DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : Maths
DPP No. : 2

Topic :-Binominal Theorem

1 **(c)**

Let $S = 1 + \frac{2 \cdot 1}{3 \cdot 2} + \frac{2 \cdot 5}{3 \cdot 6}\left(\frac{1}{2}\right)^2 + \frac{2 \cdot 5 \cdot 8}{3 \cdot 6 \cdot 9}\left(\frac{1}{2}\right)^3 + ...$

$$= 1 + \frac{\frac{2}{3}}{1}\left(\frac{1}{2}\right) + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}{2!}\left(\frac{1}{2}\right)^2 + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)\left(\frac{8}{3}\right)}{3!}\left(\frac{1}{2}\right)^3 + ...$$

$$= \left(1 - \frac{1}{2}\right)^{-2/3} = \left(\frac{1}{2}\right)^{-2/3} = 2^{2/3} = 4^{1/3}$$

$$\left[\because (1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + ...\right]$$

2 **(a)**

rth term in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$ is

$$T_r = {}^{15}C_{r-1}(3x)^{15-r+1}\left(\frac{-2}{x^2}\right)^{r-1}$$

$$= {}^{15}C_{r-1}(3)^{15-r+1}(-2)^{r-1}(x)^{15-3r+3}$$

For the term independent of x, put

$15 - 3r + 3 = 0 \Rightarrow r = 6$

3 **(a)**

We have, $\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)(C_r + C_s)$

$$= \sum_{r=0}^{n}\sum_{s=0}^{n}(rC_r + rC_s + sC_r + sC_s)$$

$$= \sum_{r=0}^{n}\left[\sum_{s=0}^{n} rC_r + r\sum_{s=0}^{n} C_s + C_s\sum_{s=0}^{n} s + \sum_{s=0}^{n} sC_s\right]$$

$$= \sum_{r=0}^{n}\left[(n+1)r \cdot C_r + r2^n + \frac{n(n+1)}{2}C_r + n \cdot 2^{n-1}\right]$$

$$= (n+1)n \cdot 2^{n-1} + (2^n)\frac{n(n+1)}{2} + \frac{n(n+1)}{2}2^n + n2^{n-1}(n+1)$$
$$= n(n+1)2^n + n(n+1)2^n$$
$$= 2n(n+1)2^n \text{ ...(i)}$$
Also, $\sum_{\mathrm{r}=0}^{n} \sum_{s=0}^{n} (\mathrm{r}+\mathrm{s})(C_\mathrm{r} + C_s)$
$$= \sum_{r=0}^{n} 4rC_r + 2 \sum_{0 \le r < s \le n} \sum (r+s)(C_r + C_s)$$
$$\therefore\ 2n(n+1)2^n = 4n \cdot 2^{n-1} + 2 \sum_{0 \le r < s \le n} \sum (r+s)(C_r + C_s)$$
$$\Rightarrow \sum_{0 \le r < s \le n} \sum (r+s)(C_r + C_s) = n^2 \cdot 2^n$$

5 **(a)**

We know,

$(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_rx^r + ...$ ...(i)

and $\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} + C_2\frac{1}{x^2} + ... + C_r\frac{1}{x^r}$

$+ C_{r+1}\frac{1}{x^{r+1}} + C_{r+2}\frac{1}{x^{r+2}} ... C_n\frac{1}{x^n}$ ...(ii)

On multiplying Eqs. (i) and (ii), equation coefficient of x^r in $\frac{1}{x^n}(1+x)^{2n}$ or the coefficient of x^{n+r} in $(1+x)^{2n}$, we get the value of required expression which is

$${}^{2n}C_{n+r} = \frac{(2n)!}{(n-r)!(n+r)!}$$

7 **(b)**

In $(x+a)^{100} + (x-a)^{100}$ n is even

$\therefore$ Total number of terms $= \frac{n}{2} + 1 = \frac{100}{2} + 1 = 51$

8 **(b)**

Given polynomial is

$(x-1)(x-2)(x-3)...(x-19)(x-20)$

$= x^{20} - (1+2+3+...+19+20)x^{19}$

$+(1 \times 2 + 2 \times 3 + ... + 19 \times 20)x^{18}$

$- ... + (1 \times 2 \times 3 \times 4 \times ... \times 19 \times 20)$

$\therefore$ Coefficient of $x^{19} = -(1+2+3+...+19+20)$

$= -\left[\frac{20}{2}(1+20)\right]$

$= -10 \times 21 = -210$

9 **(c)**

We know that,

${}^{15}C_0 + {}^{15}C_1 + {}^{15}C_2 + ... + {}^{15}C_{15} = 2^{15}$

$\Rightarrow 2\left({}^{15}C_8 + {}^{15}C_9 + \dots + {}^{15}C_{15}\right)2^{15}$ $\left[\because {}^{n}C_r = {}^{n}C_{n-r}\right]$
$\Rightarrow {}^{15}C_8 + {}^{15}C_9 + \dots + {}^{15}C_{15} = 2^{14}$

10 **(c)**
The number of terms in the expansion of $(a+b+c)^n$
$$= \frac{(n+1)(n+2)}{2}$$

11 **(c)**
We have,
$$T_{r+1} = {}^{5}C_r (y^2)^{5-r}\left(\frac{c}{y}\right)^r = {}^{5}C_r y^{10-3r}\, c^r$$
This will contain y, if $10 - 3r = 1 \Rightarrow r = 3$
$\therefore$ Coefficient of $y = {}^{5}C_3\, c^3 = 10\, c^3$

12 **(b)**
$\because (0.99)^{15} = (1-0.01)^{15}$
$= 1 - {}^{15}C_1(0.01) + {}^{15}C_2(0.01)^2 - {}^{15}C_3(0.01)^3 + \dots$
We want to answer correct upto 4 decimal places and as such, we have left further expansion.
$$= 1 - 15(0.01) + \frac{15 \cdot 14}{1 \cdot 2}(0.0001) - \frac{15 \cdot 14 \cdot 13}{1 \cdot 2 \cdot 3}(0.000001) + \dots$$
$= 1 - 0.15 + 0.0105 - 0.000455 + \dots$
$= 1.0105 - 0.150455$
$= 0.8601$

13 **(b)**
By hypothesis, $2^n = 4096 = 2^{12} \Rightarrow n = 12$
Since, n is even, hence greatest coefficient
$$= {}^{n}C_{n/2} = {}^{12}C_6 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = 924$$

14 **(c)**
Given that, ${}^{n}C_{r-1} = {}^{n}C_{r+1}$
$$\Rightarrow \frac{n!}{(n-r+1)(n-r)(n-r-1)!(r-1)!}$$
$$= \frac{n!}{(n-r-1)!(r+1)(r)(r-1)!}$$
$\Rightarrow r^2 + r = n^2 - nr + n - nr + r^2 - r$
$\Rightarrow n^2 - 2nr - 2r + n = 0$
$$\Rightarrow (n-2r)(n+1) = 0 \Rightarrow r = \frac{n}{2}$$

15 **(d)**

It is given that

${}^nC_1\, x^{n-1}a^1 = 240$ ...(i)

${}^nC_2\, x^{n-2}\, a^2 = 720$ ...(ii)

${}^nC_3\, x^{n-3}\, a^3 = 1080$ ...(iii)

From (i), (ii) and (iii)

$$\frac{\left({}^nC_2\right)^2 x^{2n-4}a^4}{{}^nC_1\,{}^nC_3\, x^{2n-4}\, a^4} = \frac{720 \times 720}{240 \times 1080}$$

$$\Rightarrow \frac{6\, n^2(n-1)^2}{4\, n^2(n-1)(n-2)} = 2$$

$$\Rightarrow \frac{3}{2}\frac{(n-1)}{(n-2)} = 2$$

$\Rightarrow 3n - 3 = 4n - 8 \Rightarrow n = 5$

16 **(d)**

$$\frac{1}{81^n}\left(1 - 10 \cdot {}^{2n}C_1 + 10^2 \cdot {}^{2n}C_2 - 10^3 \cdot {}^{2n}C_3 + \dots + 10^{2n}\right)$$

$$= \frac{1}{(81)^n}(1-10)^{2n} = 1$$

17 **(b)**

We have,

$(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots. + a_{2n}x^{2n}$

On differentiating both sides, we get

$n(1 - 1 + 1)^{n-1}(1 + 2x) = a_1 + 2a_2x + 3a_3x^2$
$+ \dots. + 2na_{2n}x^{2n-1}$

On putting $x = -1$ we get

$n(1 - 1 + 1)^{n-1}(1 - 2) = a_1 - 2a_2 + 3a_3 - \dots - 2na_{2n}$

$\Rightarrow a_1 - 2a_2 + 3a_3 - \dots - 2na_{2n} = -n$

18 **(a)**

Since $(n+1)^{\text{th}}$ term is the middle term in the expansion of $(1+x)^{2n}$

$\therefore$ Coefficient of the middle term

$$= {}^{2n}C_n = \frac{(2n)!}{n!n!}$$

$$= \frac{(1 \cdot 3 \cdot 5 \dots (2n-1)(2 \cdot 4 \cdot 6 \dots (2n-2)(2n))}{n!n!}$$

$$= \frac{1 \cdot 3 \cdot 5 \dots. (2n-1)2^n\, n!}{n!n!} = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)2^n}{n!}$$

19 **(a)**

We have,

$$(1+x)^{10}\left(1+\frac{1}{x}\right)^{12} = \frac{(1+x)^{22}}{x^{12}}$$

$\therefore$ Constant term in $(1+x)^{10}\left(1+\frac{1}{x}\right)^{12}$

= Coefficient of x^{12} in $(1+x)^{22}$

$= {}^{22}C_{12} = {}^{22}C_{10}$

20 **(b)**

Given, $a_n = na_{n-1}$

For $n = 2$

$a_2 = 2a_1 = 2 \quad (\because a_1 = 1 \text{given})$

$a_3 = 3a_2 = 3(2) = 6$

$a_4 = 4(a_3) = 4(6) = 24$

$a_5 = 5(a_4) = 5(24) = 120$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	A	C	A	D	B	B	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	B	C	D	D	B	A	A	B