DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :1

Topic :- CO-ORDINATE GEOMETRY

1 **(a)**

$a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2 (\cos^2 A - \cos^2 B)$

$= a^2(1 - \sin^2 B - 1 + \sin^2 C) + b^2(1 - \sin^2 C - 1 + \sin^2 A)$

$+ c^2 (1 - \sin^2 A - 1 + \sin^2 B)$

$= a^2(\sin^2 C - \sin^2 B) + b^2(\sin^2 A - \sin^2 C) + c^2 (\sin^2 B - \sin^2 A)$

$= k^2a^2(c^2 - b^2) + k^2b^2(a^2 - c^2) + k^2c^2 (b^2 - c^2)$

$= 0$

2 **(a)**

Let $\sin A = 3k, \sin B = 4k, \sin C = 5k$

$\because \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = p \quad$ [say]

$\Rightarrow \frac{3k}{a} = \frac{4k}{b} = \frac{5k}{c} = p$

$\Rightarrow a = 3\left(\frac{k}{p}\right), b = 4\left(\frac{k}{p}\right), c = 5\left(\frac{k}{p}\right)$

$\Rightarrow a = 3l, b = 4l, c = 5l \quad \left[\text{let } l = \frac{k}{p}\right]$

$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$= \frac{16 + 25 - 9}{2 \times 4 \times 5} = \frac{32}{40} = \frac{4}{5}$

$\therefore \cos B = \frac{c^2 + a^2 - b^2}{2ac}$

$= \frac{25 + 9 - 16}{2 \times 3 \times 5} = \frac{18}{30} = \frac{3}{5}$

Now, $\cos A : \cos B = \frac{4}{5} : \frac{3}{5} = 4:3$

4 **(b)**

Slope of perpendicular to the line joining the points

$(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta) = -\frac{\cos\alpha - \cos\beta}{\sin\alpha - \sin\beta}$

$= \tan\frac{\alpha + \beta}{2}$

Hence, equation of perpendicular is

$y = \tan\left(\frac{\alpha + \beta}{2}\right)x \quad$ ...(i)

Now, on solving the equation of line with Eq. (i), we get

$$\left[\frac{a}{2}(\cos\alpha + \cos\beta), \frac{a}{2}(\sin\alpha + \sin\beta)\right]$$

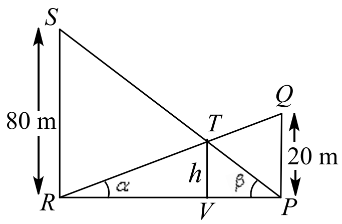
5 **(a)**

Area of $\frac{\Delta PBC}{\Delta ABC} = \left[\frac{\{-3(-2-y)+4(y-5)+x(5+2)\}}{\{6(5+2)-3(-2-3)+4(3-5)\}}\right]$

$= \left|\frac{7x+7y-14}{49}\right| = \left|\frac{x+y-2}{7}\right|$

6 **(b)**

Let PQ and RS be the poles of height 20 m and 80 m subtending angles α and β at R and P respectively. Let h be the height of the point T, the intersection of QR and PS



Then, $PR = h\cot\alpha + h\cot\beta$

$= 20\cot\alpha = 80\cot\beta$

$\Rightarrow \cot\alpha = 4\cot\beta$

$\Rightarrow \frac{\cot\alpha}{\cot\beta} = 4$

Again, $h\cot\alpha + h\cot\beta = 20\cot\alpha$

$\Rightarrow (h-20)\cot\alpha = -h\cot\beta$

$\Rightarrow \frac{\cot\alpha}{\cot\beta} = \frac{h}{20-h} = 4$

$\Rightarrow h = 80 - 4h$

$\Rightarrow h = 16$ m

8 **(a)**

Since, α, β, γ are the roots of the equation

$x^3 - 3px^2 + 3qx - 1 = 0$

$\therefore \alpha + \beta + \gamma = 3p$

$\alpha\beta + \beta\gamma + \gamma\alpha = 3q$

and $\alpha\beta\gamma = 1$

Let $G(x, y)$ be the centroid of the given triangle

$\therefore x = \frac{\alpha+\beta+\gamma}{3} = p$

and $y = \frac{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}}{3}$

$= \frac{\beta\gamma+\gamma\alpha+\alpha\beta}{3\alpha\beta\gamma} = q$

Hence, coordinates of the centroid of triangle are (p, q)

9 **(d)**

Let $O(0,0)$ be the orthocenter, $A(h,k)$ be the third vertex and $B(-2,3)$ and $C(5,-1)$ the other two vertices. Then, the slope of the line through A and O is $\frac{k}{h}$, while the line through B and C has the slope $\frac{(-1-3)}{(5+2)}=-\frac{4}{7}$. By the property of the orthocenter, these two lines must be perpendicular, so we have

$\left(\frac{k}{h}\right)\left(-\frac{4}{7}\right)=-1 \Rightarrow \frac{k}{h}=\frac{7}{4}$ ...(i)

Also, $\frac{5-2+h}{3}+\frac{-1+3+k}{3}=7$

$\Rightarrow h+k=16$ ...(ii)

Which is not satisfied by the points given in the options (a), (b) or (c)

10 **(b)**

Let (h,k) be the point

According to question,

$4\sqrt{(h-h)^2+k}=h^2+k^2$

$\Rightarrow 4|k|=h^2+k^2$

Locus of the point is

$4|y|=x^2+y^2 \Rightarrow x^2+y^2-4|y|=0$

12 **(a)**

Given points are $P(4,-2)$,$A(2,-4)$ and $B(7,1)$

Suppose P divides AB in the ratio λ :1. Then,

$$\frac{7\lambda+2}{\lambda+1}=4 \Rightarrow \lambda=\frac{2}{3}$$

Thus, P divides AB internally in the ratio 2 :3

The coordinates of the point dividing AB externally in the ratio 2 :3 are

$$\left(\frac{2\times 7-3\times 2}{2-3},\frac{2\times 1-3\times -4}{2-3}\right)=(-8,-14)$$

Hence, the harmonic conjugate of R with respect to A and B is $(-8,-14)$

13 **(c)**

If O is the origin and $P(x_1,y_1)$,$Q(x_2,y_2)$ are two points, then

$OP\times OQ\cos\angle POQ = x_1x_2+y_1y_2$

$\therefore OP\times OQ\times\sin\angle POQ$

$=\sqrt{OP^2\times OQ^2-OP^2\times OQ^2\times\cos^2\angle POQ}$

$=\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)-(x_1x_2+y_1y_2)^2}$

$=\sqrt{(x_1y_2-x_2y_1)^2}=|x_1y_2-x_2y_1|$

14 **(c)**

$$\cos B=\frac{(3)^2+(5)^2-(4)^2}{2\times 3\times 5}=\frac{3}{5}$$

$$\Rightarrow \sin B=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$$

$\therefore \sin 2B = 2\sin B\cos B$

$$=2\times\frac{4}{5}\times\frac{3}{5}=\frac{24}{25}$$

16 **(d)**

Given that, $\angle A = 45°, \angle B = 75°$

$\angle c = 180° - 45° - 75° = 60°$

$\therefore \; a + c\sqrt{2} = k(\sin A + \sqrt{2}\sin C)$

$= k(\sin 45° + \sqrt{2}\sin 60°)$

$= k\left(\frac{1}{\sqrt{2}} + \sqrt{2}\frac{\sqrt{3}}{2}\right) = k\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)$ ...(i)

And $k = \frac{b}{\sin B}$

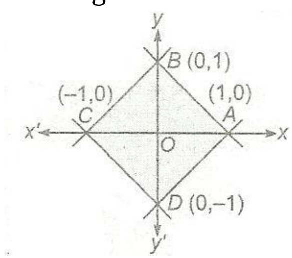
$= \dfrac{b}{\sin 75°} = \dfrac{2\sqrt{2}b}{\sqrt{3}+1}$

On putting the value of k in Eq. (i), we get

$a + c\sqrt{2} = 2b$

18 **(d)**

From figure $ABCD$ is s square



Whose diagonals AC and BD are of length 2 unit

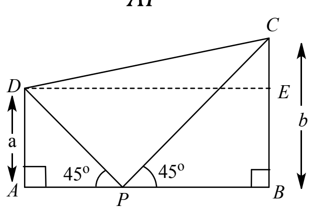
Hence, required area $= \frac{1}{2} AC \times BD$

$= \frac{1}{2} \times 2 \times 2 = 2$ sq units

19 **(c)**

In ΔAPD,

$\tan 45° = \dfrac{a}{AP} \Rightarrow AP = a$



and in $\Delta\, BPC$,

$\tan 45° = \dfrac{b}{PB}$

$\Rightarrow PB = b$

$\therefore DE = a + b$ and $CE = b - a$

In ΔDEC,

$DC^2 = DE^2 + EC^2$

$= (a + b^2) + (b - a^2)$

$= 2(a^2 + b^2)$

20 **(b)**

If the axes are rotated through 30°, we have

$$x = X\cos 30° - Y\sin 30° = \frac{\sqrt{3}X - 4}{2}$$

and, $y = X\sin 30° + Y\cos 30° = \dfrac{X + \sqrt{3}Y}{2}$

Substituting these values in $x^2 + 2\sqrt{3}\,xy - y^2 = 2a^2$, we get

$(\sqrt{3}X - Y)^2 + 2\sqrt{3}(\sqrt{3}X - Y)(X + \sqrt{3}Y) - (X + \sqrt{3}Y)^2 = 8a^2$

$\Rightarrow X^2 - Y^2 = a^2$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	B	A	B	D	A	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	C	C	B	D	C	D	C	B