DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

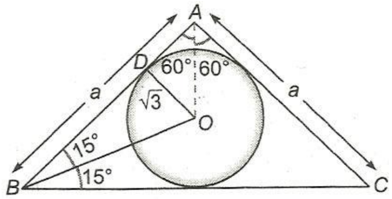
SUBJECT : MATHS
DPP NO. :8

Topic :- CO-ORDINATE GEOMETRY

1 **(c)**

Let $AB = AC$ and $\angle A = 120°$

$\therefore$ Area of triangle $= \frac{1}{2}a^2 \sin 120°$



Where, $a = AD + BD$

$= \sqrt{3}\tan 30° + \sqrt{3}\cot 15°$

$= 1 + \sqrt{3}\left(\frac{1 + \tan 45° \tan 30°}{\tan 45° - \tan 30°}\right) = 1 + \sqrt{3}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

$\therefore \ a = 4 + 2\sqrt{3}$

$\Rightarrow$ Area of triangle $= \frac{1}{2}(4 + 2\sqrt{3})^2\left(\frac{\sqrt{3}}{2}\right) = 12 + 7\sqrt{3l}$

2 **(b)**

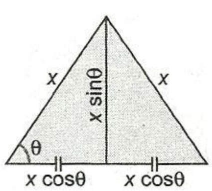
Area $= \frac{1}{2} \times$ base $\times$ altitude

$= \frac{1}{2} \times (2x\cos\theta) \times (x\sin\theta)$

$= \frac{1}{2}x^2 \sin 2\theta$

(Since, maximum value of $\sin 2\theta$ is 1)

$\therefore$ Maximum area $= \frac{1}{2}x^2$



3 **(d)**

Here, $\tan\frac{A}{2}\tan\frac{C}{2}=\frac{1}{3}$

$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}\cdot\frac{(s-b)(s-a)}{s(s-c)}}=\frac{1}{3}$

$\Rightarrow \frac{s-b}{s}=\frac{1}{3} \Rightarrow 2s=3b$

$\Rightarrow 2b=a+c$

$\Rightarrow a,b,c$ are in AP

4 **(d)**

Let the vertices of a triangle are $P(2,1)$, $Q(5,2)$ and $R(3,4)$ and $A(x,y)$ be the circumcentre of ΔPQR

$\therefore AP^2=AQ^2$

$\Rightarrow (2-x)^2+(1-y)^2=(5-x)^2+(2-y)^2$

$\Rightarrow 4+x^2-4x+1+y^2-2y=25+x^2-10x+4+y^2-4y$

$\Rightarrow 6x+2y=24$

$\Rightarrow 3x+y=12$ ...(i)

and $AP^2=AR^2$

$\Rightarrow (2-x)^2+(1-y)^2=(3-x)^2+(4-y)^2$

$\Rightarrow 4+x^2-4x+1+y^2-2y=9+x^2-6x+16+y^2-8y$

$\Rightarrow 2x+6y=20$

$\Rightarrow x+3y=10$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x=\frac{13}{4}$ and $y=\frac{9}{4}$

$\therefore$ Circumcentre is $\left(\frac{13}{4},\frac{9}{4}\right)$

5 **(c)**

$(a+b+c)(b+c-a)=kbc$

$\Rightarrow 2s(2s-2a)=kbc$

$\Rightarrow \frac{s(s-a)}{bc}=\frac{k}{4}$

$\Rightarrow \cos^2\left(\frac{A}{2}\right)=\frac{k}{4}$

$\because 0<\cos^2\left(\frac{A}{2}\right)<1$

$\therefore 0<\frac{k}{4}<1$

$\Rightarrow 0<k<4$

6 **(d)**

$\because$ Area of $\Delta PBC=\frac{1}{2}\left\|\begin{matrix}\alpha & \beta & 1\\ -3 & 5 & 1\\ 4 & -2 & 1\end{matrix}\right\|=\frac{1}{2}|7\alpha+7\beta-14|$

Also, Area of $\Delta ABC = \frac{1}{2}\left\|\begin{matrix} 6 & -3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{matrix}\right\|$

$= \frac{1}{2}|42 - 21 - 14| = \frac{7}{2}$

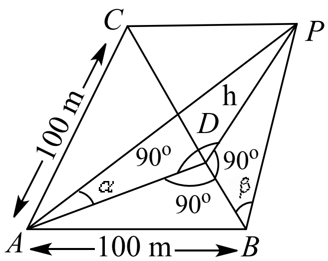
$\therefore \quad \frac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC} = \frac{\frac{7}{2}|\alpha + \beta - 2|}{\frac{7}{2}}$

$= |\alpha + \beta - 2|$

7 **(b)**

Let DP is clock tower standing at the middle point D of BC

Let $\angle PAD = \alpha = \cot^{-1} 3.2 \Rightarrow \cot\alpha = 3.2$



and $\angle PBD = \beta = \operatorname{cosec}^{-1} 2.6$

$\Rightarrow \operatorname{cosec}\beta = 2.6$

$\therefore \cot\beta = \sqrt{(\operatorname{cosec}^2\beta - 1)}$

$= \sqrt{5.76} = 2.4$

In ΔPAD and PBD,

$AD = h\cot\alpha = 3.2\,h$

and $BD = h\cot\beta = 2.4\,h$

In ΔABD, $AB^2 = AD^2 + BD^2$

$\Rightarrow 100^2 = [(3.2)^2 + (2.4)^2]h^2 = 16h^2$

$\Rightarrow h = \frac{100}{4} \Rightarrow h = 25$ m

8 **(c)**

$a\cot A + b\cot B + c\cot C$

$= \frac{a}{\sin A}\cos A + \frac{b}{\sin B}\cos B + \frac{c}{\sin C}\cos C$

$= 2R(\cos A + \cos B + \cos C)$

$= 2R\left(1 + \frac{r}{R}\right) = 2(r + R)$

10 **(a)**

Given pair of lines are rotated about the origin by $\pi/6$ in the anti-clockwise sense.

$\therefore \quad x = x'\cos\frac{\pi}{6} - y'\sin\frac{\pi}{6} = \frac{\sqrt{3}x' - y'}{2}$

and $y = x'\sin\frac{\pi}{6} + y'\cos\frac{\pi}{6} = \frac{x' + \sqrt{3}y'}{2}$

on putting the values of x and y in given pair of lines, we get

$$\sqrt{3}\left(\frac{\sqrt{3}x'-y'}{2}\right)^2-4\left(\frac{\sqrt{3}x'-y'}{2}\right)\left(\frac{x'+\sqrt{3}y'}{2}\right)+\sqrt{3}\left(\frac{x'+\sqrt{3}y'}{2}\right)^2=0$$

$\Rightarrow \sqrt{3}\left(3x'^2+y'^2-2\sqrt{3}x'y'\right)-4\left(\sqrt{3}x'^2 3x'y'-x'y'-\sqrt{3}y'^2\right)+\sqrt{3}\left(x'^2+3y'^2+2\sqrt{3}x'y'\right)=0$

$\Rightarrow 3\sqrt{3}x'^2+\sqrt{3}y'^2-6x'y'-4\sqrt{3}x'^2-8x'y'+4\sqrt{3}y'^2+\sqrt{3}x'^2+3\sqrt{3}y'^2+6x'y'=0$

$\Rightarrow 8\sqrt{3}y'^2-8x'y'=0$

$\Rightarrow \sqrt{3}\,y'^2-x'y'=0$

$\therefore$ Required equation is $\sqrt{3}y^2-xy=0$

11 **(c)**

Using sine rule,

$\frac{\sin A}{a}=\frac{\sin B}{b}$

$\Rightarrow \frac{\frac{2}{3}}{2}=\frac{\sin B}{3}$

$\Rightarrow \sin B=1$

$\Rightarrow B=90°$

12 **(a)**

since, b, c and a are in AP

By sine rule, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$\Rightarrow a=\frac{b}{\sin B}=\frac{c}{\sin C}$ $[\because \angle A=90°]$

$\Rightarrow \sin B=\frac{b}{a}, \sin C=\frac{c}{a}$

13 **(d)**

$2a^2+4b^2+c^2=4ab+2ac$

$\Rightarrow a^2+(2b)^2-4ab+a^2+c^2-2ac=0$

$\Rightarrow (a-2b)^2+(a-c)^2=0$

$\Rightarrow a=2c=c$

$\cos B=\frac{a^2+c^2-b^2}{2ac}$

$=\frac{c^2+c^2-\left(\frac{c}{2}\right)^2}{2\times c\times c}=\frac{2c^2-\frac{c^2}{4}}{2c^2}$

$\Rightarrow \cos B=\frac{7}{8}$

14 **(b)**

Given, M divides AB in the ratio $b:a$(externally)

$\therefore x=\frac{ba\cos\beta-ba\cos\alpha}{b-a}$

and $y=\frac{ab\sin\beta-ab\sin\alpha}{b-a}$

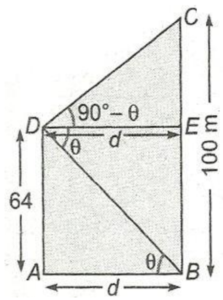
$\Rightarrow \frac{x}{y}=\frac{\cos\beta-\cos\alpha}{\sin\beta-\sin\alpha}$

$$\Rightarrow \frac{x}{y} = \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)}$$

$$\Rightarrow x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\left(\frac{\alpha+\beta}{2}\right) = 0$$

15 **(a)**

In ΔDAB, $\tan\theta = \frac{64}{d}$



$\Rightarrow d = 64\cot\theta$ ...(i)

In ΔCDE, $\tan(90° - \theta) = \frac{(100-64)}{d}$

$\Rightarrow d = 36\tan\theta$ ...(ii)

On multiplying Eqs. (i) and (ii), we get

$d^2 = 36 \times 64 \Rightarrow d = 48$

16 **(b)**

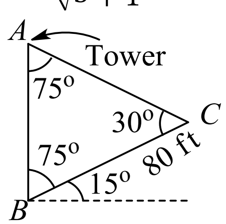
Let BC be the declivity and BA be the tower

$\therefore$ In $\Delta\, ABC$, on applying sine rule

$$\frac{BC}{\sin 75°} = \frac{AB}{\sin 30°}$$

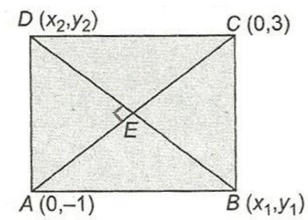
$$\Rightarrow AB = \frac{80\sin 30°}{\sin 75°}$$

$$= \frac{40 \times 2\sqrt{2}}{\sqrt{3}+1} = 40(\sqrt{6}-\sqrt{2})\text{ft}$$



17 **(c)**

Let the points be $B(x_1, y_1)$ and $D(x_2, y_2)$ and coordinates of mid point of BD are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



And coordinates of mid point of AC are (0, 1)

We know that mid point of both the diagonals lie on the same point E.

$\therefore \frac{x_1+x_2}{2}=0$ and $\frac{y_1+y_2}{2}=1$

$\Rightarrow x_1+x_2=0$ ...(i)

and $y_1+y_2=2$ (ii)

also, slope of $BD \times$ slope of $AC=-1$

$$\frac{(y_1-y_2)}{(x_1-x_2)} \times \frac{(3+1)}{(0-0)}=-1$$

$\Rightarrow y_1-y_2=0$ ...(iii)

On solving Eqs. (ii) and (iii), we get

$y_1=1, y_2=1$

Now, slope of $AB \times$ slope of $BC=-1$

$$\Rightarrow \frac{(y_1+1)}{(x_1-0)} \times \frac{(y_1-3)}{(x_1-0)}=-1$$

$\Rightarrow (y_1+1)(y_1-3)=-x_1^2$

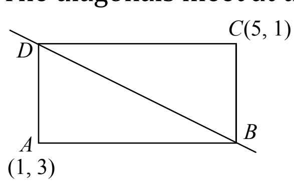
$\Rightarrow 2(-2)=-x_1^2 \quad [\because y_1=1]$

$\Rightarrow x_1= \pm 2$

$\therefore$ The required points are (2, 1) and (−2, 1)

18 **(a)**

The diagonals meet at the mid point of AC, *ie* at (3, 2) which lies on $y=2x+c$



$\therefore c=-4$

Let $B=(\alpha, 2\alpha-4)$

$\because AB \perp BC$

$$\Rightarrow \left(\frac{2\alpha-7}{\alpha-1}\right)\left(\frac{2\alpha-5}{\alpha-5}\right)=-1$$

$\therefore \alpha^2-6\alpha+8=0$

$\Rightarrow \alpha=2,4$

The other two vertices are (2, 0) and (4, 4)

19 **(c)**

Given, $r_3-r=r_1+r_2$

$$\Rightarrow 4R \sin\frac{C}{2}\left(\cos\frac{A}{2}\cos\frac{B}{2}-\sin\frac{B}{2}\sin\frac{A}{2}\right)$$

$$= 4R\cos\frac{C}{2}\left[\sin\frac{A}{2}\cos\frac{B}{2} + \cos\frac{A}{2}\sin\frac{B}{2}\right]$$

$$\Rightarrow \sin\frac{C}{2}\left[\cos\left(\frac{A+B}{2}\right)\right] = \cos\frac{C}{2}\left[\sin\left(\frac{A+B}{2}\right)\right]$$

$$\Rightarrow \sin\frac{C}{2}\left[\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\right] = \cos\frac{C}{2}\left[\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)\right]$$

$$\left[\because\ A+B+C=\pi \Rightarrow \frac{A}{2}+\frac{B}{2}=\frac{\pi}{2}-\frac{C}{2}\right]$$

$$\Rightarrow \sin^2\frac{C}{2} = \cos^2\frac{C}{2} \Rightarrow \tan\frac{C}{2} = 1$$

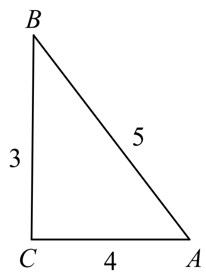
$$\Rightarrow \angle C = \frac{\pi}{2}$$

We know, $A+B+C=\pi \Rightarrow A+B=\frac{\pi}{2}$

20 **(d)**

Given $a = 3, b = 4, c = 5$

$\Rightarrow c^2 = a^2 + b^2$



Therefore, it is a right angled triangle at C

$$\therefore\ R = \frac{1}{2}c = \frac{5}{2}$$

and $r = \frac{\Delta}{s} = \frac{\frac{1}{2}\times 3\times 4}{\frac{12}{2}} = 1$

$\therefore$ Distance between incentre and circumcentre

$$= \sqrt{R^2 - 2Rr}$$

$$= \sqrt{\left(\frac{5}{2}\right)^2 - 2.\frac{5}{2}.1}$$

$$= \sqrt{\frac{5}{2}}\sqrt{\frac{5}{2}-2} = \frac{\sqrt{5}}{2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	D	D	C	D	B	C	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	D	B	A	B	C	A	C	D