DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 4

Topic :- CO-ORDINATE GEOMETRY

1 **(b)**

$2ac\sin\left(\frac{A-B+C}{2}\right)$

$=2ac\sin\left(\frac{180°-2B}{2}\right)$

$=2ac\sin(90°-B)=2ac\cos B=a^2+c^2-b^26$

2 **(d)**

$(x+1)^2+y^2+(x-2)^2+y^2=2[(x-1)^2+y^2]$

On simplification, we get $2x+3=0$

3 **(a)**

We know that centroid divides the line segment joining orthocenter and circumcentre in the ratio 2:1. Since, the coordinates of orthocenter and circumcentre are (1, 1) and (3, 2) respectively

$\therefore$ The coordinates of centroid are

$\left(\frac{2.3+1.1}{2+1},\frac{2.2+1.1}{2+1}\right)=\left(\frac{7}{3},\frac{5}{3}\right)$

4 **(c)**

Given equation are

$x\cot\theta+y\operatorname{cosec}\theta=2$ ...(i)

And $x\,cosec\,\theta+y\cot\theta=6$ ...(ii)

On squaring and subtracting Eq. (i) from Eq. (ii), we get

$x^2(\operatorname{cosec}^2\theta-\cot^2\theta)+y^2(\cot^2\theta-\operatorname{cosec}^2\theta)=(6)^2-(2)^2$

$\Rightarrow x^2-y^2=32$

It represents an equation of hyperbola

5 **(c)**

Since, $\cot A+\cot C=2\cot B$

$\Rightarrow \frac{\cos A}{\sin A}+\frac{\cos C}{\sin C}=\frac{2\cos B}{\sin B}$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc(ka)}+\frac{a^2+b^2-c^2}{2ab(kc)}=2\frac{a^2+c^2-b^2}{2ac(kb)}$

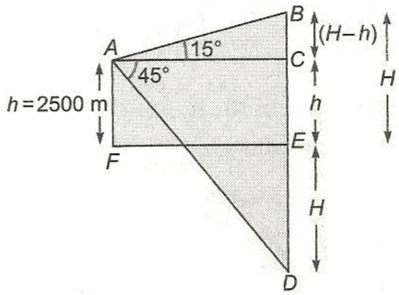
$\Rightarrow a^2+c^2=2b^2$

Hence, a^2, b^2, c^2 are in AP

6 **(a)**

In ΔABC, $AC = (H - h)\cot 15°$ ...(i)

and in ΔACD, $AC = (H + h)\cot 45°$...(ii)



From Eqs. (i) and (ii).

$(H - h)\cot 15° = (H + h)\cot 45°$

$$\Rightarrow H = \frac{h(\cot 15° + 1)}{\cot 15° - 1}$$

$$\therefore\ H = \frac{2500(2 + \sqrt{3} + 1)}{(2 + \sqrt{3} - 1)}$$

$$= \frac{2500(3 + \sqrt{3})}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}$$

$$= 2500\sqrt{3}\text{ m}$$

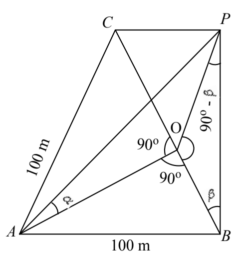
7 **(b)**

Let OP be the clock tower standing at the mid point O of side BC of ΔABC. Let $\alpha = \angle PAO = \cot^{-1} 3.2$ and $\beta = \angle PBO = \operatorname{cosec}^{-1} 2.6$

Then, $\cot\alpha = 3.2$ and $\operatorname{cosec}\beta = 2.6$

$\therefore \cot\beta = \sqrt{\operatorname{cosec}^2\beta - 1} = \sqrt{(2.6)^2 - 1} = 2.4$

In ΔPAO and ΔPBO,we have



$AO = h\cot\alpha = 3.2h$

and $BO = h\cot B = 2.4\ h$

In ΔABO, $AB^2 = OA^2 + OB^2$

$\Rightarrow 100^2 = (3.2h)^2 + (2.4\ h)^2$

$\Rightarrow 100^2 = 16h^2$

$\Rightarrow h^2 = 625 \Rightarrow h = 25$ m

8 **(d)**

$$\because \cos 30° = \frac{3 + 1 - a^2}{2\sqrt{3}}\ \left[\because \cos A = \frac{b^2 + c^2 - a^2}{2bc}\right]$$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{4-a^2}{2\sqrt{3}} \Rightarrow a^2 = 1$

$\Rightarrow \; a = 1$

Here, we see side b is largest, so $\angle B$ must be greatest

$\therefore$ By sine rule, $\frac{b}{\sin B} = \frac{a}{\sin A}$

$\Rightarrow \dfrac{\sqrt{3}}{\sin B} = \dfrac{1}{\sin 30°}$

$\Rightarrow \sin B \; = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \angle B = 120°$

9 **(c)**

Let each side of equilateral triangle $= a$

$\therefore \; \Delta = \dfrac{\sqrt{3}}{4}a^2, \qquad S = \dfrac{3a}{2}$

Now, $r = \frac{\Delta}{5} = \frac{\sqrt{3}}{4}a^2 \cdot \frac{2}{3a} = \frac{a}{2\sqrt{3}}$

$R = \dfrac{abc}{4\Delta} = \dfrac{a^3}{\sqrt{3}a^2} = \dfrac{a}{\sqrt{3}}$

$r_1 = \dfrac{\Delta}{s-a} = \dfrac{\sqrt{3}}{4}a^2 \cdot \dfrac{2}{a} = \dfrac{\sqrt{3}}{2}a$

$\therefore \; R{:}r_1{:}r_1 = \dfrac{a}{\sqrt{3}} : \dfrac{a}{2\sqrt{3}} : \dfrac{\sqrt{3}}{2}a$

$= 2{:}1{:}3$

10 **(d)**

Given, $r_1 = 2r_2 = 3r_3$

$\therefore \; \dfrac{\Delta}{s-a} = \dfrac{2\Delta}{s-b} = \dfrac{3\Delta}{s-c} = \dfrac{\Delta}{k}$ [say]

Then, $s-a = k, s-b = 2k, s-c = 3k$

$\Rightarrow 3s - (a+b+c) = 6k \; \Rightarrow \; s = 6k$

$\therefore \; \dfrac{a}{5} = \dfrac{b}{4} = \dfrac{c}{3} = k$

$\therefore \; \dfrac{a}{b} + \dfrac{b}{c} + \dfrac{c}{a} = \dfrac{5}{4} + \dfrac{4}{3} + \dfrac{3}{5} = \dfrac{191}{60}$

11 **(c)**

Let sides of the triangle are $4x, 5x, 6x$

$s = \dfrac{4x+5x+6x}{2} = \dfrac{15}{2}x$

$\Delta = \sqrt{\dfrac{15}{2} \times \left(\dfrac{15}{2}x - 4x\right)\left(\dfrac{15}{2}x - 5x\right)\left(\dfrac{15}{2}x - 6x\right)}$

$= \sqrt{\dfrac{15}{2}x \times \dfrac{7}{2}x \times \dfrac{5}{2}x \times \dfrac{3}{2}x}$

$= \dfrac{15\sqrt{7}x^2}{4}$

Circumradius, $R=\frac{4x\times 5x\times 6x}{4\times\frac{15\sqrt{7}x^2}{4}}$

$=\frac{8}{\sqrt{7}}x$

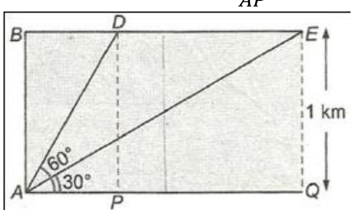
Inradius, $r=\frac{\frac{15\sqrt{7}}{4}x^2}{\frac{15}{2}x}$

$=\frac{\sqrt{7}}{2}x$

$\frac{R}{r}=\frac{\frac{8x}{\sqrt{7}}}{\frac{\sqrt{7}x}{2}}=\frac{16}{7}$

12 **(c)**

In ΔDAP, $\tan 60^\circ=\frac{1}{AP}$ $\quad[\because EQ=DP=1]$



$\Rightarrow\ AP=\frac{1}{\sqrt{3}}$

In ΔEAQ, $\tan 30^\circ=\frac{EQ}{AP+PQ}$

$\Rightarrow\frac{1}{\sqrt{3}}+PQ=\sqrt{3}$

$\Rightarrow PQ=\frac{2}{\sqrt{3}}$ km

$\therefore$ Speed of plane $=\frac{\text{Distance}}{\text{Time}}$

$$=\frac{\frac{2}{\sqrt{3}}}{\frac{10}{60\times 60}}=240\sqrt{3}\text{ km/h}$$

14 **(a)**

Area of triangle $=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$

= A rational number if vertices have integral coordinates only

If triangle is equilateral, then area

$=\frac{\sqrt{3}}{4}[(x_1-x_2)^2+(y_1-y_2)^2]$

=irrational quantity

So, triangle cannot be equilateral

15 **(d)**

$$\frac{b-c}{a}=\frac{k(\sin B+\sin C)}{k\sin A}$$

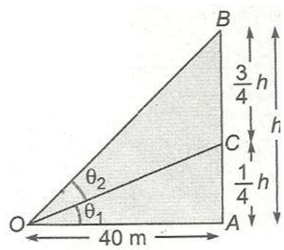
$$=\frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$\Rightarrow \frac{b+c}{a}=\frac{\cos\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}}$$

Similarly, $\frac{b-c}{a}=\frac{\sin\left(\frac{B-C}{2}\right)}{\cos\frac{A}{2}}$

16 **(b)**

Given, $\theta=\tan^{-1}\frac{3}{5} \Rightarrow \tan\theta_2=\frac{3}{5}$ ...(i)



In ΔAOC,$\tan\theta_1=\frac{AC}{AO}=\frac{h}{160}$ ...(ii)

and in ΔAOB,$\tan(\theta_1+\theta_2)=\frac{h}{40}$

$$\Rightarrow \frac{\tan\theta_1+\tan\theta_2}{1-\tan\theta_1\tan\theta_2}=\frac{h}{40}$$

$\Rightarrow \frac{\frac{h}{160}+\frac{3}{5}}{1-\frac{h}{160}\times\frac{3}{5}}=\frac{h}{40}$ [from Eqs. (i) and (ii)]

$$\Rightarrow \frac{5[h+96]}{800-3h}=\frac{h}{40}$$

$\Rightarrow h^2-200h+6400=0$

$\Rightarrow (h-160)(h-40)=0$

$\Rightarrow h=160$ or $h=40$

Hence, height of the vertical pole is 40 m

17 **(c)**

Let $A(x_1,y_1)$ and $B(x_2,y_2)$ be two points. Let C and D be the points of internal and external division of AB in the ratio $\lambda:1$. Then, the coordinates of C and D are

$\left(\frac{\lambda x_2+x_1}{\lambda+1},\frac{\lambda y_2+y_1}{\lambda+1}\right)$ and $\left(\frac{\lambda x_2-x_1}{\lambda-1},\frac{\lambda y_2-y_1}{\lambda-1}\right)$ respectively

$$\therefore AC=\frac{\lambda}{\lambda+1}AB \text{ and } AD=\frac{\lambda}{\lambda-1}AB$$

Clearly, $\frac{1}{AC}+\frac{1}{AD}=\frac{2}{AB}\Rightarrow AC,AB,AD$ are in H.P.

19 **(d)**

We have,

$A + B + C = 180°$

$\Rightarrow 3B = 180$ [$\because A, B, C$ are in AP]

$\Rightarrow B = 60°$

$\Rightarrow \cos B = \frac{1}{2}$

$\Rightarrow \frac{AB^2 + BC^2 - AC^2}{2AB.BC} = \frac{1}{2}$

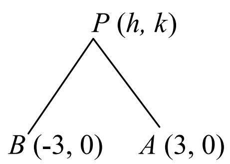
$\Rightarrow 36 + 49 - AC^2 = 6 \times 7 \Rightarrow AC^2 = 43 \Rightarrow AC = \sqrt{43}$

20 **(a)**

Let the point be $P(h,k)$

It is given that difference of the distance from points $A(3, 0)$ and $B(-3, 0)$ is 4 *ie*, $PA - PB = 4$

$\Rightarrow \sqrt{(h-3)^2 + k^2} - \sqrt{(h+3)^2 + k^2} = 4$



$\Rightarrow \sqrt{(h-3)^2 + k^2} = 4 + \sqrt{(h+3)^2 + k^2}$

On squaring both sides, we get

$(h-3)^2 + k^2 = 16 + (h+3)^2 + k^2 + 8\sqrt{(h+3)^2 + k^2}$

$\Rightarrow h^2 + 9 - 6h + k^2 = 16 + h^2 + 9 + 6h + k^2 + 8\sqrt{(h+3)^2 + k^2}$

$\Rightarrow -6h = 16 + 6h + 8\sqrt{(h+3)^2 + k^2}$

$\Rightarrow -8\sqrt{(h+3)^2 + k^2} = 12h + 16$

Again , squaring both sides, we get

$64(h+3)^2 + k^2 = (12h + 16)^2$

$\Rightarrow 64(h^2 + 9 + 6h + k^2) = 144h^2 + 256 + 2.16.12h$

$\Rightarrow 64(h^2 + 9 + 6h + k^2) = 16(9h^2 + 16 + 24h)$

$\Rightarrow 4(h^2 + 9 + 6h + k^2) = 9h^2 + 16 + 24h$

$\Rightarrow 4h^2 + 36 + 24h + 4k^2 = 9h^2 + 16 + 24h$

$\Rightarrow 5h^2 - 4k^2 = 20$

$\Rightarrow \frac{h^2}{4} - \frac{k^2}{5} = 1$

Hence, the locus of points P is $\frac{x^2}{4} - \frac{y^2}{5} = 1$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	A	C	C	A	B	D	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	A	A	D	B	C	B	D	A

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