DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : Maths
DPP No. : 1

Topic :-Binominal Theorem

1 **(b)**

We have, $(1+x)^{15}=C_0+C_1x+C_2x^2+...+C_{15}x^{15}$

$\Rightarrow \frac{(1+x)^{15}-1}{x}=C_1+C_2x+...C_{15}x^{14}$

On differentiating both sides w.r.t. x, we get

$\frac{x\cdot 15(1+x)^{14}-(1+x)^{15}+1}{x^2}=C_2+2C_3x+...+14C_{15}x^{13}$

On putting $x=1$, we get

$C_2+2C_3+...+14C_{15}=15.2^{14}-2^{15}+1$

$=13.2^{14}+1$

2 **(b)**

It is given that

${}^{2n}C_1, {}^{2n}C_2$ and ${}^{2n}C_3$ are A.P.

$\therefore 2\cdot {}^{2n}C_2={}^{2n}C_1+{}^{2n}C_3$

$2\cdot\frac{(2\,n)\,!}{(2\,n-2)\,!2\,!}=\frac{(2\,n)!}{(2\,n-1)!}+\frac{(2\,n)!}{(2\,n-3\,!3\,!)}$

$\Rightarrow 2\,\frac{(2\,n)(2\,n-1)}{2}=2\,n+\frac{(2\,n)(2\,n-1)(2\,n-2)}{3\,!}$

$\Rightarrow 6(2\,n-1)=6+(2\,n-1)(2\,n-2)$

$\Rightarrow 12\,n-6=6+4\,n^2-6\,n+2$

$\Rightarrow 4\,n^2-18\,n+14=0 \Rightarrow 2\,n^2-9\,n+7=0$

3 **(d)**

$\frac{1+2x}{(1-2x)^2}=(1+2x)(1-2x)^{-2}$

$=(1+2x)\left(1+\frac{2}{1!}(2x)+\frac{2\cdot 3}{2!}(2x)^2+...+\frac{2\cdot 3...r}{(r-1)!}(2x)^{r-1}+\frac{2\cdot 3\cdot 4...(r+1)(2x)^r}{r!}\right)$

The coefficient of $x^r=2\frac{r!}{(r-1)!}2^{r-1}+\frac{(r-1)!}{r!}2^r$

$=r2^r+(r+1)2^r=2^r(2r+1)$

4 **(c)**

Given, $A = {}^{30}C_0 \cdot {}^{30}C_{10} - {}^{30}C_1 \cdot {}^{30}C_{11} + {}^{30}C_2 \cdot {}^{30}C_{12} + \ldots + {}^{30}C_{20} \cdot {}^{30}C_{30}$

$= $ coefficient of x^{20} in $(1+x)^{30}(1-x)^{30}$

$= $ coefficient of x^{20} in $(1+x^2)^{30}$

$= $ coefficient of x^{20} in $\sum_{r=0}^{30} (-1)^r\ {}^{30}C_r(x^2)^r$

$= (-1)^{10} \cdot {}^{30}C_{10}$ {for coefficient of x^{20}, let $r = 10$}

$= {}^{30}C_{10}$

5 **(a)**

We have,

$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \ldots + a_{2n} x^{2n} = (1 - x + x^2)^n$

Putting $x = 1$ and -1, we get

$(a_0 + a_2 + a_4 + \ldots) + (a_1 + a_3 + a_5 + \ldots) = 1$ ...(i)

And,

$(a_0 + a_2 + a_4 + \ldots) - (a_1 + a_3 + a_5 \ldots) = 3^n$ ...(ii)

Adding (i) and (ii), we get

$$a_0 + a_2 + a_4 + \ldots = \frac{3^n + 1}{2}$$

6 **(a)**

We know that ,

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$

On integrating both sides, from 0 to 1, we get

$$\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1 = \left[C_0 x + \frac{C_1 x^2}{2} + \frac{C_2 x^3}{3} + \ldots + \frac{C_n x^{n+1}}{n+1}\right]_0^1$$

$$\Rightarrow \frac{2^{n+1} - 1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1}$$

7 **(a)**

7^{th} term from the beginning in the expansion of $\left(2^{1/3} + \frac{1}{3^{1/3}}\right)^x$ is given by

$$T_7 = {}^xC_6 (2^{1/3})^{x-6} \left(\frac{1}{3^{1/3}}\right)^6$$

7^{th} term from the end in the expansion of $\left(2^{1/3} + \frac{1}{3^{1/3}}\right)^x$ is the $(x + 1 - 7 + 1)^{\text{th}} = (x-5)^{\text{th}}$ term from the beginning. Therefore,

$$T_{x-5} = {}^xC_{x-6} (2^{1/3})^6 \left(\frac{1}{3^{1/3}}\right)^{x-6}$$

We have,

$$\frac{T_7}{T_{x-5}} = \frac{1}{6}$$

$\Rightarrow 6\ T_7 = T_{x-5}$

$\Rightarrow 6\ {}^xC_6\ 2^{\frac{x-6}{3}} 3^{-2} = {}^xC_{x-6}\ 2^2 3^{-\left(\frac{x-6}{3}\right)}$

$\Rightarrow 2^{\frac{x-9}{3}} = 3^{-\left(\frac{x-9}{3}\right)}$

$\Rightarrow 6^{\frac{x-9}{3}} = 1 \Rightarrow x-9=0 \Rightarrow x=9$

9 **(b)**

$\because\ T_{r+1} = {}^{10}C_r(x\sin^{-1}\alpha)^{10-r}\left(\frac{\cos^{-1}\alpha}{x}\right)^r$

$= {}^{10}C_r(\sin^{-1}\alpha)^{10-r}(\cos^{-1}\alpha)^r x^{10-2r}$

$\therefore$ For the term independent of x,

$10-2r=0 \Rightarrow r=5$

$T_{5+1} = {}^{10}C_5(\sin^{-1}\alpha)^5(\cos^{-1}\alpha)^5$

$= {}^{10}C_5(\sin^{-1}\alpha\cos^{-1}\alpha)^5$

Let $f(\alpha) = \sin^{-1}\alpha . \cos^{-1}\alpha$

$= \sin^{-1}\alpha\left(\frac{\pi}{2} - \sin^{-1}\alpha\right)$

Put $\sin^{-1}\alpha = t$

$\therefore f(\alpha) = t\left(\frac{\pi}{2} - t\right)$

$= -\left\{t^2 - \frac{\pi}{2}t\right\}$

$= -\left\{\left(t - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}\right\}$

$= \frac{\pi^2}{16} - \left(t - \frac{\pi}{4}\right)^2$

$\therefore\ f(a) = \frac{\pi^2}{16} - \left(\sin^{-1}\alpha - \frac{\pi}{4}\right)^2$

Maximum value of $f(\alpha)$ is $\frac{\pi^2}{16}$, when $\sin^{-1}\alpha = \frac{\pi}{4}$

Also, $-1 \le \alpha \le 1$

$\therefore\ -\frac{\pi}{2} \le \sin^{-1}\alpha \le \frac{\pi}{2}$

Minimum value $f(\alpha) = \frac{\pi^2}{16} - \left(-\frac{\pi}{2} - \frac{\pi}{4}\right)^2 = -\frac{\pi^2}{2}$

$\therefore$ Range is $\left[{}^{10}C_5\left(-\frac{\pi^2}{2}\right)^5,\ {}^{10}C_5\left(\frac{\pi^2}{16}\right)^5\right]$

$ie, \left[-\frac{{}^{10}C_5\pi^{10}}{2^5},\ \frac{{}^{10}C_5\pi^{10}}{2^{20}}\right]$

10 **(c)**

Let $A = \binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11}x$

$+ \binom{30}{2}\binom{30}{12} - \ldots + \binom{30}{20}\binom{30}{30}$

or $A = {}^{30}C_0 . {}^{30}C_{10} - {}^{30}C_1 . {}^{30}C_{11}$

$+ {}^{30}C_2 . {}^{30}C_{12} - \ldots + {}^{30}C_{20} . {}^{30}C_{30}$

$= \text{coefficient of } x^{20} \text{ in } (1+x)^{30}(1-x)^{30}$
$= \text{coefficient of } x^{20} \text{ in } (1-x^2)^{30}$
$$= \text{coefficient of } x^{20} \text{ in } \sum_{r=0}^{30} (-1)^r\, {}^{30}C_r (x^2)^r$$
$= (-1)^{10}\, {}^{30}C_{10}$ (for coefficient of x^{20}, let $r = 10$)
$= {}^{30}C_{10}$

11 **(b)**

The rth term of $(a+2n)^n$ is ${}^nC_{r-1}(a)^{n-r+1}(2x)^{r-1}$
$$= \frac{n!}{(n-r+1)!(r-1)!} a^{n-r+1}(2x)^{r-1}$$
$$= \frac{n(n-1)\dots(n-r+2)}{(r-1)!} a^{n-r+1}(2x)^{r-1}$$

12 **(a)**

We have, $(1+t^2)^{12}(1+t^{12})(1+t^{24})$
$= (1 + {}^{12}C_1 t^2 + {}^{12}C_2 t^4 + \dots + {}^{12}C_6 t^{12} + \dots + {}^{12}C_{12} t^{24} + \dots)(1 + t^{12} + t^{24} + t^{36})$
$\therefore$ Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$
$= {}^{12}C_6 + {}^{12}C_{12} + 1 = {}^{12}C_6 + 2$

13 **(c)**

We have,
$$\frac{(1+x)^2}{(1-x)^3} = (x^{2v} + 2x + 1)(1-x)^{-3}$$
$$\Rightarrow \frac{(1+x)^2}{(1+x)^3} = x^2(1-x)^{-3} + 2x(1-x)^{-3} + (1-x)^{-3}$$
$\therefore$ Coeff. of x^n in $\frac{(1+x)^2}{(1-x)^3}$
$= \text{Coeff. of } x^n \text{ in } x^2(1-x)^{-3} + \text{Coeff. of } x^n \text{ in } 2x(1-x)^{-3} + \text{Coeff. of } x^n \text{ in } (1-x)^{-3}$
$= \text{Coeff. of } x^{n-2} \text{ in } (1-x)^{-3} + 2 \,.\, \text{Coeff. of } x^{n-1} \text{ in } (1-x)^{-3} + \text{Coeff. of } x^n \text{ in } (1-x)^{-3}$
$= {}^{n-2+3-1}C_{3-1} + 2 \cdot {}^{n-1+3-1}C_{3-1} + {}^{n+3-1}C_{3-1}$
$= {}^nC_2 + 2 \cdot {}^{n+1}C_2 + {}^{n+2}C_2$
$$= \frac{n(n-1)}{2} + 2\frac{(n+1)\,n}{2} + (n+2)\frac{(n+1)}{2}$$
$$= \frac{1}{2}(n^2 - n + 2^2 + 2n + n^2 + 3n + 2) = 2n^2 + 2n + 1$$

14 **(c)**

On substituting $x = 1$ in $(1 + x - 3x^2)^{3148}$, then sum of coefficient
$= (1 + 1 - 3)^{3148} = (-1)^{3148} = 1$

15 **(c)**

$aC_0 - (a+d)C_1 + (a+2d)C_2 - (a+3d)C_3 + \dots + (-1)^n(a+nd)C_n$
$$= \sum_{r=0}^{n} (a+rd)(-1)^r\, {}^nC_r$$

$$= a\sum_{r=0}^{n} (-1)^r\ {}^nC_r - dn\sum_{r=1}^{n-1} {}^{n-1}C_{r-1}\,(-1)^{r-1}$$

$$= a \times 0 - dn \times 0 = 0$$

16 **(b)**

We have,

$$\frac{18^3 + 7^3 + 3\cdot 18\cdot 7\cdot 25}{3^6 + 6\cdot 243\cdot 2 + 15\cdot 81\cdot 4 + 20\cdot 27\cdot 8 + 15\cdot 9\cdot 16 + 6\cdot 3\cdot 32 + 64}$$

$$= \frac{18^3 + 7^3 + 3\cdot 18\cdot 7\cdot (18+7)}{{}^6C_0\, 3^6 + {}^6C_1\, 3^5\cdot 2^1 + {}^6C_2\, 3^4\cdot 2^2 + {}^6C_3\, 3^3\, 2^3 + {}^6C_4\, 3^2\, 2^4 + {}^6C_5\, 3^1 2^5 + {}^6C_6\, 3^0\, 2^6}$$

$$= \frac{(18+7)^3}{(3+2)^6} = \frac{5^6}{5^6} = 1$$

17 **(b)**

We have,

$$(1+x^2)^5(1+x)^4$$

$$= ({}^5C_0 + {}^5C_1\, x^2 + {}^5C_2\, x^4 + \ldots)$$

$$\times ({}^4C_0 + {}^4C_1\, x + {}^4C_2\, x^2 + {}^4C_3\, x^3 + {}^4C_4\, x^4)$$

$\therefore$ Coefficient of x^5 in $\{(1+x^2)^5(1+x)^4\}$

$$= {}^5C_2 \times {}^5C_1 + {}^4C_3 \times {}^5C_1 = 60$$

18 **(d)**

$$({}^nC_0)^2 + ({}^nC_1)^2 + ({}^nC_2)^2 + \ldots + ({}^nC_5)^2$$

$$= ({}^5C_0)^2 + ({}^5C_1)^2 + ({}^5C_2)^2 + ({}^5C_3)^2 + ({}^5C_4)^2 + ({}^5C_5)^2$$

$$= 1 + 25 + 100 + 100 + 25 + 1 = 252$$

19 **(c)**

Let

$$S = (1+x)^{1000} + 2\,x(1+x)^{999} + 3\,x^2(1+x)^{998} + \ldots + 1000\,x^{999}(1+x) + 1001\,x^{1000} \quad \ldots($$

$$\therefore \frac{x}{1+x}S = x(1+x)^{999} + 2\,x^2(1+x)^{998} + \ldots + 1000\,x^{1000} + \frac{1001\,x^{1001}}{1+x} \ldots\text{(ii)}$$

Subtracting (ii) from (i), we get

$$\left(1 - \frac{x}{x+1}\right)S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots + x^{1000} - \frac{1001\,x^{1001}}{1+x}$$

$$\Rightarrow S = (1+x)^{1001} + x\,(1+x)^{1000} + x^2(1+x)^{999} + \ldots + x^{1000}(1+x) - 1001\,x^{1001}$$

$$\Rightarrow S = (1+x)^{1001}\frac{\left\{1 - \left(\frac{x}{1+x}\right)^{1001}\right\}}{\left\{1 - \frac{x}{1+x}\right\}} - 1001\,x^{1001}$$

$$\Rightarrow S = (1+x)^{1002}\left\{1 - \left(\frac{x}{1+x}\right)^{1001}\right\} - 1001\,x^{1001}$$

$$\Rightarrow S = (1+x)^{1002} - x^{1001}(1+x) - 1001\,x^{1001}$$

$\therefore$ Coefficient of x^{50} in S is ${}^{1002}C_{50}$

20 **(d)**

$$\frac{1}{(x-1)^2(x-2)} = \frac{1}{-2(1-x)^2\left(1-\frac{x}{2}\right)}$$

$$= -\frac{1}{2}\left[(1-x)^{-2}\left(1-\frac{x}{2}\right)^{-1}\right]$$

$$= -\frac{1}{2}\left[(1+2x+\ldots)\left(1+\frac{x}{2}+\ldots\right)\right]$$

$\therefore$ Coefficient of constant term is $-\frac{1}{2}$.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	D	C	A	A	A	C	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	C	C	C	B	B	D	C	D