DPP

DAILY PRACTICE PROBLEMS

Class : XIth **Solutions** **Subject : Maths**

Date : **DPP No. : 3**

Topic :-Binominal Theorem

1 **(a)**

Since, $x(1+x)^n = xC_0 + C_1x^2 + C_2x^3 + ... + C_nx^{n+1}$

On differentiating w.r.t. x, we get

$(1+x)^n + nx(1+x)^{n-1} = C_0 + 2C_1x + 3C_2x^2 + ... + (n+1)C_nx^n$

Put $x = 1$, we get

$C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n = 2^n + n2^{n-1}$

$= 2^{n-1}(n+2)$

2 **(c)**

Let T_{r+1} denote the $(r+1)^{th}$ term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$. Then,

$T_{r+1} = {}^nC_r\, x^{3n-5r}(-1)^r$

For this term to contain x^5, we must have

$3n - 5r = 5 \Rightarrow r = \dfrac{3n-5}{5}$

$\therefore$ Coefficient of $x^5 = {}^nC_{\frac{3n-5}{5}}(-1)^{\frac{3n-5}{5}}$

Similarly,

Coefficient of $x^{10} = {}^nC_{\frac{3n-10}{5}}(-1)^{\frac{3n-10}{5}}$

Now,

Coefficient of $x^5 +$ Coefficient of $x^{10} = 0$

$\Rightarrow {}^nC_{\frac{3n-5}{5}}(-1)^{\frac{3n-5}{5}} + {}^nC_{\frac{3n-10}{5}}(-1)^{\frac{3n-10}{5}} = 0$

$\Rightarrow {}^nC_{\frac{3n-5}{5}} = {}^nC_{\frac{3n-10}{5}}$

$\Rightarrow \dfrac{3n-5}{5} + \dfrac{3n-10}{5} = n$

$\Rightarrow 6n - 15 = 5n$

$\Rightarrow n = 15$

3 **(b)**

$(1+x+x^2+x^3)^6 = (1+x)^6(1+x^2)^6$

$= \left({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + {}^6C_3x^3 + {}^6C_4x^4 + {}^6C_5x^5 + {}^6C_6x^6\right) \times \left({}^6C_0 + {}^6C_1x^2 + {}^6C_2x^4 + {}^6C_3x^6 + {}^6C_4x^8 + {}^6C_5x^{10} + {}^6C_6x^{12}\right)$

$\therefore$ Coefficient of x^{14} in $(1+x+x^2+x^3)^6$

$= {}^6C_2 \cdot {}^6C_6 + {}^6C_4 \cdot {}^6C_5 + {}^6C_6 \cdot {}^6C_4$

$= 15 + 90 + 15 = 120$

4 **(c)**

The 14th term from the end in the expansion of $(\sqrt{x} - \sqrt{y})^{17}$ is the $(18-14+1)^{\text{th}}$ i.e. 5th term from the beginning and is given by

${}^{17}C_4(\sqrt{x})^{13}(-\sqrt{y})^4 = {}^{17}C_4\, x^{13/2}y^2$

5 **(d)**

Put $x = 1$, we get

$(1+2+3+\ldots+n)^2 = \sum n^3$

6 **(d)**

We have,

$(1+x+x^2)^n = a_0 + a_1x + a_2x^3 + \ldots. + a_{2n}x^{2n}$

On differentiating both sides, we get

$n(1+x+x^2)^{n-1}(1+2x) = a_1 + 2a_2x + 3a_3x^2$
$+\ldots + 2na_{2n}x^{2n-1}$

Now, on putting $x = 1$, we get

$n(3)^{n-1} \cdot (3) = a_1 + 2a_2 + 3a_3 + \ldots + 2na_{2n}$

$\Rightarrow a_1 + 2a_2 + 3a_3 + \ldots. + 2na_{2n} = n \cdot 3^n$

7 **(c)**

There are total $(n+1)$ factors, let $P(x) = 0$

Let $\left(x + {}^nC_0\right)\left(x + 3\,{}^nC_1\right)\left(x + 5\,{}^nC_2\right)\ldots\left[x + (2n+1)\,{}^nC_n\right]$

$= a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$

Clearly, $a_n = 1$

and roots of the equation $P(x) = 0$ are $-{}^nC_0, -3\,{}^nC_1, \ldots$

Sum of roots $= -a_{n-1}/a_n$

$= -{}^nC_0 - 3\,{}^nC_1 - 5\,{}^nC_2 \ldots$

$\Rightarrow a_{n-1} = (n+1)2^n$

8 **(b)**

${}^{n-2}C_r + 2 \cdot {}^{n-2}C_{r-1} + {}^{n-2}C_{r-2}$

$= \left({}^{n-2}C_r + {}^{n-2}C_{r-1} \right) + \left({}^{n-2}C_{r-1} + {}^{n-2}C_{r-2} \right)$
$= {}^{n-1}C_r + {}^{n-1}C_{r-1} \ \left(\therefore {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r \right)$
$= {}^{n}C_r$

9 **(d)**

$\because \dfrac{1}{(x-1)^2(x-2)} = \dfrac{1}{-2(1-x)^2\left(1-\frac{x}{2}\right)}$

$= -\dfrac{1}{2}\left[(1-x)^{-2}\left(1-\dfrac{x}{2}\right)^{-1}\right]$

$= -\dfrac{1}{2}\left[(1+2x+\dots)\left(1+\dfrac{x}{2}+\dots\right)\right]$

$\therefore$ Coefficient of constant term is $-\frac{1}{2}$

10 **(b)**

In the expansion of $\left(x^2 + \dfrac{a}{x}\right)^5$, the general term is

$T_{r+1} = {}^{5}C_r (x^2)^{5-r}\left(\dfrac{a}{x}\right)^r = {}^{5}C_r \cdot a^r \cdot x^{10-3r}$

For the coefficient of x, put

$10 - 3r = 1 \Rightarrow r = 3$

$\therefore$ Coefficient of $x = {}^{5}C_3 a^3 = 10a^3$

12 **(b)**

Coefficient of x^r in the expansion of $(1+x)^{10}$ is ${}^{10}C_r$ and it is maximum for $r = \frac{10}{2} = 5$

Hence, Greatest coefficient $= {}^{10}C_5 = \frac{10\,!}{(5\,!)^2}$

13 **(c)**

Given expansion is $\left(\frac{a}{x} + bx\right)^{12}$

$\therefore$ General term, $T_{r+1} = {}^{12}C_r \left(\frac{a}{x}\right)^{12-r}(bx)^r$

$= {}^{12}C_r (a)^{12-r} b^r x^{-12+2r}$

For coefficient of x^{-10}, put

$-12 + 2r = -10$

$\Rightarrow r = 1$

Now, the coefficient of x^{-10} is

${}^{12}C_1 (a)^{11} (b)^1 = 12a^{11}b$

15 **(a)**

We have,

$$T_{r+1} = {}^{21}C_r \left(\sqrt[3]{\frac{a}{\sqrt{b}}}\right)^{21-r} \left(\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^{r}$$

$$\Rightarrow T_{r+1} = {}^{21}C_r\, a^{7-\frac{r}{2}} b^{\frac{2}{3}r-\frac{7}{2}}$$

Since the powers of a and b are the same

$$\therefore 7 - \frac{r}{2} = \frac{2}{3}r - \frac{7}{2} \Rightarrow r = 9$$

16 **(b)**

$$(1-x)^{-4} = 1.x^0 + 4x^1 + \frac{4.5}{2}x^2 + \dots$$

$$= \left[\frac{1.2.3}{6}x^0 + \frac{2.3.4}{6}x + \frac{3.4.5}{6}x^2 + \frac{4.5.6}{6}x^3 + \dots + \frac{(r+1)(r+2)(r+3)}{6}x^r + \dots\right]$$

Therefore, $T_{\mathrm{r}+1} = \frac{(r+1)(r+2)(r+3)}{6}x^r$

17 **(a)**

We have,

$$y = \frac{1}{3} + \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \dots$$

$$\Rightarrow y + 1 = 1 + \frac{1}{3} + \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \dots$$

Comparing the series on RHS with

$1 + n\,x + \frac{n(n-1)}{2\,!}x^2 + \dots$, we get

$$n\,x = \frac{1}{3} \qquad \dots\text{(i)}$$

and, $\frac{n(n-1)}{2}x^2 = \frac{1}{6} \quad \dots$(ii)

Dividing (ii) by square of (i), we get

$$\frac{n-1}{2\,n} = \frac{9}{6} \Rightarrow n = -\frac{1}{2}$$

$$\Rightarrow x = -\frac{2}{3} \qquad \left[\text{putting } n = -\frac{1}{2} \text{ in (i)}\right]$$

$$\therefore y + 1 = (1+x)^n$$

$$\Rightarrow y + 1 = \left(1 - \frac{2}{3}\right)^{-1/2}$$

$$\Rightarrow y + 1 = \left(\frac{1}{3}\right)^{-1/2}$$

$$\Rightarrow (y+1)^2 = \left(\frac{1}{3}\right)^{-1} \Rightarrow y^2 + 2\,y + 1 = 3 \Rightarrow y^2 + 2\,y = 2$$

18 **(b)**

$S(k) = 1 + 3 + 5 .... + (2k - 1) = 3 + k^2$

Put $k = 1$ in both sides, we get

LHS $= 1$ and RHS $= 3 + 1 = 4$

$\Rightarrow$ LHS $\neq$ RHS

Put $(k + 1)$ in both sides in the place of k, we get

LHS $= 1 + 3 + 5 .... + (2k - 1) + (2k + 1)$

RHS $= 3 + (k + 1)^2 = 3 + k^2 + 2k + 1$

Let LHS = RHS

Then, $1 + 3 + 5 .... + (2k - 1) + (2k + 1)$

$= 3 + k^2 + 2k + 1$

$\Rightarrow 1 + 3 + 5 + ... + (2k - 1) = 3 + k^2$

If $S(k)$ is true, then $S(k + 1)$ is also true.

Hence, $S(k) \Rightarrow S(k + 1)$

19 **(b)**

The general term in the expansion of $(5^{1/6} + 2^{1/8})^{100}$ is given by

$T_{r+1} = {}^{100}C_r (5^{1/6})^{100-r} (2^{1/8})^r$

As 5 and 2 are relatively prime, T_{r+1} will be rational, if

$\frac{100 - r}{6}$ and $\frac{r}{8}$ are both integers *ie*, if $100 - r$ is a multiple of 6 and r is a multiple of 8. As $0 \leq r \leq 100$, multiples of 8 upto 100 and corresponding value of $100 - r$ are

$r = 0, 8, 16, 24, ..., 88, 96$

ie, $100 - r = 100, 92, 84, 76, ..., 12, 4$

Out of $100 - r$, multiples of 6 are 84, 60, 36, 12

$\therefore$ There are four rational terms

Hence, number of irrational terms is $101 - 4 = 97$

20 **(b)**

We have,

$$T_r = {}^{10}C_{r-1} \left(\frac{x}{3}\right)^{10-r+1} \left(-\frac{2}{x^2}\right)^{r-1}$$

$$\Rightarrow T_r = {}^{10}C_{r-1} \left(\frac{1}{3}\right)^{11-r} (-2)^{r-1}\, x^{13-3r}$$

For this term to contain x^4, we must have

$13 - 2r = 4 \Rightarrow r = 3$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	C	D	D	C	B	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	C	A	A	B	A	B	B	B