DPP

DAILY PRACTICE PROBLEMS

Class : XIth **Solutions** **Subject : Maths**

Date : **DPP No. : 9**

Topic :-Binominal Theorem

1 **(a)**

$(2x^2 - x - 1)^5 = a_0 + a_1x + a_2x^2 + \dots + a_{10}x^{10}$

On putting $x = 0$, we get

$-1 = a_0$

On putting $x = 1$, we get

$0 = a_0 + a_1 + a_2 + \dots + a_{10}$ ...(i)

On putting $x = -1$, we get

$(2 + 1 - 1)^5 = a_0 - a_1 + a_2 - \dots + a_{10}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$0 + (2)^5 = 2(a_0 + a_2 + \dots + a_{10})$

$\Rightarrow$ $16 - 1 = a_2 + \dots + a_{10}$

$\Rightarrow a_2 + a_3 + \dots + a_{10} = 15$

2 **(b)**

We have,

${}^{2n}C_r = {}^{2n}C_{r+2} \Rightarrow r + r + 2 = 2n \Rightarrow n = r + 1$

3 **(d)**

$\therefore$ coefficient of x^{100} in the expansion of

$\sum_{j=0}^{200}(1+x)^j$ will be $\sum_{j=0}^{200} {}^{j}C_{100}$

$= [{}^{100}C_{100} + {}^{101}C_{100} + {}^{102}C_{100} + \dots + {}^{200}C_{100}]$

$[\because {}^{n}C_n + {}^{n+1}C_n + {}^{n+2}C_n + \dots + {}^{2n-1}C_n = {}^{2n}C_{n+1}]$

$= \binom{201}{100}$

4 **(b)**

We have,

$\frac{1}{n\,!} + \frac{1}{2\,!(n-2)\,!} + \frac{1}{4\,!(n-4)\,!} + \dots$

$$= \frac{1}{n\,!}\left\{\frac{n\,!}{n\,!} + \frac{n\,!}{2\,!(n-2)\,!} + \frac{n\,!}{4'!(n-4)\,!} + \ldots\right\}$$

$$= \frac{1}{n\,!}\{\,{}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\} = \frac{2^{n-1}}{n\,!}$$

5 **(c)**

General term $T_{r+1} = {}^{10}C_r\left(\frac{x}{2}\right)^{10-r}\left(-\frac{3}{x^2}\right)^r$

$$= {}^{10}C_r \cdot \frac{x^{10-3r} \cdot (-1)^r \cdot 3^r}{2^{10-r}}$$

For the coefficient of x^4put

$10 - 3r = 4$

$\Rightarrow \quad r = 2$

Hence, coefficient of x^4 is

$${}^{10}C_2 \cdot \frac{3^2}{2^8} = \frac{405}{256}$$

6 **(a)**

Given, $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$

Also, $(x+1)^n = C_n + C_{n-1}x + C_{n-2}x^2 + \ldots + C_0x^n$

On multiplying both equations and comparing coefficient of x^{n-1} on both sides, we get

$C_0C_2 + C_1C_2 + C_2C_3 + \ldots + C_{n-1}C_n = {}^{2n}C_{n-1}$

$$= \frac{(2n)!}{(n-1)!(n+1)!}$$

7 **(c)**

Now, $7^9 = (8-1)^9 = -1(1-8)^9$

$= -1 + {}^9C_1 8 - {}^9C_2 8^2 + \ldots + {}^9C_9 8^9$ and $9^7 = (1+8)^7$

$= 1 + {}^7C_1 8 + {}^7C_2 8^2 + {}^7C_3 8^3 + \ldots + {}^7C_7 8^7$

$\therefore\ 7^9 + 9^7 = 8({}^9C_1 + {}^7C_1) + 8^2({}^7C_2 - {}^9C_2) + \ldots$

$= 8(9+7) + 8^2(21-36) + \ldots$

$= 64 \times 2 + 64(-15) + \ldots$

Hence, it is divisible by 64

8 **(d)**

Let $f = (8 - 3\sqrt{7})^{10}$, here $0 < f < 1$

$\therefore\ (8 + 3\sqrt{7})^{10} + (8 - 3\sqrt{7})^{10}$ is an integer hence, this is the value of n

9 **(d)**

We have, $\left(3x - \frac{1}{2x}\right)^8$

$\therefore$ Ninth term $T_9 = 8_{c_8}(3x)^{8-8}\left(\frac{-1}{2x}\right)^8$

$= \frac{1}{256x^8}$

10 **(b)**

The general term in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$ is given by

$T_{r+1} = {}^{n-3}C_r(x)^{n-3-r}\left(\frac{1}{x^2}\right)^r$

$= {}^{n-3}C_r x^{n-3-3r}$

As x^{2k} occurs in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$, we must have $n - 3 - 3r = 2k$ for some non-negative integer r

$\Rightarrow 3(1 + r) = n - 2k$

$\Rightarrow n - 2k$ is a multiple of 3

11 **(c)**

Let T_{r+1} denote the $(r+1)^{\text{th}}$ term in the expansion of $\left(7^{1/3} + 5^{1/2}x\right)^{600}$. Then,

$T_{r+1} = {}^{600}C_r\,(7^1)^{600-r}(5^{1/3}x)^r = {}^{600}C_r 7^{200-\frac{r}{3}} \times 5^{\frac{r}{2}} \times x^r$

Here, $0 \le r \le 600$

For $200 - \frac{r}{3}$ and $\frac{r}{2}$ to be integers, we must have

$\frac{r}{3}$ and $\frac{r}{2}$ as integers, and $0 \le r \le 600$

$\Rightarrow r$ is multiple of 2 and 3 both and $0 \le r \le 600$

$\Rightarrow r$ is a multiple of 6 and $0 \le r \le 600$

$\Rightarrow r = 0,6,12,\ldots,600$

Hence, there are 101 terms with integral coefficients

12 **(b)**

We have,

$(xy + yz + zx)^6 = \sum_{r+s+t=6} \frac{6!}{r!s!t!}(xy)^r(yz)^s(zx)^t$

$= \sum_{r+s+t=6} \frac{6!}{r!s!t!} x^{r+t}y^{r+s}z^{s+t}$

If the general term in the above expansion contains $x^3y^4z^5$, then

$r + t = 3, r + s = 4$ and $s + t = 5$

Also, $r + s + t = 6$

On solving these equations, we get

$r = 1, s = 3, t = 2$

$\therefore$ Coefficient of $x^3y^4z^5 = \dfrac{6!}{1!3!2!} = 60$

13 **(c)**

We have,

$$(1+2x+x^2)^n = \sum_{r=0}^{2n} a_r\, x^r$$

$$\Rightarrow \{(1+x)^2\}^n = \sum_{r=0}^{2n} a_r\, x^r$$

$$\Rightarrow (1+x)^{2n} = \sum_{r=0}^{2n} a_r\, x^r$$

$$\Rightarrow \sum_{r=0}^{2n} {}^{2n}C_r\, x^r = \sum_{r=0}^{2n} a_r\, x^r \Rightarrow a_r = {}^{2n}C_r$$

14 **(a)**

Given, ${}^{20}C_4 + {}^{20}C_3 + {}^{20}C_3 + {}^{20}C_2 - {}^{22}C_{18}$

$= {}^{21}C_4 + {}^{20}C_3 + {}^{20}C_2 - {}^{22}C_{18}$

$= {}^{22}C_4 - {}^{22}C_{18} = {}^{22}C_{18} - {}^{22}C_{18} = 0$

15 **(d)**

We have,

$y = 3\,x + 6\,x^2 + 10\,x^3 + \dots$

$\Rightarrow 1 + y = (1 + 3\,x + 6\,x^2 + 10\,x^3 + \dots)$

$\Rightarrow 1 + y = (1-x)^{-3}$

$\Rightarrow (1-x) = (1+y)^{-1/3}$

$\Rightarrow x = 1 - (1+y)^{-1/3}$

$\Rightarrow x = \dfrac{1}{3}\,y - \dfrac{1\cdot 4}{3^2\cdot 2}y^2 + \dfrac{1\cdot 4\cdot 7\cdot}{3^3\,3\,!}y^3 \dots$

16 **(c)**

We know that,

$(x+a)^n + (x-a)^n = 2[\,{}^nC_0x^n + {}^nC_2x^{n-2}a^2 + \dots.]$

Here, $n = 5$, $x = x$ and $a = (x^3-1)^{1/2}$

$\therefore [x + (x^3-1)^{1/2}]^5[x - (x^3-1)^{1/2}]^5$

$= 2[\,{}^5C_0x^5 + {}^5C_2x^3(x^3-1) + {}^5C_4x(x^3-1)^2]$

$= 2[x^5 + 10x^3(x^3-1) + 5\,x(x^3-1)^2]$

$\therefore$ Given expression is a polynomial of degree 7.

17 **(c)**

We have,

$C_0^2+3\cdot C_1^2+5\cdot C_2^2+\ldots+(2n+1)C_n^2$
$=\{C_0^2+C_1^2+C_2^2+\ldots+C_n^2\}$
$+\{2\,C_1^2+4\cdot C_2^2+6\cdot C_3^2+\ldots+2\,n\,C_n^2\}$ ...(i)
We have,
$(1+x)^{2n}=(1+x)^n(1+x)^n$
$\Rightarrow(1+x)^{2n}=(C_0+C_1x+C_2x^2+\ldots+C_nx^n)\times(C_0x^n+C_1x^{n-1}+\ldots+C_{n-1}\,x+C_n)$
On equating the coefficient of x^n on both sides, we get
${}^{2n}C_n=C_0^2+C_1^2+C_2^2+\ldots C_n^2$ ...(ii)
Also,
$n(1+x)^{n-1}(1+x)^n=(C_1+2\,C_2\,x+3\,C_3\,x^2+\ldots+n\,C_n\,x^{n-1})$
$\times(C_0\,x^n+C_1\,x^{n-1}+C_2\,x^{n-2}+\ldots+C_n)$
On equating the coefficient of x^{n-1} on both sides, we get
$n\cdot{}^{2n-1}C_{n-1}=(C_1^2+2\,C_2^2+3\,C_3^2+\ldots+n\,C_n^2)$
$\Rightarrow 2n\cdot{}^{2n-1}C_{n-1}=2\,C_1^2+4\,C_2^2+6\,C_3^2+\ldots++2\,n\,C_n^2$...(iii)
From (i),(ii) and (iii), we obtain
$C_0^2+3\cdot C_1^2+5\,C_2^2+\ldots+(2n+1)C_n^2$
$=\frac{2n}{n}\,{}^{2n-1}C_{n-1}+2n\cdot{}^{2n-1}C_{n-1}=2(n+1)\,{}^{2n-1}C_{n-1}$

18 **(d)**
Here ${}^{n-1}C_r=(k^2-3)\,{}^nC_{r+1}$
$\Longrightarrow {}^{n-1}C_r=(k^2-3)\frac{n}{r+1}\,{}^{n-1}C_r$
$\Longrightarrow k^2-3=\frac{r+1}{n}$
$\left[\text{since, } n-1\ge r\Longrightarrow\frac{r+1}{n}\le 1\text{and } n,r\ge 0\right]$
$\Longrightarrow 0<k^2-3\le 1\Longrightarrow 3<k^2\le 4$
$\Longrightarrow k\in[-2,-\sqrt{3})\cup(\sqrt{3},2]$

19 **(a)**
Let $f(x)=(x+y)^{100}+(x-y)^{100}$
Here, $n=100$, which is even.
$\therefore$ Total number of terms
$=\frac{n+2}{2}=\frac{100+2}{2}$
$=51$

20 **(c)**
We know that
$C_0^2+C_1^2+C_2^2+C_3^2+\ldots+C_n^2=\frac{(2\,n)\,!}{n\,!n\,!}$ ...(i)
and, $C_0^2-C_1^2+C_2^2-C_3^2+\ldots-C_n^2=0$, when n is odd ...(ii)

subtracting (ii) from (i), we get

$$2\left(C_1^2 + C_3^2 + C_5^2 + \ldots + C_n^2\right) = \frac{(2\,n)\,!}{(n\,!)^2}$$

$$\Rightarrow C_1^2 + C_3^2 + C_5^2 + \ldots + C_n^2 = \frac{(2n)\,!}{2(n\,!)^2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	B	B	D	B	D	B	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	A	B	B	B	B	B	B	C