A \tan B$

$\therefore \tan A + \tan B = \sqrt{3}(1 - \tan A \tan B)$ ...(ii)

Also, $\tan A + \tan B > 0$

$\Rightarrow \sqrt{3}(1 - \tan A \tan B) > 0$

$\Rightarrow \tan A \tan B < 1$ ...(iii)

From Eq. (i) and (ii),

$$\frac{\sqrt{3}(1 - \tan A \tan B)}{2} \geq \sqrt{(\tan A \tan B)}$$

Let $\tan A \tan B = \lambda$

$\therefore \sqrt{3}(1 - \lambda) \geq 2\sqrt{\lambda}$

$\Rightarrow 3\lambda^2 - 10\lambda + 3 \geq 0$

$\Rightarrow (3\lambda - 1)(\lambda - 3) \geq 0$

$\because \lambda - 3 < 0$ [from Eq. (iii)]

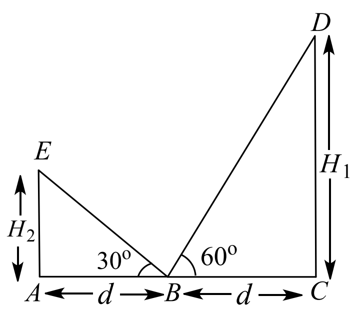
$\therefore 3\lambda - 1 \leq 0$

$\Rightarrow \lambda \leq \frac{1}{3}$

$\Rightarrow \tan A \tan B \leq \frac{1}{3}$

10 **(c)**

In ΔBCD, $\tan 60° = \frac{H_1}{d}$



$\Rightarrow H_1 = d \tan 60°$

and in ΔABE, $\tan 30° = \frac{H_2}{d}$

$\Rightarrow H_2 = d \tan 30°$

$$\therefore \frac{H_1}{H_2} = \frac{\tan 60°}{\tan 30°} = \frac{\sqrt{3}}{1/\sqrt{3}} = \frac{3}{1}$$

11 **(c)**

We have, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$$\Rightarrow \cos 60° = \frac{a^2 + b^2 - c^2}{2ab}$$

$\Rightarrow a^2 + b^2 - c^2 = ab$

$\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$

On dividing by $(a + c)(b + c)$ and add 2 on both sides, we get

$$1+\frac{b}{a+c}+1+\frac{a}{b+c}=3$$
$$\Rightarrow\ \frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$$

12 **(a)**

$(a+c)^2-b^2=3ac \Rightarrow\ a^2+c^2-b^2=ac$

But $\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{1}{2}\Rightarrow\ \angle B=\frac{\pi}{3}=60°$

13 **(c)**

Given, a^2, b^2, c^2 are in AP

$\Rightarrow\ \sin^2 B-\sin^2 A\ =\ \sin^2 C-\sin^2 B$

$\Rightarrow\ \sin(B+A)\sin(B-A)\ =\ \sin(C+B)\sin(C-B)$

$\Rightarrow\ \sin C(\sin B\cos A\ -\ \cos B\sin A)$

$=\ \sin A\,(\sin C\cos B\ -\ \cos C\sin B\,)$

On dividing by $\sin A\sin B\sin C$, we get

$2\cot B\ =\ \cot A\ +\ \cot C$

$\Rightarrow \cot A, \cot B, \cot C$ are in AP

14 **(c)**

Given, $\sin A\sin B=\frac{ab}{c^2}$

$$\Rightarrow c^2=\frac{ab}{\sin A\sin B}=\left(\frac{a}{\sin A}\right)\left(\frac{b}{\sin B}\right)$$
$$\Rightarrow c^2=\left(\frac{c}{\sin C}\right)^2$$
$$\because\ \left(\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}\right)$$

$\Rightarrow\ \sin^2 C\ =1$

$\Rightarrow\ c=90°$

Hence, $\Delta\,ABC$ is a right angled triangle

15 **(a)**

Let a, b, c be the sides of triangle, then

$$a+b+c=\frac{6}{3}(\,\sin A\ +\ \sin B\ +\ \sin C\,)$$
$$\Rightarrow\ a+b+c=2(\,\sin A\ +\ \sin B\ +\ \sin C\,)$$
$$\Rightarrow\ \frac{a}{2}=\ \sin A$$

But $a=1$

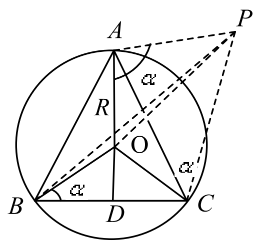
$$\therefore\ \sin A\ =\frac{1}{2}\Rightarrow \angle A=\frac{\pi}{6}$$

16 **(a)**

The centroid of $\Delta ABC=\left(\frac{2+8+5}{3}, \frac{3+10+5}{3}\right)=(5, 6)$

17 **(d)**

Since, the tower OP makes equal angle at the vertices of the triangle, therefore foot of the tower is the circumcentre



In ΔOAP, $\tan\alpha = \frac{OP}{OA}$

$\Rightarrow OP = OA\tan\alpha$

$\Rightarrow OP = R\tan\alpha$

18 **(a)**

In $\Delta\, ABE$,

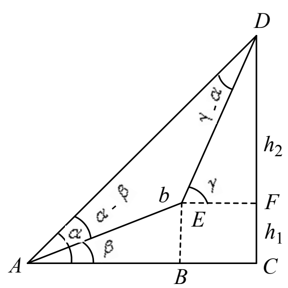
$$\sin\beta = \frac{BE}{b}$$

$\Rightarrow BE = h_1 = b\sin\beta$

Using sine rule in ΔAED,

$$\frac{\sin(\alpha-\beta)}{ED} = \frac{\sin(\gamma-\alpha)}{b}$$

$$\Rightarrow ED = \frac{b\sin(\alpha-\beta)}{\sin(\gamma-\alpha)}$$



Now, in ΔFED,

$$\sin\gamma = \frac{h_2}{ED}$$

$$\Rightarrow h_2 = \frac{b\sin(\alpha-\beta)\sin\gamma}{\sin(\gamma-\alpha)}$$

$\therefore$ Total height, CD

$$= h_1 + h_2 = b\sin\beta + \frac{b\sin(\alpha-\beta)\sin\gamma}{\sin(\gamma-\alpha)}$$

$$= \frac{b[\sin\beta\sin(\gamma-\alpha) + \sin(\alpha-\beta)\sin\gamma]}{\sin(\gamma-\alpha)}$$

$$= \frac{b[\sin\beta\{\sin\gamma\cos\alpha - \cos y\sin\alpha\} + \sin\gamma\{\sin\alpha\cos\beta\cos\alpha\}]}{\sin(\gamma-\alpha)}$$

$$= \frac{b[\sin\beta\sin\gamma\cos\alpha - \sin\beta\cos\gamma\sin\alpha + \{\sin\gamma\sin\alpha\cos\beta - \sin\gamma\sin\beta\cos\alpha\}]}{\sin(\gamma-\alpha)}$$

$$= \frac{b \sin \alpha \sin(\gamma - \beta)}{\sin(\gamma - \alpha)}$$

19 **(c)**

Given, $a = 1, b = 2, \angle C = 60°$

$\therefore$ Area of triangle $= \frac{1}{2} ab \sin C$

$$= \frac{1}{2} \times 1 \times 2 \times \sin 60°$$

$$= \frac{\sqrt{3}}{2} \text{ sq unit}$$

20 **(c)**

The given points are collinear

If $\begin{vmatrix} t_1 & 2at_1 + at_1^3 & 1 \\ t_2 & 2at_2 + at_2^3 & 1 \\ t_3 & 2at_3 + at_3^3 & 1 \end{vmatrix} = 0$

$$\Rightarrow \quad a \begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 & 2t_2 + t_2^3 & 1 \\ t_3 & 2t_3 + t_3^3 & 1 \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$, we get

$$\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 - t_1 & 2(t_2 - t_1) + (t_2^3 - t_1^3) & 0 \\ t_3 - t_1 & 2(t_3 - t_1) + (t_3^3 - t_1^3) & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad (t_2 - t_1)(t_3 - t_1) \begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ 1 & 2 + t_2^2 + t_1^2 + t_1 t_2 & 0 \\ 1 & 2 + t_3^2 + t_1^2 + t_3 t_1 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad (t_2 - t_1)(t_3 - t_1)(t_3 - t_2)(t_3 + t_2 + t_1) = 0$

$\Rightarrow \; t_1 + t_2 + t_3 = 0$

$[\because t_1 \neq t_2 \neq t_3]$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	D	D	C	B	A	A	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	C	C	A	A	D	A	C	C