DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : Maths
DPP No. : 8

Topic :-Binominal Theorem

1 **(c)**

$(1-a\,x)^{-1}(1-bx)^{-1}$

$=(a^0+a\,x+a^2x^2+\ldots(b^0+bx+b^2x^2+\ldots.)$

Hence, $a_n=$ coefficient of x^n in $(1-ax)^{-1}(1-bx)^{-1}$

$a^0b^n+ab^{n-1}+\ldots..+a^nb^0$

$$=a^0b^n\left(1+\frac{a}{b}+\left(\frac{a}{b}\right)^2+\ldots.+\left(\frac{a}{b}\right)^n\right)$$

$$=a^0b^n\left(\frac{\left(\frac{a}{b}\right)^{n+1}-1}{\frac{a}{b}-1}\right)$$

$$=\frac{a^{n+1}-b^{n+1}}{a-b}=\frac{b^{n+1}-a^{n+1}}{b-a}$$

2 **(c)**

Given, $(1+2x+x^2)^5=\sum_{k=0}^{15}\ a_kx^k$

$\Rightarrow(1+x)^{10}=a_0x^0+a_1x+a_2x^2+\ldots+a_{15}x^{15}$

$\Rightarrow {}^{10}C_0+{}^{10}C_1x+{}^{10}C_2x^2+\ldots+{}^{10}C_{10}x^{10}$

$=a_0+a_1x+a_2x^2+a_3x^3+\ldots+a_{15}x^{15}$

On equating the coefficient of constant and even power of x, we get

$a_0=10_{C_0}, a_2=10_{C_2},$

$a_4={}^{10}C_4, \ldots..a_{10}={}^{10}C_{10}, a_{12}=a_{14}=0$

$$\therefore\ \sum_{k=0}^{7}\ a_{2k}={}^{10}C_0+{}^{10}C_2+{}^{10}C_4+{}^{10}C_6$$

$+{}^{10}C_8+{}^{10}C_{10}+0+0$

$=2^{10-1}=2^9=512$

3 **(b)**

Since, n is even, therefore $\left(\frac{n}{2}+1\right)$th term is the middle term.

$\therefore T_{\frac{n}{2}+1} = {}^nC_{n/2}(x^2)^{n/2}\left(\frac{1}{x}\right)^{n/2}$

$= 924x^6$ (given)

$\Rightarrow x^{n/2} = x^6 \Rightarrow n = 12$

4 **(b)**

We have, $(1+x^2)^5(1+x)^4$

$= ({}^5C_0 + {}^5C_1x^2 + {}^5C_2x^4 + \dots)({}^4C_0 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4)$

The coefficient of x^5 in $[(1+x^2)^5(1+x)^4]$

$= {}^5C_2 . {}^4C_1 + {}^4C_3 . {}^5C_1 = 10.4 + 4.5 = 60$

5 **(d)**

$(1+x+x^2+x^3)^n = \{(1+x)^n(1+x^2)^n\}$

$= (1 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n)(1 + {}^nC_1x^2 + {}^nC_2x^4 + \dots + {}^nC_nx^{2n})$

Therefore the coefficient of $x^4 = {}^nC_2 + {}^nC_2\,{}^nC_1 + {}^nC_4$

$= {}^nC_4 + {}^nC_2 + {}^nC_1\,{}^nC_2$

6 **(b)**

Let $a = {}^nC_{r-1}, b = {}^nC_r, c = {}^nC_{r+1}$

and $d = {}^nC_{r+2}$

$\therefore\ a + b = {}^{n+1}C_r, b + c = {}^{n+1}C_{r+1}, c + d = {}^{n+1}C_{r+2}$

$$\Rightarrow \frac{a+b}{a} = \frac{{}^{n+1}C_r}{{}^nC_{r-1}} = \frac{n+1}{r} \Rightarrow \frac{a}{a+b} = \frac{r}{n+1}$$

and $\frac{b+c}{b} = \frac{{}^{n+1}C_{r+1}}{{}^nC_r} = \frac{n+1}{r+1} \Rightarrow \frac{b}{b+c} = \frac{r+1}{n+1}$

and $\frac{c+d}{c} = \frac{{}^{n+1}C_{r+2}}{{}^nC_{r+1}} = \frac{n+1}{r+2} \Rightarrow \frac{c}{c+d} = \frac{r+2}{n+1}$

$\therefore\ \frac{a}{a+b}, \frac{b}{b+c}, \frac{c}{c+d}$ are in AP

$\because$ AM $> GM$

$$\Rightarrow \frac{b}{b+c} > \sqrt{\frac{ac}{(a+b)(c+d)}}$$

or $\left\{\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right\} > 0$

7 **(d)**

We have,

$$T_{r+1} = {}^6C_r(\sqrt{x^5})^{6-r}\left(\frac{3}{\sqrt{x^3}}\right)^r$$

$\Rightarrow T_{r+1} = {}^6C_r\, x^{15-\frac{5}{2}r-\frac{3}{2}r}\, 3^r = {}^6C_r\, x^{15-4r}\, 3^r$

This will contain x^3, if $15-4r=3 \Rightarrow r=3$

$\therefore$ Coefficient of $x^3 = {}^6C_3 \cdot 3^3 = 540$

8 **(b)**

General term, $T_{r+1} = {}^{11}C_r \frac{a^{11-r}}{b^r}(-1)^r x^{11-3r}$

For the coefficient of x^{-7}, put

$11-3r=-7 \Rightarrow r=6$

$$\therefore \text{Coefficient of } x^{-7} = {}^{11}C_6 \frac{a^5}{b^6} = \frac{462a^5}{b^6}$$

9 **(a)**

We have,

Coefficient of x^5 in $(x+3)^6 = {}^6C_1 \times 3^1 = 18$

10 **(d)**

$A_r =$ Coefficient of x^r in $(1+x)^{10} = {}^{10}C_r$

$B_r =$ Coefficient of x^r in $(1+x)^{20} = {}^{20}C_r$

$C_r =$ Coefficient of x^r in $(1+x)^{30} = {}^{30}C_r$

$$\therefore \sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r) = \sum_{r=1}^{10} A_rB_{10}B_r, - \sum_{r=1}^{10} A_rC_{10}A_r$$

$$= \sum_{r=1}^{10} {}^{10}C_r\, {}^{20}C_{10}\, {}^{20}C_r \sum_{r=1}^{10} {}^{10}C_r\, {}^{30}C_{10}\, {}^{10}C_r\text{l}$$

$$\sum_{r=1}^{10} {}^{10}C_{10-rl}\, {}^{20}C_{10}\, {}^{20}C_r - \sum_{r=1}^{10} {}^{10}C_{10-r}\, {}^{30}C_{10}\, {}^{10}C_r l$$

$$= {}^{20}C_{10} \sum_{r=1}^{10} {}^{10}C_{10-r}\, {}^{20}C_r - {}^{30}C_{10} \sum_{r=1}^{10} {}^{10}C_{10-r}\, {}^{10}C_r$$

$$= {}^{20}C_{10}({}^{30}C_{10}-1) - {}^{30}C_{10}({}^{20}C_{10}-1)$$

$$= {}^{20}C_{10}({}^{30}C_{10}-1) - {}^{30}C_{10}({}^{20}C_{10}-1)$$

$$= {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$$

11 **(a)**

$\because$ Coefficient of $x^p$is ${}^{p+q}C_p$ and coefficient of x^q is ${}^{(p+q)}C_q$

$\therefore$ Both the coefficients are equal

12 **(c)**

In the expansion of $(1+x)^{2n}$, the general term

$= {}^{2n}C_k x^k, 0 \le k \le 2n$

As given for $r > 1, n > 2,\ {}^{2n}C_{3r} = {}^{2n}C_{r+2}$
$\Rightarrow$Either $3r = r + 2$ or $3r = 2n - (r + 2)$ $\left(\because {}^{n}C_{r} = {}^{n}C_{n-r}\right)$
$\Rightarrow r = 1$ or $n = 2r + 1$
We take the relation only
$n = 2r + 1$ $(\because r > 1)$

13 **(a)**

The general term in the expansion of $\left(x \sin^{-1}\alpha + \frac{\cos^{-1}\alpha}{x}\right)^{10}$ is given by

$$T_{r+1} = {}^{10}C_r (x \sin^{-1}\alpha)^{10-r}\left(\frac{\cos^{-1}\alpha}{x}\right)^r$$

$$\Rightarrow T_{r+1} = {}^{10}C_r (\sin^{-1}\alpha)^{10-r}(\cos^{-1}\alpha)^r\, x^{10-2r} \quad \text{...(i)}$$

This will be independent of x, if
$10 - 2r = 0 \Rightarrow r = 5$
Putting $r = 5$ in (i), we get

$$T_6 = {}^{10}C_5 (\sin^{-1}\alpha \cos^{-1}\alpha)^5$$

$$\Rightarrow T_6 = {}^{10}C_5 \left\{\sin^{-1}\alpha\left(\frac{\pi}{2} - \sin^{-1}\alpha\right)\right\}^5$$

$$\Rightarrow T_6 = {}^{10}C_5 \left\{\frac{\pi}{2}\sin^{-1}\alpha - (\sin^{-1}\alpha)^2\right\}^5$$

$$\Rightarrow T_6 = {}^{10}C_5 \left\{\frac{\pi^2}{16} - \left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2\right\}^5$$

Now,

$$-\frac{\pi}{2} \le \sin^{-1}\alpha \le \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{2} \le -\sin^{-1}\alpha \le \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4} \le \left(\frac{\pi}{4} - \sin^{-1}\alpha\right) \le \frac{3\pi}{4}$$

$$\Rightarrow 0 \le \left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2 \le \frac{9\pi^2}{16}$$

$$\Rightarrow -\frac{9\pi^2}{16} \le -\left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2 \le 0$$

$$\Rightarrow -\frac{\pi^2}{2} \le \frac{\pi^2}{16} - \left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2 \le \frac{\pi^2}{16}$$

$$\Rightarrow -\ {}^{10}C_5\left(\frac{\pi^2}{2}\right)^5 \le {}^{10}C_5\left\{\frac{\pi^2}{16} - \left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2\right\}^5 \le {}^{10}C_5\left(\frac{\pi^2}{16}\right)^5$$

$$\Rightarrow -\frac{{}^{10}C_5\,\pi^{10}}{2^5} \le T_6 \le {}^{10}C_5\frac{\pi^{10}}{2^{20}}$$

14 **(b)**

In the expansion of $(3 + 7x)^{29}$

$$T_{r+1} = {}^{29}C_r \cdot 3^{29-r} \cdot (7x)^r$$
$$= \left({}^{29}C_r \times 3^{29-r} \times 7^r\right) x^r$$

Let a_r= coefficient of $(r+1)$th term

$$= {}^{29}C_r \times 3^{29-r} \times 7^r$$

and a_{r-1} = coefficient of rth term

$$= {}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$$

According to question $a_r = a_{r-1}$

$\Longrightarrow {}^{29}C_r \times 3^{29-r} \times 7^r = {}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$

$\Longrightarrow \dfrac{{}^{29}C_r}{{}^{29}C_{r-1}} = \dfrac{3}{7} \Longrightarrow \dfrac{30-r}{r} = \dfrac{3}{7}$

$\Longrightarrow 210 - 7r = 3r \Longrightarrow r = 21$

16 **(b)**

In the expansion of $(1+x)^{50}$ the sum of the coefficient of odd powers

$= C_1 + C_3 + C_5 + \dots = 2^{50-1} = 2^{49}$

17 **(b)**

It is given that the coefficients of r^{th} and $(r+1)^{\text{th}}$ term in the expansion of $(3+7x)^{29}$ are equal

$\therefore {}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1} = {}^{29}C_r \times 3^{29-r} \times 7^r$

$\Rightarrow {}^{29}C_{r-1} \times 3 = {}^{29}C_r \times 7$

$\Rightarrow \dfrac{3}{30-r} = \dfrac{7}{r} \Rightarrow r = 21$

18 **(b)**

We have,

${}^{2n}C_p = {}^{2n}C_{p+2} \Rightarrow p + p + 2 = 2n \Rightarrow p = n - 1$

19 **(b)**

We have

$(1+x)^n = \sum_{r=0}^{n} a_r x^r \Rightarrow a_r = {}^nC_r$

Now,

$\left(1 + \dfrac{a_1}{a_0}\right)\left(1 + \dfrac{a_1}{a_1}\right) \dots \left(1 + \dfrac{a_n}{a_{n-1}}\right)$

$= \prod_{r=1}^{n} \left(1 + \dfrac{a_r}{a_{r-1}}\right)$

$= \prod_{r=1}^{n} \left(\dfrac{a_{r-1} + a_r}{a_{r-1}}\right)$

$$= \prod_{r=1}^{n} \left(\frac{{}^{n}C_{r} + {}^{n}C_{r-1}}{{}^{n}C_{r-1}} \right)$$

$$= \prod_{r=1}^{n} \frac{{}^{n+1}C_{r}}{{}^{n}C_{r-1}}$$

$$= \prod_{r=1}^{n} \frac{n+1}{r} \qquad \left[\because\ {}^{n+1}C_{r} = \frac{n+1}{r}\ {}^{n}C_{r-1} \right]$$

$$= (n+1)^{n} \left(\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \ldots \times \frac{1}{n} \right) = \frac{(n+1)^{n}}{n\,!}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	B	B	D	A	D	D	A	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	D	C	A	B	B	B	C	B