DPP

DAILY PRACTICE PROBLEMS

Class : XIth | Solutions | Subject : Maths

Date : | | DPP No. : 4

Topic :-Binominal Theorem

1 **(c)**

We have, $32^{32}=(2^5)^{32}=2^{160}=(3-1)^{160}$

$={}^{160}C_0 3^{160}-{}^{160}C_1\cdot 3^{159}+\ldots-{}^{160}C_{159}\cdot 3+{}^{160}C_{160}3^{\circ}$

$=3m+1$, where $m\in N$

$32^{(32)^{(32)}}=(32)^{3m+1}$

$=(2^5)^{3m+1}=2^{15m+5}$

$=2^{3(5m+1)}\cdot 2^2=(2^3)^{5m+1}\cdot 2^2$

$=(7+1)^{5m=1}\times 4$

$=\{{}^{5m+1}C_0 7^{5m+1}+{}^{5m+1}C_1 7^{5m}+\ldots+{}^{5m+1}C_{5\text{m}+1}7+{}^{5m+1}C_{5\text{m}+1}.7^0\}\times 4$

$=(7n+1)\times 4,$

where $n={}^{5m+1}C_0 7^{5m+1}+\ldots+{}^{5m+1}C_{5\text{m}}\cdot 7$

$28n+4$

Thus, when $32^{(32)^{(32)}}$ is divided by 7, the remainder is 4

2 **(c)**

We have,

$$\left[2^{\log_2\sqrt{9^{x-1}+7}}+\frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right]^7$$

$$=\left[\sqrt{9^{x-1}+7}+\frac{1}{(3^{x-1}+1)^{1/5}}\right]^7$$

$$\therefore T_6={}^7C_5\left(\sqrt{9^{x-1}+7}\right)^{7-5}\left[\frac{1}{(3^{x-1}+1)^{1/5}}\right]^5$$

$$={}^7C_5(9^{x-1}+7)\frac{1}{(3^{x-1}+1)}$$

$$\Rightarrow 84={}^7C_5\frac{(9^{x-1}+7)}{(3^{x-1}+1)}$$

$$\Rightarrow 9^{x-1}+7=4(3^{x-1}+1)$$

$\Rightarrow \dfrac{3^{2x}}{9} + 7 = 4\left(\dfrac{3^x}{3} + 1\right)$

$\Rightarrow 3^{2x} - 12(3^x) + 27 = 0$

$\Rightarrow y^2 - 12y + 27 = 0$ (put $y = 3^x$)

$\Rightarrow (y-3)(y-9) = 0$

$\Rightarrow y = 3, 9 \Rightarrow 3^x = 3, 9 \Rightarrow x = 1, 2$

3 **(d)**

Here, $P(1) = 2$ and from the equation

$P(k) = k(k+1) + 2$

$\Longrightarrow \quad P(1) = 4$

So, $P(1)$ is not true

Hence, mathematical induction is not applicable.

4 **(b)**

We have,

$(1 + 2\,x + x^2)^{20} = \{(1+x)^2\}^{20} = (1+x)^{40}$

Clearly, $(1+x)^{40}$ contains 41 terms

Hence, $(1 + 2x + x^2)^{20}$ contains 41 terms

5 **(d)**

The series of binomial coefficient is

$^{15}C_8$

$\underset{\text{decreasing value}}{\xleftarrow{^{15}C_0, ^{15}C_1, ^{15}C_2, \dots, ^{15}C_7}} \quad \downarrow \quad \underset{\text{decreasing value}}{\xrightarrow{^{15}C_9, \dots, ^{15}C_9, ^{15}C_{15}}}$

From the above discussion, we can say that decreasing series is $^{15}C_7,\ ^{15}C_6,\ ^{15}C_5$.

6 **(c)**

For $n = 1$, $10^n + 3 \cdot 4^{n+2} + 5$

$= 10 + 3 \cdot 4^3 + 5 = 207$ This is divisible by 9.

$\therefore$ By induction, the result is divisible by 9.

7 **(d)**

$\dfrac{^8C_0}{6} - {}^8C_1 + {}^8C_2 \cdot 6 - {}^8C_3 \cdot 6^2 + \dots + {}^8C_8 \cdot 6^7$

$= \dfrac{1}{6}\left[{}^8C_0 - 6\,{}^8C_1 + 6^2\,{}^8C_2 - 6^3\,{}^8C_3 + \dots + 6^8\,{}^8C_8\right]$

$= \dfrac{1}{6}[(1-6)^8] = \dfrac{5^8}{6}$

8 **(a)**

In the expansion of $(1+x)^n$,it is given that ${}^nC_1, {}^nC_2, {}^nC_3$ are in AP

$\Rightarrow 2\ ^nC_2 = \ ^nC_1 + \ ^nC_3$

$\Rightarrow 2 \cdot \frac{n(n-1)}{1 \cdot 2} = \frac{n}{1} + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}$

$\Rightarrow 6(n-1) = 6 + (n-2)(n-1)$

$\Rightarrow 6n - 6 = 6 + n^2 - 3n + 2$

$\Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow (n-2)(n-7) = 0$

$\Rightarrow n = 2, 7$

But $n = 2$ is not acceptable because, when $n = 2$, there are only three terms in the expansion of $(1+x)^2$

$\therefore\ n = 7$

10 **(a)**

$(1+x)^n = \ ^nC_0 + \ ^nC_1x + \ ^nC_2x^2 + ... + \ ^nC_nx^n$ ...(i)

On differentiating both sides w. r. t. x, we get

$n(1+x)^{n-1} = \ ^nC_1 + 2\ ^nC_2x + ... + n\ ^nC_nx^{n-1}$ ...(ii)

On putting $x = 1$ in Eq. (ii), we get

$n(2)^{n-1} = \ ^nC_1 + 2\ ^nC_2 + ... + n\ ^nC_n$ ...(iii)

On putting $x = -1$ in Eq. (ii) we get

$0 = \ ^nC_1 - 2\ ^nC_2 + 3\ ^nC_3 - ...(-1)^{n-1} \cdot \ ^nC_n$...(iv)

On adding Eqs. (iii) and (iv), we get

$n2^{n-1} = 2(\ ^nC_1 + 3\ ^nC_3 + ..)$

$\Longrightarrow \ ^nC_1 + 3\ ^nC_3 + 5\ ^nC_5 + ... = \frac{n}{2} \cdot 2^{n-1} = n2^{n-2}$

11 **(a)**

Given expression is $\left(x + x^{\log 10^x}\right)^5$

$\therefore T_3 = \ ^5C_2 \cdot x^3\left(x^{\log 10^x}\right)^2 = 10^6$ (given)

Put $x = 10$, then $10^4.10^2 = 10^6$ is satisfied. Hence, $x = 10$.

12 **(c)**

Given, $^nC_0 - \frac{1}{2}\ ^nC_1 + \frac{1}{3}\ ^nC_2 - ... + (-1)^{\text{n}}\frac{^nC_{\text{n}}}{n+1}$

At $n = 1$, $^1C_0 - \frac{1}{2}\ ^1C_1 = 1 - \frac{1}{2} = \frac{1}{2}$

At $n = 2$, $^2C_0 - \frac{1}{2}\ ^2C_1 + \frac{1}{3}\ ^2C_2 = 1 - 1 + \frac{1}{3} = \frac{1}{3}$

Which is satisfied only in option (c)

13 **(b)**

$8^{2n}-(62)^{2n+1}=(1+63)^n-(63-1)^{2n+1}$

$=(1+63)^n+(1-63)^{2n+1}$

$=\left(1+{}^nC_1 63+{}^nC_2(63)^2+\ldots+(63)^n\right)$

$+(1-{}^{(2n+1)}C_1 63+{}^{(2n+1)}C_2(63)^2+\ldots+(-1)(63)^{(2n+1)})$

$=2+63[{}^nC_1+{}^nC_2(63)+\ldots+(63)^{n-1}{}_-{}^{(2n+1)}C_1$

$+{}^{(2n+1)}C_2(63)-\ldots-(63)^{(2n)}]$

$\therefore$ Remainder is 2.

14 **(a)**

We have,

$T_{r+1}={}^{20}C_r\times 4^{\frac{20-r}{3}}\times 6^{-\frac{r}{4}}$

$\Rightarrow T_{r+1}={}^{20}C_r 2^{\frac{160-11r}{12}} 3^{-\frac{r}{4}}, r=0,1,2,\ldots,20$

This term will be rational if $\frac{160-11\,r}{12}$ and $\frac{r}{4}$ are rational numbers

Now, $\frac{r}{4}$ is rational if $r=0,4,8,12,16,20$

Clearly, $\frac{160-11r}{12}$ is rational for $=8,16$ and 20

Hence, there are only 3 rational terms

15 **(c)**

We have,

$\left(x^2+1+\frac{1}{x^2}\right)^n=\frac{1}{x^{2n}}(1+x^2+x^4)^n=\frac{1}{t^n}(1+t+t^2)^n$, where $t=x^2$

Clearly, $(1+t+t^2)^n$ is a polynomial of degree $2n$

Hence, there are $(2n+1)$ terms

16 **(b)**

$(19)^{2005}+(11)^{2005}-(9)^{2005}$

$=(10+9)^{2005}+(10+1)^{2005}-(9)^{2005}$

$=(9^{2005}+{}^{2005}C_1(9)^{2004}\times 10+\ldots..)+({}^{2005}C_0+{}^{2005}C_1 10+\ldots)-(9)^{2005}$

$=({}^{2005}C_1 9^{2005}\times 10+$ multipale of 10)+(1+multipal of 10)

$\therefore$ Unit digit=1

17 **(b)**

In the expansion of $(x+2y)^6$,

$\left(\frac{6}{2}+1\right)$th term is the middle term.

$\therefore\quad T_4=T_{3+1}={}^6C_3 x^{6-3}(2y)^3$

$=\quad 8({}^6C_3)(xy)^3$

$\therefore$ Coefficient of middle term

$=8({}^6C_3)$

18 **(c)**

General terms, $T_{r+1} = (1)^r \ {}^{15}C_r \ \ (x^4)^{15-r} \cdot \left(\frac{1}{x^3}\right)^r$

$$= (-1)^r \, {}^{15}C_r \ \cdot x^{60-7r}$$

For the coefficient of x^{-17}, put $60 - 7r = -17$

$\Rightarrow \qquad 60 + 17 = 7r \Rightarrow r = 11$

Now, coefficient of $x^{-17} = (-1)^{11} {}^{15}C_{11} \ = -\,{}^{15}C_{11}$

19 **(b)**

$$\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{2\left(1-\frac{x}{4}\right)^{1/2}}$$

$$= \frac{\left[\left[1 + \frac{1}{2}(-3x) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}(-3x)^2 + \ldots\right] + \left[1 + \frac{5}{3}(-x) + \frac{5}{3}\cdot\frac{2}{3}\cdot\frac{1}{2}(-x)^2 + \ldots\right]\right]}{2\left[1 + \frac{1}{2}\left(-\frac{x}{4}\right) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}\left(-\frac{x}{4}\right)^2 + \ldots\right]}$$

$$= \frac{2\left[1 - \frac{19}{12}x - \frac{41}{144}x^2 - \ldots\right]}{2\left[1 - \frac{x}{8} - \frac{1}{128}x^2 - \ldots\right]}$$

$$= \left[1 - \frac{19}{12}x - \frac{41}{144}x^2 - \ldots\right]\left[1 - \frac{x}{8} - \frac{1}{128}x^2 \ldots\right]^{-1}$$

$$= 1 - \frac{35}{24}x + \ldots$$

On neglecting higher powers of x, we get

$$a + bx = 1 - \frac{35}{24}x$$

$$\Rightarrow a = 1, b = -\frac{35}{24}$$

20 **(b)**

${}^{18}C_{15} + 2\left({}^{18}C_{16}\right) + {}^{17}C_{16} + 1 = {}^{n}C_3$

$\Rightarrow {}^{18}C_{15} + {}^{18}C_{16} + {}^{18}C_{16} + {}^{17}C_{16} + {}^{17}C_{17} = {}^{n}C_3$

$\Rightarrow \qquad {}^{19}C_{16} + {}^{18}C_{16} + {}^{18}C_{17} = {}^{n}C_3$

$\Rightarrow \qquad {}^{19}C_{16} + {}^{19}C_{17} = {}^{n}C_3$

$\Rightarrow {}^{20}C_{17} = {}^{n}C_3 \Rightarrow {}^{20}C_3 = {}^{n}C_3 \Rightarrow n = 20$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	D	B	D	C	D	A	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	A	C	B	B	C	B	B