DPP

DAILY PRACTICE PROBLEMS

Class : XIth | Solutions | Subject : Maths
Date : | | DPP No. : 10

Topic :-Binominal Theorem

1 **(a)**

$$\sum_{k=0}^{10} {}^{20}C_k = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_{10}$$

On putting $x = 1$and $n = 20$in $(1+x)^n$

$$= {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$$

We get

$$2^{20} = 2\left({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_9\right) + {}^{20}C_{10}$$

$$\Rightarrow 2^{19} = \left({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_9\right) + \frac{1}{2}\,{}^{20}C_{10}$$

$$\Rightarrow 2^{19} = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_{10} - \frac{1}{2}\,{}^{20}C_{10}$$

$$\Rightarrow {}^{20}C_0 + {}^{20}C_1 + \dots + {}^{20}C_{10} = 2^{19} + \frac{1}{2}\,{}^{20}C_{10}$$

2 **(b)**

$$(7.995)^{1/3} = (8 - 0.005)^{1/3}$$

$$= (8)^{1/3}\left[1 - \frac{0.005}{8}\right]^{1/3}$$

$$= 2\left[1 - \frac{1}{3} \times \frac{0.005}{8} + \frac{\frac{1}{3}\left(\frac{1}{3} - 1\right)}{2 \cdot 1}\left(\frac{0.005}{8}\right)^2 + \dots\right]$$

$$= 2\left[1 - \frac{0.005}{24} - \frac{\frac{1}{3} \times \frac{1}{3}}{1} \times \frac{(0.005)^2}{64} + \dots\right]$$

$= 2(1 - 0.000208)$ (neglecting other terms)

$= 2 \times 0.999792$

$= 1.9996$

3 **(b)**

It is given that ${}^mC_1, {}^mC_2$ and ${}^mC_3$ are in A.P.

$\therefore 2\ {}^mC_2 = {}^mC_1 + {}^mC_3$

$\Rightarrow m^2 - 9\,m + 14 = 0$

$\Rightarrow m = 2, 7$

For $m = 2$, there are only three terms. Therefore, $m = 7$.

Now,

$\Rightarrow 21 = {}^7C_5\left\{\sqrt{2^{\log_{10}(10-3^x)}}\right\}^{7-5}\left\{\sqrt[5]{2^{(x-2)\log_{10}3}}\right\}^5$

$\Rightarrow 21 = 21 \cdot 2^{\log_{10}(10-3^x)} \cdot 2^{(x-2)\log_{10}3}$

$\Rightarrow 1 - 2^{\log_{10}(10-3^x)+(x-2)\log_{10}3}$

$\Rightarrow 2^0 = 2^{\log_{10}[(10-3^x)\cdot 3^{x-2}]}$

$\Rightarrow (10 - 3^x)3^{x-2} = 1$

$\Rightarrow 3^{2\,x-2} - 10.3^{x-2} + 1 = 0$

$\Rightarrow 3^{2x} - 10.3^x + 9 = 0$

$\Rightarrow (3^x - 1)(3^x - 9) = 0$

$\Rightarrow 3^x = 1, 3^x = 9 \Rightarrow x = 0, 2$

4 **(b)**

Given, ${}^nC_{12} = {}^nC_6$

or ${}^nC_{n-12} = {}^nC_6$

$\Longrightarrow n - 12 = 6 \Longrightarrow n = 18$

$\therefore\ {}^nC_2 = {}^{18}C_2 = 153$

5 **(b)**

Let $(r + 1)$th term be the coefficient of x^0 in the expansion of $\left(x - \frac{1}{x}\right)^6$.

$\therefore T_{r+1} = {}^6C_r x^{6-r}\left(-\frac{1}{x}\right)^r$

$= (-1)^r\,{}^6C_r x^{6-2r}$

Since, this term is a constant term.

$\therefore 6 - 2r = 0 \Longrightarrow r = 3$

$\therefore T_4 = (-1)^3\,{}^6C_3 = -20$

6 **(d)**

General term, $T_{r+1} = {}^{15}C_r(x^3)^{15-r}\left(\frac{2}{x^2}\right)^r$

$= {}^{15}C_r x^{45-5r}(2)^r$

For term independent of x, put $45 - 5r = 0 \Longrightarrow r = 9$

$\therefore$ Independent term $= T_{9+1} = T_{10}$

7 **(b)**

We have, $T_{r+1} = {}^{21}C_r\left(\frac{a^{1/3}}{b^{1/6}}\right)^{21-r}\left(\frac{b^{1/2}}{a^{1/6}}\right)^r$

$= {}^{21}C_r \frac{a^{7-(r/3)}}{b^{7/2-r/6}} \cdot \frac{b^{r/2}}{a^{r/6}}$

$= {}^{21}C_r a^{7-(r/2)} b^{2r/3-7/2}$

Since, exponents of a and b in the $(r+1)$th term are equal

$\therefore\ 7 - \frac{r}{2} = \frac{2r}{3} - \frac{7}{2}$

$\Rightarrow \frac{21}{2} = \frac{7}{6} r \Rightarrow r = 9$

8 **(b)**

$\left(\frac{1}{x}+1\right)^n (1+x)^n = \frac{1}{x^n}(1+x)^{2n}$

$= \frac{1}{x^n}\left(1 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \dots + {}^{2n}C_{n-1} x^{n-1} + \dots + {}^{2n}C_{2n} x^{2n}\right)$

The coefficient of $\frac{1}{x}$ is ${}^{2n}C_{n-1}$.

9 **(d)**

$x = (\sqrt{3}+1)^5 = (\sqrt{3})^5 + {}^5C_1(\sqrt{3})^4 + {}^5C_2(\sqrt{3})^3$

$+ {}^5C_3(\sqrt{3})^2 + {}^5C_4(\sqrt{3}) + {}^5C_5$

$= 9\sqrt{3} + 45 + 30\sqrt{3} + 30 + 5\sqrt{3} + 1$

$= 76 + 44\sqrt{3}$

$\therefore\ [x] = \left[(\sqrt{3}+1)^5\right] = [76 + 44\sqrt{3}]$

$= [76] + [44 \times 1.732]$

$= 76 + [76.2]$

$= 76 + 76 = 152$

10 **(a)**

We have,

$2\ C_0 + \frac{2^2}{2} C_1 + \frac{2^3}{2} C_2 + \dots + \frac{2^{11}}{11} C_{10}$

$= \sum_{r=0}^{10} {}^{10}C_r \frac{2^{r+1}}{r+1}$

$= \frac{1}{11} \sum_{r=0}^{10} \frac{11}{r+1}\ {}^{10}C_r\ 2^{r+1}$

$= \frac{1}{11} \sum_{r=0}^{10} {}^{11}C_{r+1} \cdot 2^{r+1}$

$= \frac{1}{11}\left({}^{11}C_1\ 2^1 + \dots + {}^{11}C_{11} \cdot 2^{11}\right)$

$$= \frac{1}{11}({}^{11}C_0 \cdot 2^0 + {}^{11}C_1 2^1 + \dots + {}^{11}C_{11} \cdot 2^{11} - {}^{11}C_0 \cdot 2^0)$$

$$= \frac{1}{11}[(1+2)^{11} - 1] = \frac{3^{11} - 1}{11}$$

11 **(a)**

Sum of coefficients of the expansion $\left(\frac{1}{x} + 2x\right)^n = 6561$

$\therefore (1+2)^n = 3^8 \Longrightarrow 3^n = 3^8 \Longrightarrow n = 8$

Now, $T_{r+1} = {}^8C_r 2^{8-r} x^{-8+2r}$

Since, this term is independent of x, then

$$-8 + 2r = 0 \Longrightarrow r = 4$$

$\therefore$ Coefficient of independent term, $T_5 = {}^8C_4 \cdot 2^4 = 16 \cdot {}^8C_4$

12 **(d)**

Sum of coefficient of odd powers of x in $(1+x)^{30}$

$= C_1 + C_3 + \dots + C_{29} = 2^{30-1} = 2^{29}$

13 **(b)**

6th term in the expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$ is

$$T_6 = {}^{10}C_5 (2x^2)^5 \left(-\frac{1}{3x^2}\right)^5$$

$$= -\frac{10!}{5!5!} \times 32 \times \frac{1}{243}$$

$$= -\frac{896}{27}$$

14 **(c)**

$${}^{47}C_4 \sum_{r=1}^{5} {}^{52-r}C_3$$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$

$= {}^{51}C_3 + {}^{51}C_4 + {}^{52}C_4$

15 **(b)**

We have,

$(1 - 2x + 3x^2 - 4x^3 + \dots)^{-n} = \{(1+x)^{-2}\}^{-n} = (1+x)^{2n}$

$\therefore$ Coefficient of x^n in $(1 - 2x + 3x^2 - 4x^3 + \dots)^{-n}$

$= $ Coefficient of x^n in $(1+x)^{2n} = {}^{2n}C_n = \frac{(2n)!}{(n!)^2}$

16 **(c)**

We have,

$(1+x)^n\left(1+\frac{1}{x}\right)^n$

$= (C_0 + C_1x + \dots + C_nx^n)\left\{C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \dots + \frac{C_n}{x^n}\right\}$

$\therefore$ Term independent of $x = C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2$

17 **(a)**

We have,

$A =$ Coeff. of x^r in the expansion of $(1+x)^n\ {}^nC_r$

$B =$ Coeff. of x^{n-r} in the expansion of $(1+x)^n = {}^nC_{n-r}$

$\because {}^nC_r = {}^nC_{n-r} \quad \therefore A = B$

18 **(b)**

We have,

$$x = \frac{\begin{bmatrix} 729 + 6(2)(243) + 15(4)(81) \\ +20(8)(27) + 15(16)(9) + 6(32)(3) + 64 \end{bmatrix}}{1 + 4(4)6(16) + 4(64) + 256}$$

$$= \frac{\begin{bmatrix} {}^6C_0(3)^6 + {}^6C_1 3^5 2 + {}^6C_2 3^4 2^2 \\ + {}^6C_3 3^3\, 2^3 + {}^6C_4 3^2 2^4 + {}^6C_5 3\, 2^5 + {}^6C_6 2^6 \end{bmatrix}}{{}^4C_0 + {}^4C_1 4 + {}^4C_2 4^2 + {}^4C_3 4^3 + {}^4C_4 4^4}$$

$\Rightarrow x = \frac{(3+2)^6}{(1+4)^4} = \frac{5^6}{5^4}$

$\Rightarrow x = 5^2$

$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = 5 - \frac{1}{5} = 4.8$

19 **(b)**

Coefficient of pth, $(p+1)$th and $(p+2)$th terms in the expansion $(1+x)^n$ are ${}^nC_{p-1}$, ${}^nC_p$, ${}^nC_{p+1}$ respectively

Since, these are in AP

$\therefore\ 2\,{}^nC_p = {}^nC_{p-1} + {}^nC_{p+1}$

$\Rightarrow 2\frac{n!}{(n-p)!p!} = \frac{n!}{(n-p+1)!(p-1)!} + \frac{n!}{(n-p-1)!(p+1)!}$

$\Rightarrow \frac{2}{(n-p)!p!} = \frac{p}{(n-p+1)(n-p)!p!} + \frac{n-p}{(n-p)!(p+1)p!}$

$\Rightarrow \frac{2}{1} = \frac{p}{(n-p+1)} + \frac{n-p}{p+1}$

$\Rightarrow n^2 - n(4p+1) + 4p^2 - 2 = 0$

20 **(a)**

$(2^{1/2} + 3^{1/5})^{10} = {}^{10}C_0 2^5 + {}^{10}C_1 2^{9/2} \cdot 3^{1/5} + \dots\dots + {}^{10}C_{10} \cdot 3^2$

Thus, sum of rational terms of above expansion $= 2^5 + 3^2 = 41$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	D	B	C	A	C	D	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	C	A	D	C	C	D	A	C