DPP

DAILY PRACTICE PROBLEMS

Class : XIth | **Solutions** | **Subject : Maths**

Date : | | **DPP No. : 7**

Topic :-Binominal Theorem

1 **(d)**

$(bc+ca+ab)^9=[bc+a(b+c)]^9$

$\therefore$ Coefficient of $a^5b^6c^7$

$=$ coefficient of $a^5b^6c^7$ in ${}^9C_5(bc)^4a^5(b+c)^5$

$=$coefficient of b^2c^3 in ${}^9C_5(b+c)^5$

$={}^9C_5\times{}^5C_3=1260$

2 **(b)**

We have, $(x-1)(x-2)(x-3)...(x-100)$

Number of terms $=100$

$\therefore$ Coefficient of x^{99} in $(x-1)(x-2)(x-3)...(x-100)$

$=(-1-2-3-...-100)$

$=-(1+2+...+100)$

$=-\dfrac{100\times101}{2}=-5050$

3 **(b)**

Given, $\sin n\theta=\sum_{r=0}^{n}b_r\sin^r\theta$

$\Rightarrow\ \sin n\theta=b_0\sin^0\theta+b_1\sin^1\theta+b_2\sin^2\theta+b_3\sin^3\theta+...+b_n\sin^n\theta$

$\Rightarrow\ \sin n\theta=b_0+b_1\sin\theta+b_2\sin^2\theta+...+b_n\sin^n\theta$

(n is an odd integer)

$\because\ \sin n\theta={}^nC_1\sin\theta\cos^{n-1}\theta-{}^nC_3\sin^3\theta\cos^{n-3}\theta+...$

$={}^nC_1\sin\theta(1-\sin^2\theta)^{(n-1)/2}-{}^nC_3\sin^3\theta(1-\sin^2\theta)^{(n-3)/2}+...$

$\therefore b_0=0,\ b_1=$ coefficient of $\sin\theta={}^nC_1=n$

($\because n-1, n-3$ are all even integers)

4 **(d)**

We have,

$\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$

$=2\left\{{}^6C_0\,x^6+{}^6C_2\,x^4\left(\sqrt{x^2-1}\right)^2+{}^6C_4\,x^2\left(\sqrt{x^2-1}\right)^4+{}^6C_6\left(\sqrt{x^2-1}\right)^6\right\}$

$= 2\{x^6 + {}^6C_2\, x^4(x^2-1) + {}^6C_2\, x^2(x^2-1)^2 + (x^2-1)^3\}$
$= [\{2\ {}^6C_2 + 1\}x^6 - 3\{{}^6C_2 + 1\}x^4 + 4\ {}^6C_2\, x^2 - 1]$
Clearly, it contains 4 terms

5 **(a)**
We know that,
$(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$
$(x-1)^n = C_0x^n - C_1x^{n-1} + C_2x^{n-2} - ... + (-1)^nC_n$
On multiplying both equations and equating the coefficient of x^n, we get
$C_0^2 - C_1^2 + C_2^2 - ... + (-1)^nC_n^2 = {}^nC_{n/2}(-1)^{n/2}(x^2)^{n/2}$
Above is possible only when $\frac{n}{2}$ is an integer *ie*, n is even and in case n is odd, then term x^n will not occur

126 **(d)**
$(1-x+x^2)^n = a_0 + a_1x + ... + a_{2n}x^{2n}$
Putting $x = -1$ and 1
Successively and adding, we get
$a_0 + a_2 + a_4 + ... + a_{2n} = \frac{3^n+1}{2}$

7 **(d)**
Now, coefficient of x^{15} in $(1+x)^{20}$
$=$ coefficient of x^{15} in $(1+x)^{15}(1+x)^5$
$\Longrightarrow {}^{20}C_{15} =$ coefficient of x^{15} in
$$({}^{15}C_0x^{15} + {}^{15}C_1x^{14} + {}^{15}C_2x^{13} + {}^{15}C_3x^{12} + {}^{15}C_4x^{11} + {}^{15}C_5x^{10})$$
$({}^5C_0x^5 + {}^5C_1x^4 + {}^5C_2x^3 + {}^5C_3x^2 + {}^5C_4x + {}^5C_5)$
$= {}^{20}C_{15} = {}^{15}C_0 \cdot {}^5C_5 + {}^{15}C_1 \cdot {}^5C_4 + {}^{15}C_2 \cdot {}^5C_3 + {}^{15}C_3 \cdot {}^5C_2 + {}^{15}C_4 \cdot {}^5C_1 + {}^{15}C_5 \cdot {}^5C_0$
$\Longrightarrow {}^{15}C_0 \cdot {}^5C_5 + {}^{15}C_1 \cdot {}^5C_4 + {}^{15}C_2 \cdot {}^5C_3 + {}^{15}C_3 \cdot {}^5C_2 + {}^{15}C_4 \cdot {}^5C_1 = {}^{20}C_{15} - {}^{15}C_5 \cdot {}^5C_0$
$$= \frac{20!}{5!15!} - \frac{15!}{5!10!}$$

8 **(a)**
The given expression is
$1 + (1+x) + (1+x)^2 + ... + (1+x)^n$ being in GP
Let $S = 1 + (1+x) + (1+x)^2 + ... + (1+x)^n$
$= \frac{(1+x)^{n+1} - 1}{(1+x) - 1} = x^{-1}[(1+x)^{n+1} - 1]$
$\therefore$ The coefficient of x^k in S
$=$ The coefficient of x^k in $[(1+x)^{n+1} - 1]$
$= {}^{n+1}C_{k+1}$

9 **(d)**

Since, in a binomial expansion of $(a-b)^n, n \geq 5$, then sum of 5th and 6th terms is equal to zero.

$$\therefore \quad {}^nC_4 a^{n-4}(-b)^4 + {}^nC_5 a^{n-5}(-b)^5 = 0$$

$$\Rightarrow \frac{n!}{(n-4)!4!} a^{n-4} b^4 - \frac{n!}{(n-5)!5!} a^{n-5} b^5 = 0$$

$$\Rightarrow \frac{n!}{(n-5)!4!} a^{n-5} \cdot b^4 \left(\frac{a}{n-4} - \frac{b}{5}\right) = 0$$

$$\Rightarrow \quad \frac{a}{b} = \frac{n-4}{5}$$

10 **(a)**

We have,

$$\left(1-\frac{1}{x}\right)^n (1-x)^n = (1-x)^{2n} \frac{(-1)^n}{x^n} = \frac{(-1)^n (1-x)^{2n}}{x^n}$$

$\therefore$ Middle term in $\left(1-\frac{1}{x}\right)^n (1-x)^n$

$$= \frac{(-1)^n}{x^n} \text{ middle term in } (1-x)^{2n}$$

$$= \frac{(-1)^n}{x^n} \times (n+1)^{th} \text{ term in } (1-x)^{2n}$$

$$= \frac{(-1)^n}{x^n} \times {}^{2n}C_n(-x)^n = {}^{2n}C_n$$

12 **(d)**

The number of terms in the expansion of $(a+b+c)^{10}$

$$= {}^{12}C_2 = \frac{11 \cdot 12}{2} = 66$$

13 **(d)**

The given expression of $\frac{1}{(4-3x)^{1/2}}$ can be rewritten as

$4^{-1/2}\left(1-\frac{3}{4}x\right)^{-1/2}$ and it is valid only when $\left|\frac{3}{4}x\right| < 1$

$$\Rightarrow -\frac{4}{3} < x < \frac{4}{3}$$

14 **(c)**

$$\because (3+2x)^{50} = 3^{50}\left(1+\frac{2x}{3}\right)^{50}$$

Here, $T_{r+1} = 3^{50}\ {}^{50}C_r\left(\frac{2x}{3}\right)^r$

and $T_r = 3^{50}\ {}^{50}C_{r-1}\left(\frac{2x}{3}\right)^{r-1}$

But $x = \frac{1}{5}$

$\therefore \frac{T_{r+1}}{T_r} \geq 1 \Rightarrow \frac{{}^{50}C_r}{{}^{50}C_{r-1}} \frac{2}{3} \cdot \frac{1}{5} \geq 1$

$\Rightarrow 102 - 2r \geq 15r \Rightarrow r \leq 6$

15 **(a)**

Given that, $(1 + ax)^n = 1 + 8x + 24x^2 + ...$

$\Rightarrow 1 + \frac{n}{1} ax + \frac{n(n-1)}{1.2} a^2x^2 + ... = 1 + 8x + 24x^2 + ...$

On comparing the coefficients of x, x^2, we get

$na = 8,\ \frac{n(n-1)}{1.2} a^2 = 24$

$\Rightarrow na(n-1)a = 48$

$\Rightarrow 8(8 - a) = 48$

$\Rightarrow 8 - a = 6$

$\Rightarrow a = 2 \Rightarrow n = 4$

16 **(b)**

$\because\ (0.99)^{15} = (1 - 0.01)^{15}$

$= 1 - {}^{15}C_1(0.01) + {}^{15}C_2(0.01)^2 - {}^{15}C_3(0.01)^3 + ...$

We want to answer correct upto 4 decimal places and as such, we have left further expansion.

$= 1 - 15(0.01) + \frac{15 \cdot 14}{1 \cdot 2}(0.0001) - \frac{15 \cdot 14 \cdot 13}{1 \cdot 2 \cdot 3}(0.000001) + ...$

$= 1 - 0.15 + 0.0105 - 0.000455 + ...$

$= 1.0105 - 0.150455$

$= 0.8601$

17 **(b)**

Given that,

$\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ...$

$= \frac{1}{n!}\left[\frac{n!}{1!(n-1)!} + \frac{n!}{3!(n-3)!} + \frac{n!}{5!(n-5)!} + ...\right]$

$= \frac{1}{n!}\left[{}^{n}C_1 + {}^{n}C_3 + {}^{n}C_5 + ...\right]$

$= \frac{2^{n-1}}{n!}$

18 **(b)**

$$\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$$

$$=\frac{\left(1+\frac{3}{2}x+\frac{\frac{3}{2}\cdot\frac{1}{2}}{2}x^2\right)-\left(1+\frac{3x}{2}+\frac{3\cdot 2}{2}\cdot\frac{x^2}{4}\right)}{(1-x)^{1/2}}$$

[neglecting higher powers of x]

$$=-\frac{3x^2}{8}(1-x)^{-1/2}$$

$$=-\frac{3x^2}{8}\left(1+\frac{1}{2}x+\frac{\frac{1}{2}\cdot\frac{3}{2}}{2}\cdot x^2\right)=-\frac{3x^2}{8}$$

[neglecting higher powers of x]

19 **(c)**

Total number of terms in the expansion of $(2x+3y-4z)^n$, is

$${}^{n+3-1}C_{3-1}={}^{n+2}C_2=\frac{(n+2)(n+1)}{2}$$

20 **(b)**

We have,

$$(1+x)^m(1+x)^n=\left(\sum_{r=0}^{m}{}^mC_r\,x^r\right)\cdot\left(\sum_{r=0}^{n}{}^nC_r\,x^r\right)$$

Equation coefficients of x^r on both sides, we get

$${}^mC_r+{}^mC_{r-1}\,{}^nC_1+{}^mC_{r-2}\,{}^nC_2+\ldots+{}^mC_1\,{}^nC_{r-1}+\ldots+{}^mC_0\,{}^nC_r={}^{m+n}C_r$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	B	B	C	A	D	C	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	B	C	C	B	A	C	B