DPP

DAILY PRACTICE PROBLEMS

Class : XIth **Solutions** **Subject : Maths**

Date : **DPP No. : 6**

Topic :-Binominal Theorem

1 **(c)**

We have,

$$(x+y+z)^{18}=\sum_{r+s+t+=18}\frac{18!}{r!s!t!}x^r\,y^s\,z^t$$

$\therefore$ Coefficient of $x^8y^6z^4=\frac{18!}{8!6!4!}=\frac{18!}{10!8!}\times\frac{10!}{6!4!}$

$= {}^{18}C_{10}\times {}^{10}C_6$

Also, Coefficient of $x^8y^6z^4=\frac{18!}{8!\,6!\,4!}=\frac{18!}{4!\,14!}\times\frac{14!}{8!\,6!}$

$= {}^{18}C_{14}\times {}^{14}C_8= {}^{18}C_4\times {}^{14}C_6$

Again,

Coefficient of $x^8y^6z^4=\frac{18!}{8!6!4!}=\frac{18!}{12!6!}\times\frac{12!}{8!4!}$

$= {}^{18}C_6\times {}^{12}C_8$

2 **(c)**

We have,

$1+x+x^2+x^3=(1+x)(1+x^2)$

$\therefore (1+x+x^2+x^3)^{11}=(1+x)^{11}(1+x^2)^{11}$

$=({}^{11}C_0+{}^{11}C_1\,x+{}^{11}C_2\,x^2+{}^{11}C_3\,x^3+{}^{11}C_4\,x^4+\dots)$

$\times({}^{11}C_0+{}^{11}C_1\,x^2+{}^{11}C_2\,x^4+\dots)$

$\Rightarrow$Coefficient of x^4 in $(1+x+x^2+x^3)^{11}$

$=$ Coefficient of x^4 in

$\{({}^{11}C_0+{}^{11}C_1\,x+{}^{11}C_2\,x^2+\dots)({}^{11}C_0+{}^{11}C_1\,x^2+{}^{11}C_2\,x^4+\dots)\}$

$= {}^{11}C_0\times {}^{11}C_2+{}^{11}C_2\times {}^{11}C_1+{}^{11}C_4\times {}^{11}C_0=990$

3 **(b)**

We have,

$a=$ Sum of the coefficients in the expansion of $(1-3x+10\,x^2)^n$

$\Rightarrow a=(1-3+10)n=8^n=2^{3n}$

$b=$ Sum of the coefficients in the expansion of $(1+x^2)^n$

$\Rightarrow b=(1+1)^n=2^n$

Clearly, $a=b^3$

4 **(b)**

Let $P(n){:}10^{n-2}\geq 81n$

For $n=4$, $10^2\ngeq 81\times 4$

For $n=5$, $10^3\geq 81\times 5$

Hence, by mathematical induction for $n\geq 5$, the proposition is true.

5 **(c)**

Given that, $T_1=\ {}^nC_0=1$ ...(i)

$T_2=\ {}^nC_1ax=6x$

$\Rightarrow \frac{n!}{(n-1!)}a=6\Rightarrow na=6$ ...(ii)

and $T_3=\ {}^nC_2(ax)^2=6x^2$

$\Rightarrow \frac{n(n-1)}{2}a^2=16$ ...(iii)

Only option (c) is satisfying Eqs. (ii) and (iii)

6 **(a)**

It is given that

$(a+b\,x)^{-2}=\frac{1}{4}-3\,x$

$\Rightarrow a^{-2}\left(1+\frac{b}{a}x\right)^{-2}=\frac{1}{4}-3\,x$

$\Rightarrow a^{-2}\left(1-\frac{2}{a}b\,x\right)=\frac{1}{4}-3\,x \quad \left[\begin{array}{c}\text{Neglecting } x^2 \text{ and}\\ \text{higher powers of } x\end{array}\right]$

$\Rightarrow a^{-2}=\frac{1}{4},\frac{-2\,b}{a^3}=-3$

Now, $a^{-2}=\frac{1}{4}\Rightarrow a^2=4\Rightarrow a=2 \quad [\because a>0]$

Putting $a=2$ in $-\frac{2\,b}{a^3}=-3$, we get $-\frac{2\,b}{8}=-3\Rightarrow b=12$

7 **(d)**

We have,

$T_{r+1}=\ {}^{15}C_r(x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r=\ {}^{15}C_r\,x^{60-7r}\,(-1)^r$

If x^{39} occurs in T_{r+1}, then

$60-7r=39\Rightarrow r=3$

$\therefore$ Coefficient of $x^{39}=\ {}^{15}C_3(-1)^3=-455$

8 **(c)**

$(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + a_3y^3 + \ldots$

On differentiating w. r. t. y, we get

$-m(1-y)^{m-1}(1+y)^n + (1-y)^m\, n(1+y)^{n-1}$

$= a_1 + 2a_2y + 3a_3y^2 + \ldots$ ...(i)

On putting $y = 0$ in Eq. (i), we get

$-m + n = a_1 = 10$ [$\because a_1 = 10$ given] ...(ii)

Again on differentiating Eq. (i) w. r. t. y, we get

$-m[-(m-1)(1-y)^{m-2}(1+y)^n + (1+y)^{m-1}n(1+y)^{n-1}]$

$+n[-m(1-y)^{m-1}(1+y)^{n-1} + (1-y)^m(n-1)(1+y)^{n-2}]$

$= 2a_2 + 6a_3y + \ldots$ ...(iii)

On putting $y = 0$ in Eq. (iii), we get

$-m[-(m-1) + n] + n[-m + (n-1)] = 2a_2 = 20$

$\Rightarrow m(m-1) - mn - mn + n(n-1) = 20$

$\Rightarrow \quad m^2 + n^2 - m - n - 2mn = 20$

$\Rightarrow \quad (m-n)^2 - (m+n) = 20$

$\Rightarrow \quad 100 - (m+n) = 20$

[using Eq. (iii)]

$\Rightarrow \quad m + n = 80$ ...(iv)

On solving Eqs. (ii) and (iv), we get

$m = 35$ and $n = 45$

9 **(c)**

Let a_1,a_2,a_3,a_4 be respectively the coefficients of $(r+1)$th, $(r+2)$th, $(r+3)$th and $(r+4)$th terms in the expansion of $(1+x)^n$. Then ,

$a_1 = {}^nC_r, a_2 = {}^nC_{r+1}, a_3 = {}^nC_{r+2}, a_4 = {}^nC_{r+3}$

Now, $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} + \frac{{}^nC_{r+2}}{{}^nC_{r+2} + {}^nC_{r+3}}$

$= \frac{{}^nC_r}{{}^{n+1}C_{r+1}} + \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}}$ $(\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$

$= \frac{{}^nC_r}{\frac{n+1}{r+1}\,{}^nC_r} + \frac{{}^nC_{r+2}}{\frac{n+1}{r+3}\,{}^nC_{r+2}}$ $\left(\because {}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-2}\right)$

$= \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2(r+2)}{n+1}$

$= 2\frac{{}^nC_{r+1}}{{}^{n+1}C_{r+2}} = 2\frac{{}^nC_{r+1}}{{}^nC_{r+1} + {}^nC_{r+2}}$

$= \frac{2a_2}{a_2 + a_3}$

10 **(d)**

$(a^2 - 6a + 11)^{10} = 1024$

$\Rightarrow \quad (a^2 - 6a + 11)^{10} = 2^{10}$

$\Rightarrow \quad a^2 - 6a + 11 = 2$

$\Rightarrow \quad a^2 - 6a + 9 = 0$

$\Rightarrow \quad (a-3)^2 = 0$

$\Rightarrow \quad a = 3$

11 **(b)**

The general termof $\left(x + \frac{2}{x^2}\right)^n$ is

$$T_{R+1} = {}^nC_R(x)^{n-R}\left(\frac{2}{x^2}\right)^R$$

$$= {}^nC_R x^{n-3R} 2^R$$

For x^{2r} occurs, it means

$n - 3R = 2r$

$\Rightarrow \quad n - 2r = 3R$

Hence, $n - 2r$ is of the form $3k$

12 **(c)**

$2^{3n} - 1 = (2^3)^n - 1$

$= 8^n - 1 = (1+7)^n - 1$

$= 1 + {}^nC_1 7 + {}^nC_2 7^2 + \dots + {}^nC_n 7^n - 1$

$= 7[{}^nC_1 + {}^nC_2 7 + \dots + {}^nC_n 7^{n-1}]$

$\therefore 2^{3n} - 1$ is divisible by 7

113 **(b)**

We have,

$(\alpha - 2 + 1)^{35} = (1-\alpha)^{35}$

$\Rightarrow (\alpha - 1)^{35} = -(\alpha - 1)^{35}$

$\Rightarrow 2(\alpha - 1)^{35} = 0 \Rightarrow \alpha = 1$

14 **(b)**

We have,

$\therefore 3^{\log_3 \sqrt{25^{x-1}+7}} \qquad [\because a^{\log_a n} = n]$

$= \sqrt{25^{x-1} + 7} = \sqrt{(5^{x-1}) + 7} = \sqrt{y^2 + 7}$, where $y = 5^{x-1}$

and,

$3^{-(1/8)\log_3(5^{x-1}+1)}$

$= 3^{\log_3 (5^{x-1}+1)^{-1/8}} = (5^{x-1}+1)^{-1/8} = (y+1)^{-1/8}$

$\therefore \left\{3^{\log_3 \sqrt{25^{x-1}}+7} + 3^{-1/8 \log_3(5^{x-1}+1)}\right\}^{10}$

$= \left[\sqrt{y^2 + 7} + (y+1)^{-1/8}\right]^{10}$

Now,

$T_9 = 180$

$\Rightarrow {}^{10}C_8\left\{\left(\sqrt{y^2+7}\right)^{10-8}\left[(y+1)^{-1/8}\right\}^8 = 180$

$\Rightarrow {}^{10}C_8(y^2+7)(y+1)^{-1} = 180$

$\Rightarrow 45\left(\frac{y^2+7}{y+1}\right) = 180$

$\Rightarrow y^2+7 = 4y+4 \Rightarrow y^2-4y+3=0$

$\Rightarrow y=1, y=3$

$\Rightarrow 5^{x-1}=1$ or, $5^{x-1}=3$

$\Rightarrow 5^x = 5$ or, $5^x = 15$

$\Rightarrow x = 1$ or, $x = \log_5 15$

$\Rightarrow x = \log_5 15$ $\quad [\because x > 1]$

15 **(c)**

The given sigma expansion

$\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m}.2^m$ can be written as

$[(x-3)+2]^{100} = (x-1)^{100} = (x-1)^{100}$

$\therefore$ Coefficient of x^{53} in $(1-x)^{100} = (-1)^{53}\,{}^{100}C_{53} = -\,{}^{100}C_{53}$

16 **(c)**

The coefficient of x in the middle term of expansion of

$(1+\alpha x)^4 = {}^4C_2\alpha^2$

The coefficient of x in the middle term of expansion of

$(1-\alpha x)^6 = {}^6C_3(-\alpha)^3$

Given, ${}^4C_2\alpha^2 = {}^6C_3(-\alpha)^3$

$\Longrightarrow \quad 6\alpha^2 = -20\alpha^3$

$\Longrightarrow \quad \alpha = \frac{-6}{20} = \frac{-3}{10}$

17 **(b)**

The general term in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$ is

$T_{r+1} = {}^{10}C_r\left(\frac{x}{3}\right)^{\frac{10-r}{2}}\left(\frac{3}{2x^2}\right)^r$

$= {}^{10}C_r 3^{\frac{-10+3r}{2}}.2^{-r}.x^{\frac{10-5r}{2}}$

For independent of x,

$\frac{10-5r}{2} = 0 \Rightarrow r = 2$

$\therefore T_3 = {}^{10}C_2 \times \left(\frac{1}{3}\right)^4\left(\frac{3}{2}\right)^2$

$= \frac{10\times 9}{2\times 1} \times \frac{1}{3\times 3\times 2\times 2} = \frac{5}{4}$

18 **(a)**

Given , $(1+x-2x^2)^6 = 1 + a_1x + a_2x^2 + ... + a_{12}x^{12}$ ...(i)

On putting $x = 1$ in Eq. (i), we get

$(1+1-2)^6 = 1 + a_1 + a_2 + ... + a_{12}$

$\Rightarrow (0)^6 = 1 + a_1 + a_2 + ... + a_{12}$ ....(ii)

On putting $x = -1$ in Eq. (i), we get

$(1-1-2)^6 = 1 - a_1 + a_2 - a_3 + ... + a_{12}$

$\Rightarrow (-2)^6 = 1 + a_1 + a_2 - a_3 + ... + a_{12}$ ...(iii)

On adding Eqs. (ii) and (iii) we get

$(-2)^6 = 2(1 + a_2 + a_4 + ... + a_{12})$

$\Rightarrow \frac{64}{2} - 1 = a_2 + a_4 + ... + a_{12}$

$\therefore a_2 + a_4 + ... + a_{12} = 31$

19 **(c)**

Since, $(1+x-3x^2)^{10} = 1 + a_1x + a_2x^2 + ... + a_{20}x^{20}$

On putting $x = -1$, we get

$(1-1-3)^{10} = 1 - a_1 + a_2 - ... + a_{20} = 3^{10}$...(i)

Again putting $x = 1$, we get

$(1+1-3)^{10} = 1 + a_1 + a_2 - ... + a_{20} = 1$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2(1 + a_2 + a_4 + ... + a_{20}) = 3^{10} + 1$

$\Rightarrow a_2 + a_4 + ... + a_{20} = \frac{3^{10}+1}{2} - 1 = \frac{3^{10}-1}{2}$

20 **(b)**

We have,

$(1+x)^{2n} = (a_0 + a_2\,x^2 + a_4\,x^4 + ...) + x(a_1 + a_3\,x^2 + a_5\,x^4 + ...)$

Replacing x by i and $-i$ respectively and multiplying, we get

$(a_0 - a_2 + a_4 ...)^2 + (a_1 - a_3 + a_5 ...)^2 = (1+i)^{2n}(1-i)^{2n}$

$\Rightarrow (a_0 - a_2 + a_4 - ...)^2 + (a_1 - a_3 + a_5 ...)^2 = 2^{2n} = 4^n$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	B	D	D	D	D	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	B	A	A	B	B	B	B	B