DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth **SUBJECT : MATHS**

DATE : **DPP NO. : 9**

Topic :- CO-ORDINATE GEOMETRY

1 **(b)**

Let P is a point on the perpendicular bisector of AB, its equation is

$(y-1)=\frac{1}{3}(x-4) \Rightarrow x-3y-1=0$

So, general point is $P(3h+1, h)$

Area $=\frac{1}{2}\begin{bmatrix} 3h+1 & h & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{bmatrix} = \pm 10$

$\Rightarrow h=2, 0$

Or position of points is $(7, 2)$ and $(1, 0)$

2 **(a)**

Given, $\angle A=\frac{\pi}{2}$, then $a^2=b^2+c^2=4^2+3^2=25$

and $\frac{a}{\sin A}=2R \Rightarrow a=2R$ and $a=5$

also, $r=\frac{\Delta}{s}=\frac{bc}{a+b+c}$ $\left(\because \Delta=\frac{bc}{2}\right)$

$\therefore \frac{R}{r}=\frac{a(a+b+c)}{2bc}=\frac{5\times 12}{2\times 4\times 3}=\frac{5}{2}$

3 **(d)**

Since A, B, C are in AP

$\Rightarrow 2B=A+C \Rightarrow \angle B=60°$

$\therefore \frac{a}{2}(2\sin C\cos C)+\frac{c}{a}(2\sin A\cos A)$

$=2k(a\cos C+c\cos A)$

$=2k(b)$

$=2\sin B$ [using, $b=a\cos C+c\cos A$]

$=\sqrt{3}$

4 **(d)**

Given equation is

$x^2-5x+6=0$

$\Rightarrow (x-3)(x-2)=0$

$\Rightarrow x=3, 2$

These are the sides of a triangle

Let $a=3, b=2, \angle C=\frac{\pi}{3}$

$\Rightarrow \cos\left(\frac{\pi}{3}\right)=\frac{3^2+2^2-c^2}{2.3.2}$ $\left[\because \cos C=\frac{a^2+b^2-c^2}{2ab}\right]$

$\Rightarrow \frac{1}{2}=\frac{13-c^2}{12} \Rightarrow c^2=7$

$\Rightarrow c=\sqrt{7}$ [sides cannot be negative]

$\therefore$ Perimeter of a triangle $=a+b+c$

$=3+2+\sqrt{7}=5+\sqrt{7}$

5 **(b)**

Given, $\angle A=60°, a=5, b=4$

$\therefore \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

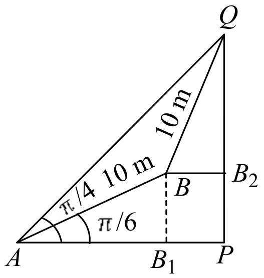
$\Rightarrow \cos 60° = \dfrac{1}{2} = \dfrac{16 + c^2 - 25}{8c}$

$\Rightarrow 4c = c^2 - 9$

$\Rightarrow c^2 - 4c - 9 = 0$

6 **(a)**

Let PQ be the height h of the tower and A, B are the points of observations



We have, $\angle QAP = \frac{\pi}{4}, \angle BAP = \frac{\pi}{6},$

$AB = 10$ m, $BQ = 10$ m

$\therefore \angle QAB = \dfrac{\pi}{12} = \angle AQB$

$\Rightarrow \angle ABQ = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

On applying cosine rule in ΔABQ, we get

$AQ^2 = AB^2 + BQ^2 - 2AB \cdot BQ \cos \dfrac{5\pi}{6}$

$= 100 + 100 + 200 \cdot \dfrac{\sqrt{3}}{2}$

$= 100(2 + \sqrt{2})$

$\Rightarrow \quad AQ = 10\sqrt{2 + \sqrt{3}}$

In ΔAPQ, $AP = AQ \cos \frac{\pi}{4} = \frac{10\sqrt{2 + \sqrt{3}}}{\sqrt{2}}$

$= 5\sqrt{4 + 2\sqrt{3}} = 5\sqrt{(\sqrt{3} + 1)^2}$

$\Rightarrow AP = 5(1 + \sqrt{3})$m

7 **(a)**

Let the points be $A = (a\cos\theta, a\sin\theta)$ and

$B = (a\cos\phi, a\sin\phi)$

$\therefore AB = \sqrt{(a\cos\theta - a\cos\phi)^2 + (a\sin\theta - a\sin\phi)^2}$

$= \sqrt{a^2\cos^2\theta + a^2\cos^2\phi - 2a^2\cos\theta\cos\phi + a^2\sin^2\theta + a^2\sin^2\phi - 2a^2\sin\theta\sin\phi}$

$= \sqrt{2a^2 - 2a^2(\cos\theta\cos\phi + \sin\theta\sin\phi)}$

$= \sqrt{2}a(\sqrt{1 - \cos(\theta - \phi)})$

$\Rightarrow 2a = \sqrt{2}a\sqrt{2}\sin\left(\dfrac{\theta - \phi}{2}\right)$

$\Rightarrow \quad \sin\left(\dfrac{\theta - \phi}{2}\right) = 1$

$\Rightarrow \quad \dfrac{\theta - \phi}{2} = n\pi \pm \dfrac{\pi}{2}$

$\Rightarrow \quad \theta - \phi = 2n\pi \pm \pi$

$\Rightarrow \theta = 2n\pi \pm \pi - \phi$

Where $n \in Z$

8 **(d)**

Area of pentagon

$$= \frac{1}{2}\begin{bmatrix} x_1y_2 + x_2y_3 + x_3y_4 + x_4y_5 \\ + x_5y_1 - (y_1x_2 + y_2x_3 + y_3x_4 \\ + y_4x_5 + y_5x_1) \end{bmatrix}$$

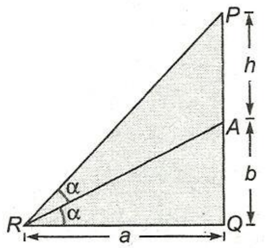
$$= \frac{1}{2}[0(0) + 12(2) + 12(7) + 6(5) + 0(0) - \{0 + 0 + 2(6) + 7(0) + 5(0)\}]$$

$$= \frac{1}{2}[(24 + 84 + 30 - 12)]$$

$= 63$ sq unit

9 **(a)**

Let the height of the flag be h



In ΔARQ,$\tan\alpha = \frac{b}{a}$ ....(i)

and in ΔPRQ, $\tan 2\alpha = \frac{h+b}{a}$ ...(ii)

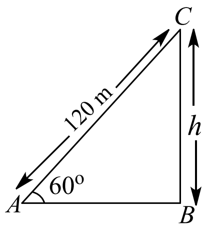
$$\Rightarrow \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{h+b}{a}$$

$$\Rightarrow \frac{2 \times \frac{b}{a}}{1 - \frac{b^2}{a^2}} = \frac{h+b}{2} \quad \text{[from Eq.(i)]}$$

$$\Rightarrow \frac{2ab}{a^2 - b^2} = \frac{h+b}{a} \Rightarrow h = \frac{b(a^2+b^2)}{(a^2-b^2)}$$

10 **(a)**

Let BC be the height of kite



In Δ ABC, $\sin 60° = \frac{h}{120}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{120}$$

$\Rightarrow$ $h = 60\sqrt{3}$ m

The height of the kite is $60\sqrt{3}$ m

11 **(b)**

$$\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$$

$$= \frac{(2R\sin A)\cos A + (2R\sin B)\cos B + 2R\sin C\,(\cos C)}{2R\sin A + 2R\sin B + 2R\sin C}$$

$$\left(\because R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = \frac{abc}{4\Delta}\right)$$

$$= \frac{R[2\sin A\cos A + 2\sin B\cos B + 2\sin C\cos C}{2R[\sin A + \sin B + \sin C]}$$

$$= \frac{1}{2}\cdot\frac{(\sin 2A + \sin 2B + \sin 2C)}{(\sin A + \sin B + \sin C)}$$

$$= \frac{4\sin A\sin B\sin C}{2[4\cos(A/2)\cos(B/2)\cos(C/2)]}$$

$$\left(\begin{array}{l}\because\ \sin 2\alpha + \sin 2\beta + \sin 2\gamma = 4\sin\alpha\sin\beta\sin\gamma \\ \text{and}\ \ \sin\alpha + \sin\beta + \sin\gamma = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}\end{array}\right)$$

$$= \frac{4\left[2\sin\frac{A}{2}\cos\frac{A}{2}\times 2\sin\frac{B}{2}\cos\frac{B}{2}\times 2\sin\frac{C}{2}\times\cos\frac{C}{2}\right]}{2\times 4\left[\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\right]}$$

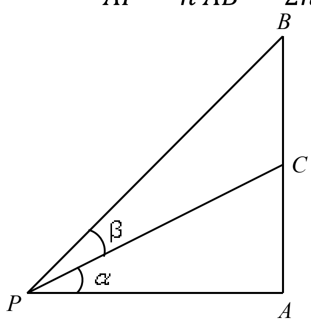
$$= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$= \frac{r}{R}\ \left[\because r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right]$$

12 **(a)**

Let $\angle APC = \alpha$. Then,

$$\tan\alpha = \frac{AC}{AP} = \frac{AC}{n\,AB} = \frac{AB}{2n\,AB} = \frac{1}{2n}$$



In $\Delta\,APB$, we have

$$\tan(\alpha+\beta) = \frac{AB}{AP} = \frac{AB}{n\,AB} = \frac{1}{n}$$

Now, $\beta = \alpha + \beta - \alpha$

$$\Rightarrow \tan\beta = \frac{\tan(\alpha+\beta) - \tan\alpha}{1 + \tan(\alpha+\beta)\tan\alpha}$$

$$\Rightarrow \tan\beta = \frac{1/n - 1/2n}{1 + 1/n\cdot 1/2n} = \frac{n}{2\,n^2+1}$$

13 **(d)**

Suppose $P(3,7)$ divides the segment joining $A(1, 1)$ and $B(6, 16)$ in the ratio $\lambda:1$. Then,

$$\frac{6\lambda+1}{\lambda+1}=3 \text{ and } \frac{16\lambda+1}{\lambda+1}=7$$

$$\Rightarrow \lambda=\frac{2}{3}$$

$\Rightarrow P$ divides AB internally in the ratio 2 :3

Thus, Q divides AB externally in the ratio 2 :3 and hence its coordinates are

$$\left(\frac{2\times6-3\times1}{2-3}, \frac{2\times16-3\times1}{2-3}\right)\equiv(-9,-29)$$

14 **(d)**

Let the angles of a triangle are $3x$, $5x$ and $10x$

$\therefore 3x+5x+10x=180° \Rightarrow x=10°$

$\therefore$ Smallest angle of a triangle $=30°$

And the greatest angle $=100°$

Required ratio $=\sin 30°:\sin 100°$

$$=\frac{1}{2}:\cos 10° = 1:2\cos 10°$$

15 **(b)**

Given, $\begin{vmatrix}1 & a & b\\ 1 & c & a\\ 1 & b & c\end{vmatrix}=0$

$\Rightarrow c^2-ab-a(c-a)+b(b-c)=0$

$\Rightarrow a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \frac{1}{2}[2a^2+2b^2+2c^2-2ab-2bc-2ca]=0$

$\Rightarrow \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2=0$

$\Rightarrow a=b=c$

$\Rightarrow \angle A=60°, \angle B=60°, \angle C=60°$

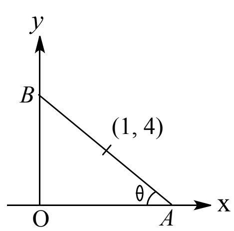
$\therefore \sin^2 A+\sin^2 B+\sin^2 C$

$=\sin^2 60°+\sin^2 60°+\sin^2 60°$

$$=\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2=\frac{9}{4}$$

17 **(c)**

Let $\angle OAB=\theta$



Then, $OA+AB=1+4\cot\theta+4+\tan\theta$

$=5+4\cot\theta+\tan\theta\geq 5+4=9$

(using AM ≥ GM)

19 **(c)**

Given $\angle A = 20°$

$\therefore\ \angle B = \angle C = 80°$

Then, $b = c$

$\therefore\ \dfrac{a}{\sin 20°} = \dfrac{b}{\sin 80°} = \dfrac{c}{\sin 80°}$

$\Rightarrow \dfrac{a}{\sin 20°} = \dfrac{b}{\cos 10°}$

$\Rightarrow a = 2b\sin 10° \quad ...(i)$

$\therefore\ a^3 + b^3 = 8b^3\sin^3 10° + b^3$

$= b^3\{2(4\sin^3 10°) + 1\}$

$= b^3\{2(3\sin 10° - \sin 30°) + 1\}$

$= b^3\{6\sin 10°\}$

$= 3b^2\{2b\sin 10°\}$

$= 3b^2a \quad$ [from Eq. (i)]

$= 3ac^2 \quad (\because b = c)$

20 **(a)**

$a(\cos^2 B + \cos^2 C) + \cos A\,(c\cos C + b\cos B)$

$= \cos B\,(a\cos B + b\cos A) + \cos C\,(a\cos C + c\cos A)$

$= (\cos B)c + (\cos C)b$

$= a$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	D	D	B	A	A	D	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	D	D	B	D	C	B	C	A