DPP
DAILY PRACTICE PROBLEMS

Class : XIth **Solutions** **Subject : Maths**
Date : **DPP No. : 5**

Topic :-Binominal Theorem

1 **(c)**
We have, $49^n + 16n - 1 = (1 + 48)^n + 16n - 1$
$= 1 + {}^nC_1(48) + {}^nC_2(48)^2 + \dots + {}^nC_n(48)^n + 16n - 1$
$= (48n + 16n) + {}^nC_2(48)^2 + {}^nC_3(48)^3 + \dots + {}^nC_n(48)^n$
$= 64n + 8^2[{}^nC_2 \cdot 6^2 + {}^nC_3 \cdot 6^3 \cdot 8 + {}^nC_4 \cdot 6^4 \cdot 8^2 + \dots + {}^nC_n \cdot 6^n \cdot 8^{n-2}]$
Hence, $49^n + 16n - 1$ is divisible by 64

2 **(b)**
We have, $(1 + x)^{50} = \sum_{r=0}^{50} {}^{50}C_r x^r$. (The sum of coefficients of odd powers of x)
$= {}^{50}C_1 + {}^{50}C_3 + \dots + {}^{50}C_{49}$
$= 2^{50-1} = 2^{49}$

4 **(b)**
Given, $\alpha = \frac{5}{2!3} + \frac{5 \cdot 7}{3!3^2} + \frac{5 \cdot 7 \cdot 9}{4!3^2} + \dots$ ...(i)
On comparing
$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$ ...(ii)
With respect to factorial, we get
$$n(n-1)x^2 = \frac{5}{3} \quad \text{....(iii)}$$
$$n(n-1)(n-2)x^3 = \frac{5 \cdot 7}{3^2} \quad \text{...(iv)}$$
and $n(n-1)(n-2)(n-3)x^4 = \frac{5 \cdot 7 \cdot 9}{3^3}$ ..(v)
on dividing Eq. (iv) by (iii) and Eq. (v) by Eq. (iv), we get
$$(n-2)x = \frac{7}{3} \quad \text{...(vi)}$$
and $(n-3)x = 3$ ...(vii)

Again, dividing Eq. (vi) by Eq. (vii), we get

$$\frac{n-2}{n-3} = \frac{7}{9}$$

$\Rightarrow \quad 9n - 18 = 7n - 21$

$\Rightarrow \quad 2n = -3 \Rightarrow n = -\frac{3}{2}$

On putting the value of n in Eq. (vi), we get

$\left(-\frac{3}{2} - 2\right)x = \frac{7}{3} \Rightarrow x = -\frac{2}{3}$

$\therefore$ From Eq. (ii),

$\left(1 - \frac{2}{3}\right)^{-3/2} = 1 + 1 + \frac{5}{2!3} + \frac{5 \cdot 7}{3!3^2} + \ldots$

$\Rightarrow 3^{3/2} - 2 = \frac{5}{2!3} + \frac{5 \cdot 7}{3!3^2} + \ldots$

$\Rightarrow \quad \alpha = 3^{3/2} - 2 \quad$ [from Eq. (i)]

Now, $\alpha^2 + 4\alpha = (3^{3/2} - 2)^2 + 4(3^{3/2} - 2)$

$= 27 + 4 - 4 \cdot 3^{3/2} + 4 \cdot 3^{3/2} - 8$

$= 23$

5 **(d)**

$\frac{1 + 2x}{(1 - 2x)^2} = (1 + 2x)(1 - 2x)^{-2}$

$= (1 + 2x)\left(1 + \frac{2}{1!}(2x) + \frac{2 \cdot 3}{2!}(2x)^2 + \ldots + \frac{2 \cdot 3 \ldots r}{(r-1)!}(2x)^r + \frac{2 \cdot 3 \cdot 4 \ldots (r+1)(2x)^r}{r!}\right)$

The coefficient of $x^r = 2\frac{r!}{(r-1)!}2^{r-1} + \frac{(r+1)!}{r!}2^r$

$= r2^r + (r+1)r^2$

$= 2^r(2r + 1)$

6 **(d)**

We have,

$\{(1 + x)(1 + y)(x + y)\}^n = (1 + x)^n(1 + y)^n(x + y)^n$

$\therefore$ Coefficient of $x^n\, y^n$ in $\{(1 + x)(1 + y)(x + y)\}^n = \sum_{r=0}^{n} \left({}^nC_r\right)^3$

7 **(d)**

We have,

$(1 + x + x^2)^n = C_0 + C_1x + C_2\, x^2 + \ldots + C_{2n}x^{2\,n}$

Replacing x by $-\frac{1}{x}$, we get

$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n = C_0 - C_1\frac{1}{x} + C_2\frac{1}{x^2} + \ldots + C_{2n}\frac{1}{x^{2\,n}}$

Now,

$C_0\, C_1 - C_1\, C_2 + C_2\, C_3 - \ldots$

$= \text{Coeff. of } x \text{ in } \{C_0 + C_1\, x + C_2\, x^2 + \ldots\}\left\{C_0 - C_1 \frac{1}{x} + C_2 \frac{1}{x^2} - \ldots\right\}$

$= \text{Coeff. of } x \text{ in } (1 + x + x^2)^n \left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n$

$= \text{Coeff. of } x^{2\,n+1} \text{ in } (1 + x + x^2)^n (x^2 - x + 1)^n$

$= \text{Coeff. of } x^{2\,n+1} \text{ in } \left[(1 + x^2)^2 - x^2\right]^2$

$= \text{Coeff. of } x^{2\,n+1} \text{ in } [1 + x^2 + x^4]^n = 0$

8 **(d)**

$\because a, b, c$ are in AP

$\Rightarrow 2b = a + c$

$\Rightarrow a - 2b + c = 0$

On putting $x = 1$, we get

Required sum $= (1 + (a - 2b + c)^2)^{1973} = (1 + 0)^{1973} = 1$

9 **(a)**

We have , $T_2 = 14a^{5/2}$

$\Longrightarrow {}^nC_1 (a^{1/13})^{n-1} (a^{3/2})^1 = 14a^{5/2}$

$\Longrightarrow na^{\frac{n-1}{13} + \frac{3}{2}} = 14a^{5/2}$

$\Longrightarrow \qquad n = 14$

$\therefore \frac{{}^nC_3}{{}^nC_2} = \frac{{}^{14}C_3}{{}^{14}C_2} = 4$

10 **(b)**

For $n > 1$, we have

$(1 + x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + {}^nC_3\, x^3 + \ldots + {}^nC_n\, x^n$

$\Rightarrow (1 + x)^n = 1 + nx + ({}^nC_2\, x^2 + {}^nC_3\, x^3 + \ldots + {}^nC_n\, x^n)$

$\Rightarrow (1 + x)^n - 1 - nx = x^2({}^nC_2 + {}^nC_3\, x + {}^nC_4\, x^2 + \ldots + {}^nC_n\, x^{n-2})$

Clearly, RHS is divisible by x^2 and x. So, LHS is also divisible by x as well as x^2

11 **(c)**

Let T_{r+1} be the $(r + 1)^{th}$ terms in the expansion of $\left(\frac{x^2}{a} - \frac{a}{x}\right)^{12}$. Then,

$T_{r+1} = {}^{12}C_r \left(\frac{x^2}{a}\right)^{12-r} \left(-\frac{a}{x}\right)^r = {}^{12}C_r\, x^{24-3r} (-1)^r\, a^{2r-12}$

For the coefficient of $x^6\, y^{-2}$, we must have

$24 - 3r = 6$ and $2r - 12 = -2$

These two equations are inconsistent

Hence, there is no term containing $x^6\, a^{-2}$

So, its coefficient is 0

12 **(d)**

$\because\ I + f + f' = (5+2\sqrt{6})^n + (5-2\sqrt{6})^n$

$= 2k$(even integer)

$\therefore\ f + f' = 1$

Now, $(I+f)f' = (5+2\sqrt{6})^n(5-2\sqrt{6})^n = (1)^n = 1$

$\Rightarrow (I+f)(1-f) = 1$

or $I = \frac{1}{(1-f)} - f$

13 **(b)**

Given equation can be rewritten as

$E = a[{}^nC_0 - {}^nC_1 + {}^nC_2 - ... + (-1)^n\, {}^nC_n] + [{}^nC_1 - (2)({}^nC_2) + (3)({}^nC_3) - ... + (-1)^n(n)$ (

$\Rightarrow E = 0 + 0 = 0$ (by properties)

14 **(a)**

Coefficient of x^{r-1} in

$(1+x)^n + (1+x)^{n+1} + ... + (1+x)^{n+k}$

$= {}^nC_{r-1} + {}^{n+1}C_{r-1} + ... + {}^{n+k}C_{r-1}$

$= {}^nC_r + {}^nC_{r-1} + {}^{n+1}C_{r-1} + ... + {}^{n+k}C_{r-1} - {}^nC_r$

$= {}^{n+k+1}C_r - {}^nC_r$

Now, $\sum_{r=0}^{n+k+1}(-1)^r a_r = \sum_{r=0}^{n+k+1}(-1)^r\, {}^{n+k-1}C_r - \sum_{r=0}^{n+k+1}(-1)^r\, {}^nC_r = 0$

15 **(a)**

We have, $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ...$

If x is replace by $-\left(1-\frac{1}{x}\right)$ and n is $-n$, then expression

becomes $\left[1-\left(1-\frac{1}{x}\right)\right]^{-n}$

$= 1 + (-n)\left[-\left(1-\frac{1}{x}\right)\right] + \frac{(-n)(-n-1)}{2!}\left[-\left(1-\frac{1}{x}\right)\right]^2 + ...$

$\Rightarrow x^n = 1 + n\left(1-\frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2 + ...$

16 **(b)**

Given expansion is $(x+a)^n$

On replacing a by ai and $-ai$ respectively, we get

$(x+ai)^n = (T_0 - T_2 + T_4 - ...) + i(T_1 - T_3 + T_5 - ...)$ ...(i)

and $(x-ai)^n = (T_0 - T_2 + T_4 - ...) + i(T_1 - T_3 + T_5 - ...)$ ...(ii)

On multiplying Eqs. (ii) and (i), we get required result

$(x^2+a^2)^n = (T_0 - T_2 + T_4 - ...)^2 + (T_1 - T_3 + T_5 - ...)^2$

17 **(b)**

Given coefficient of $(2x+1)$th term = coefficient of $(r+2)$th term

$\Rightarrow {}^{43}C_{2r} = {}^{43}C_{r+1}$

$\Rightarrow 2r + (r+1) = 43$ or $2r = r + 1$

$\Rightarrow r = 14$ or $r = 1$

18 **(b)**

We have,

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_n x^n$ ...(i)

and $\left(1+\frac{1}{x}\right)^n = C_0 + C_1 \frac{1}{x} + C_2 \left(\frac{1}{x}\right)^2 + ... + C_n \left(\frac{1}{x}\right)^n$ ...(ii)

On multiplying Eqs. (i) and (ii) and taking the coefficient of constant terms in right hand side

$= C_0^2 + C_1^2 + C_2^2 + ... + C_n^2$

In right hand side $(1+x)^n\left(1+\frac{1}{x}\right)^n$ or in $\frac{1}{x^n}(1+x)^{2n}$ or term containing x^n in $(1+x)^{2n}$.

Clearly the coefficient of x^n in $(1+x)^{2n}$ is equal to ${}^{2n}C_n = \frac{(2n)!}{n!n!}$

19 **(b)**

We have,

$$\frac{C_k}{C_{k-1}} = \frac{{}^nC_k}{{}^nC_{k-1}} = \frac{n-k+1}{k}$$

$$\therefore \sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2$$

$$= \sum_{k=1}^{n} k^3 \frac{(n-k+1)^2}{k^2} = \sum_{k=1}^{n} k(n-k+1)^2$$

$$= (n+1)^2 \left(\sum_{k=1}^{n} k\right) - 2(n+1)\left(\sum_{k=1}^{n} k^2\right) + \left(\sum_{k=1}^{n} k^3\right)$$

$$= (n+1)^2 \frac{n(n+1)}{2} - \frac{2(n+1)n(n+1)(2n+1)}{6} + \left\{\frac{n(n+1)}{2}\right\}^2$$

$$= \frac{n(n+1)^2}{12}\{6(n+1) - 4(2n+1) + 3n\}$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

20 **(b)**

Let

$S = 1 \times 2 \times 3 \times 4 + 2 \times 3 \times 4 \times 5 + 3 \times 4 \times 5 \times 6 + ... + n(n+1)(n+2)(n+3)$

$$\Rightarrow S = \sum_{r=1}^{n} r(r+1)(r+2)(r+3)$$

$$\Rightarrow S = \sum_{r=1}^{n} \frac{(r+3)!}{(r-1)!}$$

$$\Rightarrow S = 4! \sum_{r=1}^{n} \frac{(r+3)!}{(r-1)!4!}$$

$$\Rightarrow S = 4! \sum_{r=1}^{n} \frac{(r+3)!}{(r-1)!4!}$$

$$\Rightarrow S = 4! \sum_{r=1}^{n} {}^{r+3}C_4$$

$$\Rightarrow S = 4! \sum_{r=12}^{n} \text{Coefficient of } x^4 \text{in } (1+x)^{r+3}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^4 \text{ in } \sum_{r=1}^{n} (1+x)^{r+3}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^4 \text{ in } (1+x)^4 \left\{ \frac{(1+x)^n - 1}{(1+x) - 1} \right\}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^5 \text{ in } \{(1+x)^{\mathrm{n}+4} - (1+x)^4\}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^5 \text{ in } (1+x)^{n+4}$$

$$\Rightarrow S = 4! \times {}^{n+4}C_5 = \frac{1}{5} n(n+1)(n+2)(n+3)(n+4)$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	B	D	D	D	D	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	B	A	A	B	B	B	B	B