DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 5

Topic :- CO-ORDINATE GEOMETRY

1 **(b)**

$\because \cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \cos^2 C = \left(\dfrac{a^2+b^2-c^2}{2ab}\right)^2$

$\Rightarrow \cos^2 C = \dfrac{[a^4+b^4+c^4+2a^2b^2-2c^2(a^2+b^2)]}{4a^2b^2}$

$\Rightarrow \cos^2 C = \dfrac{1}{2}$ [$\because$ $a^4+b^4+c^4 = 2c^2(a^2+b^2)$ given]

$\Rightarrow \cos C = \pm\dfrac{1}{\sqrt{2}}$

$\Rightarrow \angle C = 45°$ or $135°$

2 **(d)**

Locus, of P is $\left|\sqrt{x^2+y^2-8y+16}-\sqrt{x^2+y^2+8y+16}\right| = 6$

On squaring, we get

$x^2+y^2-2 = \sqrt{x^2+y^2+8y+16}\sqrt{x^2+y^2-8y+16}$

$\Rightarrow \quad (x^2+y^2-2)^2 = (x^2+y^2+16)^2-(8y)^2$

On simplification, we get

$\dfrac{y^2}{9}-\dfrac{x^2}{7} = 1$

3 **(a)**

Given, $\sin\frac{A}{2}\sin\frac{C}{2} = \sin\frac{B}{2}$

$\Rightarrow \sqrt{\dfrac{(s-b)(s-c)}{bc}}\sqrt{\dfrac{(s-a)(s-b)}{ab}} = \sqrt{\dfrac{(s-a)(s-c)}{ac}}$

$\Rightarrow \dfrac{s-b}{b} = 1$

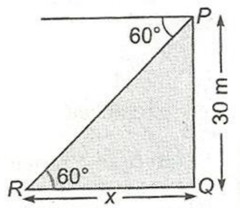
$\Rightarrow s = 2b$

4 **(c)**

In $\Delta\, PRQ$

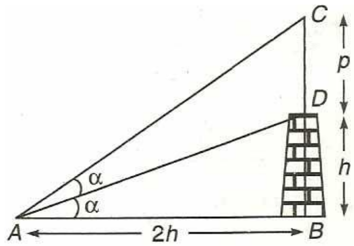
$\tan 60° = \dfrac{30}{x}$

$\Rightarrow x = 10\sqrt{3}$ m



5 **(a)**

In ΔABD, $\tan\alpha = \frac{h}{2h}$



$\Rightarrow \tan\alpha = \frac{1}{2}$ ...(i)

In ΔABC, $\tan 2\alpha = \frac{h+p}{2h}$

$$\Rightarrow \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{h+p}{2h}$$

$$\Rightarrow \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{h+p}{2h}$$

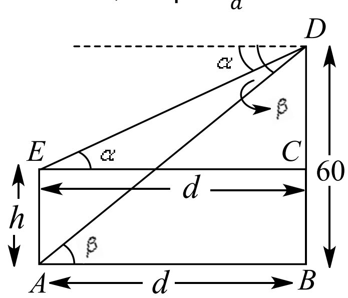
$$\Rightarrow \frac{4}{3} = \frac{h+p}{2h}$$

$\Rightarrow 8h = 3h + 3p$

$\Rightarrow 5h = 3p \Rightarrow p = \frac{5h}{3}$ m

6 **(d)**

In $\Delta\ ABD$, $\tan\beta = \frac{60}{d}$



$\Rightarrow d = 60\cot\beta$ ...(i)

In $\Delta\ DEC$, $\tan\alpha = \frac{DC}{EC}$

$\Rightarrow DC = d\tan\alpha$

$\Rightarrow 60 - h = d\tan\alpha$ $(\because BC = EA = h)$

$\Rightarrow 60 - h = 60\cot\beta\tan\alpha$ [from Eq.(i)]

$\Rightarrow\ h = 60\left(1 - \frac{\cos\beta}{\sin\beta}\cdot\frac{\sin\alpha}{\cos\alpha}\right)$

$\Rightarrow\ h = \frac{60\sin(\beta-\alpha)}{\cos\alpha\sin\beta}$

$\Rightarrow \frac{60\sin(\beta-\alpha)}{x} = \frac{60\sin(\beta-\alpha)}{\cos\alpha\sin\beta}$ (given)

$\Rightarrow\ x = \cos\alpha\sin\beta$

7 **(c)**

We know that, in triangle larger side has an larger angle opposite to it. Since, angles $\angle A$, $\angle B$ and $\angle C$ are in AP

$\Rightarrow\ 2B = A + C$

$\because\ \ A + B + C = \pi$

$\Rightarrow\ B = 60°$

$\therefore\ \cos B\ = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow\ \cos 60°\ = \frac{1}{2} = \frac{100 + a^2 - 81}{20a}$

$\Rightarrow a^2 + 19 = 10a$

$\Rightarrow\ a^2 - 10a + 19 = 0$

$\therefore\ a = \frac{10 \pm \sqrt{100 - 76}}{2} = 5 \pm \sqrt{6}$

8 **(c)**

Here, $\frac{1}{\sin^2\frac{A}{2}}, \frac{1}{\sin^2\frac{B}{2}}, \frac{1}{\sin^2\frac{C}{2}}$ are in AP

$\Rightarrow \frac{1}{\sin^2\frac{C}{2}} - \frac{1}{\sin^2\frac{B}{2}} = \frac{1}{\sin^2\frac{B}{2}} - \frac{1}{\sin^2\frac{A}{2}},$

$\Rightarrow \frac{ab}{(s-a)(s-b)} - \frac{ac}{(s-a)(s-c)} = \frac{ac}{(s-a)(s-c)} - \frac{bc}{(s-b)(s-c)}$

$\Rightarrow \left(\frac{a}{s-a}\right)\left(\frac{b(s-c) - c(s-b)}{(s-b)(s-c)}\right)$

$= \left(\frac{c}{s-c}\right)\left(\frac{a(s-b) - b(s-a)}{(s-a)(s-b)}\right)$

$\Rightarrow\ ab + bc = 2ac \Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$

Hence, a, b, c are in HP

10 **(c)**

Let the vertices of triangle be $P(1, 1)$, $Q(-1, -1)$ and $R(-\sqrt{3}, \sqrt{3})$

$\therefore PQ = \sqrt{(1+1)^2 + (1+1)^2} = 2\sqrt{2}$

$QR = \sqrt{(-\sqrt{3}+1)^2 + (\sqrt{3}+1)^2}$

$= \sqrt{3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3}} = 2\sqrt{2}$

and $\ RP = \sqrt{(-\sqrt{3}-1)^2 + (\sqrt{3}-1)^2}$

$= \sqrt{3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3}}$

$= 2\sqrt{2}$

$\Rightarrow \quad PQ = QR = RP$

$\therefore$ Triangle is an equilateral triangle

11 **(a)**

Let the third vertex be (a, b)

$\therefore$ Area of $\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & b & 1 \\ 6 & 8 & 1 \end{vmatrix} = \frac{1}{2}|[8a - 6b]|$

As (a, b) are integers, so we take

$(0, 0), (1, 1), (1, 2)$

At $(0, 0)$, $\Delta = 0$, it is not possible

At $(1, 1) \Delta = 1$

At $(1, 2)$, $\Delta = 2$

Here, we see that minimum area is 1

12 **(d)**

$$\left(\cot\frac{A}{2} + \cot\frac{B}{2}\right)\left(a\sin^2\frac{B}{2} + b\sin^2\frac{A}{2}\right)$$

$$= \left(\frac{\cos\frac{A}{2} + \sin\frac{B}{2} + \cos\frac{B}{2} + \sin\frac{A}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}}\right)\left(a\sin^2\frac{B}{2} + b\sin^2\frac{A}{2}\right)$$

$$= \frac{\sin\left(\frac{A+B}{2}\right)\left(a\sin^2\frac{B}{2} + b\sin^2\frac{A}{2}\right)}{\sin\frac{A}{2}\sin\frac{B}{2}}$$

$$= \left(\cos\frac{C}{2}\right)\left(a\frac{\sin\frac{B}{2}}{\sin\frac{A}{2}} + b\frac{\sin\frac{A}{2}}{\sin\frac{B}{2}}\right)$$

$$= \sqrt{\frac{s(s-c)}{ab}}\left(a\frac{\sqrt{\frac{(s-a)(s-c)}{ac}}}{\sqrt{\frac{(s-b)(s-c)}{bc}}} + b\frac{\sqrt{\frac{(s-b)(s-c)}{bc}}}{\sqrt{\frac{(s-a)(s-c)}{ac}}}\right)$$

$$= \sqrt{\frac{s(s-c)}{ab}}\left(\sqrt{\left(\frac{s-a}{s-b}\right)ab} + \sqrt{\left(\frac{s-b}{s-a}\right)ab}\right)$$

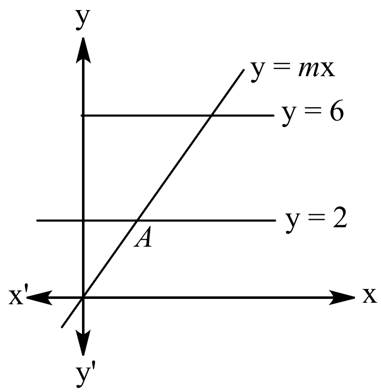
$$= \sqrt{s(s-c)}\left(\frac{s-a+s-b}{\sqrt{(s-a)(s-b)}}\right)$$

$$= \sqrt{s(s-c)}\left(\frac{2s-a-b}{\sqrt{(s-a)(s-b)}}\right)$$

$$= c\sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = c\cot\frac{C}{2}$$

13 **(c)**

Given lines $y = mx, y = 2, y = 6$



Coordinates of points A and B are $\left(\frac{2}{m}, 2\right), \left(\frac{6}{m}, 6\right)$ respectively

$\therefore \quad AB = \sqrt{\left(\frac{2}{m} - \frac{6}{m}\right)^2 + (2-6)^2} < 5 \quad$ [given]

$\Rightarrow \left(\frac{2}{m} - \frac{6}{m}\right)^2 + (4)^2 < 25$

$\Rightarrow \left(\frac{2}{m} - \frac{6}{m}\right)^2 < 9 \Rightarrow -3 < \frac{2}{m} - \frac{6}{m} < 3$

$\Rightarrow -\frac{4}{3} > m > \frac{4}{3}$

$\therefore \quad m \in \left]-\infty, -\frac{4}{3}\right[\cup \left]\frac{4}{3}, \infty\right[$

14 **(c)**

By sine rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad$ (say)

$\therefore (b-c)\sin A + (c-a)\sin B + (a-b)\sin C$

$= (b-c)ak + (c-a)bk + (a-b)kc$

$= k[ab - ac + bc - ab + ac - bc]$

$= 0$

15 **(a)**

Given that, $a = 3, b = 5, c = 6$

Now, $s = \frac{a+b+c}{2} = 7$

$\therefore \ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{7(7-3)(7-5)(7-6)}$

$= \sqrt{7 \cdot 4 \cdot 2 \cdot 1} = 2\sqrt{14}$

$\therefore \ r = \frac{\Delta}{s} = \frac{2\sqrt{14}}{7} = \sqrt{\frac{8}{7}}$

16 **(a)**

$\cos B = \frac{3^2 + 4^2 - 5^2}{2(3)(4)} = \frac{9 + 16 - 25}{2(3)(4)} = 0$

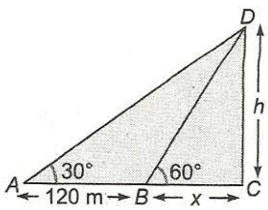
$\Rightarrow \angle B = 90°$

$\therefore\ \sin\frac{B}{2}+\cos\frac{B}{2}=\sin 45^\circ+\cos 45^\circ$

$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$

17 **(b)**

In ΔCAD, $\tan 30^\circ=\frac{CD}{AC}$



$\Rightarrow\frac{1}{\sqrt{3}}=\frac{h}{120+x}$

$\Rightarrow\sqrt{3}h=120+x$ ...(i)

and in ΔCBD, $\tan 60^\circ=\frac{CD}{BC}$

$\Rightarrow\sqrt{3}=\frac{h}{x}\Rightarrow h=\sqrt{3}x$ ...(ii)

From Eqs. (i) and (ii), we get, $x=60$ m

On putting $x=60$ in Eq.(i), we get

$h=60\sqrt{3}$ m

18 **(c)**

Area of triangle $=\frac{1}{2}[x(1-2)+1(2-0)+0(0-1)]$

$=\frac{1}{2}[-x+2+0]=4$ [given]

$\Rightarrow\ 2-x=8\ \Rightarrow x=-6$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	A	C	A	D	C	C	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	C	C	A	A	B	C	C	C